ANALOG AND DIGITAL COMMUNICATION

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Preface

Analog and digital communication is an essential subject in the field of engineering stream. This book is written according to the syllabus of analog and digital communication of AKTU. The book is written in simple language to explain the fundamental concepts of communication.

The book is divided into small chapters and arranged sequentially to ensure the smooth flow of the subject. An attempt is made to explain the topics with suitable numerical examples as far as possible. In the last section of the chapters, a large number of solved numerical examples are covered. Multiple choice questions along with theoretical and numerical problems are included as exercise practice at the end of every chapter.

This book aims to provide basic concepts of communication to the students, and therefore, I hope this book will benefit both teachers and students.

ACKNOWLEDGEMENT

Book writing was my great desire, but I would never imagine writing this book without my best friend and better half Ms. Richa Gupta. She sacrifices her time and continuously encourages me to move towards a new horizon. I am eternally grateful to my mother Ms. Mangla Devi who taught me the lesson of discipline without which no one can imagine achieving anything in life.

I am very thankful to my elder brothers Mr. Alok Gupta and Mr. Rajendra Gupta who provided me with a roof over my head and became the father figure I desperately needed. Moreover, the love and respect of my younger brothers Mr. Abinash Gupta (Deputy Manager, NHPC) and Mr. Vikas Gupta (Geophysicist, ONGC) are motivations for me to complete this book-writing journey.

A very special thanks to my Ph.D. supervisors Prof. Paulson Samuel (MNNIT Allahabad) and Dr. Deepak Kumar (MNNIT Allahabad), who delighted me with their light of knowledge. I am also grateful to Prof. Raghuvir Kumar (Director General, BNCET), Prof. Ashutosh Dwivedi (Director, BNCET), Dr. Abhishek Mishra (HOD-EC, BNCET), who provided me a platform to complete my desire.

Further, I am very thankful to my friends and colleagues, Dr. Akbar Ahmad (Dean, MI College, Maldives), Mr. C. N. Singh (Associate Prof., HBTU Kanpur), Mr. Prabhat Shukla (Asst. Professor, UCER Allahabad), Dr. S. K. Kushwaha (Asst. Professor, GHRIET Nagpur), Dr. B. M. Reddy, Dr. Praveen Mathur, Mr. S. K. Singh and Mr. Jitendra Srivastava.

I am thankful to my publisher International Institute for Academic Research and Development (IIFARD) for publishing my book in very short span of time.

In last but not least, I am thankful to all those who have been a part of my journey of book-writing.

DEDICATED TO

My Father

(Late) Hirday Narayan Gupta

Syllabus

UNIT-1: **(Chapters: 1, 2, 3)**

Elements of communication system and its limitations, Amplitude modulation and detection, Generation and detection of DSB-SC, SSB and vestigial side band modulation, carrier acquisition AM transmitters and receivers, Superhetrodyne Receiver, IF amplifiers, AGC circuits, Frequency Division multiplexing

UNIT-2: **(Chapters: 4, 5)**

Angle Modulation: Basic definition, Narrow-Band and wideband frequency modulation, transmission bandwidth of FM signals, Generation and detection of frequency modulation, Generation and detection of Phase Modulation. Noise: External noise, internal noise, noise calculations, signal to noise ratio.

UNIT-3: **(Chapters: 6, 7)**

Pulse Modulation: Introduction, sampling process, Analog Pulse Modulation Systems, Pulse Amplitude Modulation (PAM), Pulse width modulation (PWM) and Pulse Position Modulation (PPM). Waveform coding Techniques: Discretization in time and amplitude, Quantization process, quantization noise, Pulse code Modulation, Differential Pulse code Modulation, Delta Modulation and Adaptive Delta Modulation.

UNIT-4: **(Chapters: 8)**

Digital Modulation Techniques: Types of digital modulation, waveforms for amplitude, frequency and phase shift keying, coherent and non-coherent methods for the generation of ASK, FSK and PSK. Comparisons of above digital modulation techniques.

UNIT-5: **(Chapters: 9)**

Time Division Multiplexing: Fundamentals, Electronic Commutator, Bit/byte interleaving, T1 carrier system, synchronization and signaling of T1, TDM and PCM hierarchy, synchronization techniques. Introduction to Information Theory: Measure of information, Entropy & Information rate, channel capacity, Hartley Shannon law, Huffman coding, Shannon Fano coding.

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Definition

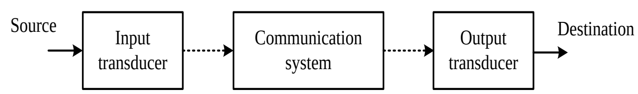
A communication system is designed for the transmission of a specific type of signal from transmitter end to receiver end.

CHAPTER 1

INTRODUCTION TO COMMUNICATION SYSTEMS

Highlights

REPRESENTATION



1.1 Introduction

The message (analog or digital) transmission from source to destination is possible through the communication system, as shown in Fig. 1.1. The input transducer converts the message signal from the source into an electrical signal whereas output transducers convert the electrical signal into the desired physical quantity. If we consider a voice communication system, the microphone (input transducer) is used at the input side and the loudspeaker (output transducer) is used at the output side. Here, the primary concern is signal transmission through the communication system. In this chapter, analog and digital communication systems with a brief introduction of their properties are presented.

Fig. 1.1 Block diagram of communication system

1.2 Communication System

A communication system is designed to transmit a specific type of signal from transmitter end to receiver end. Since the two important distinct categories of the signal are analog and digital signals, therefore, according to the nature of the signal, the communication system is divided as:

1. Analog communication system
2. Digital communication system

Usually, any communication system consists of three essential elements 1) A transmitter, 2) Transmission or communication channel and 3) A receiver.

For the communication system, the terms '**signal**' and '**message**' are interchangeable. The message signal is also termed the baseband signal.

1.3 Elements of a Communication System

The block diagram of a communication system is shown in Fig. 1.2. If we consider the voice communication system, the input and output transducers are a microphone and a loudspeaker, respectively. The functions of each element are as follows:

(a) Source: The message signal is originated by the source. In voice communication, the source is a human voice. The input transducer is used to convert this signal into an electrical signal.

(b) Transmitter: The transmitter modifies the input signal characteristics suited for efficient transmission through the transmission channel. The signal shaping (encoding) and the modulation operations (AM, FM or PM) are performed at the transmitter end.

(c) **Channel:** The modulated signal is transmitted through a transmission medium (communication channel) which works as a bridge between the transmitter and the receiver. The transmission medium may be an optical fiber, waveguide or co-axial etc. the transmission medium introduces various impairments such as noise, distortion or attenuation in the modulated signal, which are further explained in this chapter.

(d) **Receiver:** The inverse function of the transmitter is performed by the receiver. All the modifications such as modulation and encoding done at the transmitter end are undone at the receiver end. Amplification and filtering are the primary operations of the receiver to compensate for the transmission loss and distortion. Further, the obtained electrical signal is sent to the output transducer to convert this electrical signal into the original message.

(e) **Destination:** The message is communicated to a specific unit which is called a destination in the field of communication.

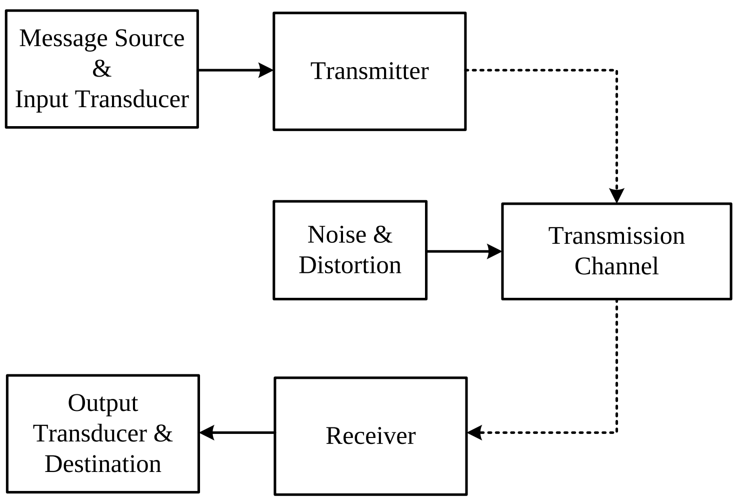


Fig. 1.2 Elements of a communication system

1.3.1 Limitations of Communication Systems

Typically, two types of limitations are faced by an engineer while designing a communication system

1. **Technological Problems:** It includes hardware availability, government regulations, economic factors etc. Theoretically, such types of problems can be solved, but perfect solutions are not possible.
2. **Fundamental Physical Limitations:** It consists of the following two types of constraints

   (a) ***Bandwidth Limitation***

   The bandwidth is a measure of speed for both signals and systems. If a signal is changing rapidly over time, it has large bandwidth. In addition, the ability of the system to track the signal variations is called transmission bandwidth. Practically, every communication system has a fixed bandwidth that limits the degree of signal variability.

(b) ***Noise Limitation***

Another limitation of the communication system is noise. The thermal noise is unwanted voltage that is generated due to the thermally initiated random motions of electrons. Usually, noise is measured relative to signal power in terms of signal to noise ratio (SNR). If the signal power is much more significant, the noise is unnoticeable. But, the signal power comes close to noise power due to transmission loss in long-distance communication. Therefore, noise lowers fidelity in analog communication and yields errors in digital communication.

1.4 Digital Communication System

Most of the modern communication system is digital. In a digital communication system, the analog signal is converted into an equivalent digital signal, and further digitally modulated signal is transmitted over the communication channel to the receiver. The basic elements of a digital communication system are shown in Fig. 1.3.

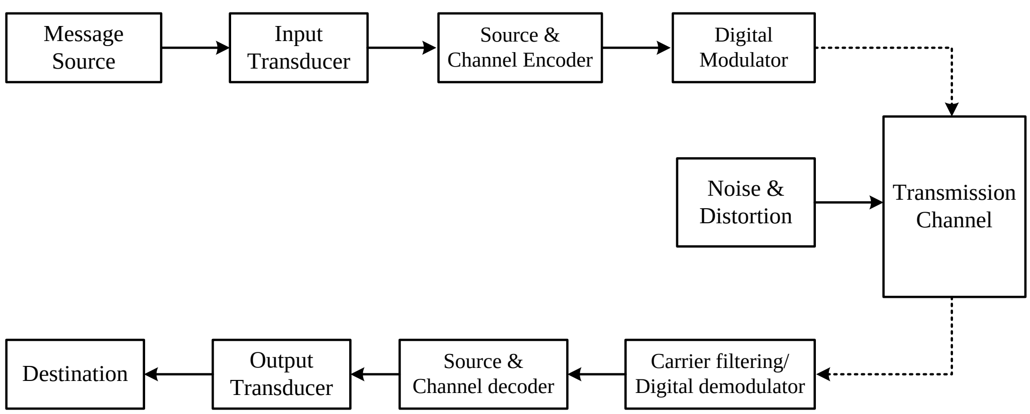


Fig. 1.3 Basic elements of digital communication system

The functions of each element of the digital communic ation system are as follows:

(i) **Source and Input Transducer:** The source generates analog signal. In voice communication, the audio signal is generated by the source and further this signal is converted into an electrical signal by an input transducer.

(ii) **Source Encoder:** The output of the input transducer is encoded into the minimum number of bits by the source encoder. This process removes the unnecessary, redundant bits and supports effective utilization of the bandwidth.

(iii) Channel Encoder: The message bits stream may be altered by the noise during the transmission. Therefore, the channel encoder adds some redundant bits to the transmitted bit stream to make an error-free transmission.

(iv) Digital Modulator: In a digital modulator, the transmitted signal or message bit stream is modulated and converted to analog from its digital sequence to make it travel through the channel or medium.

(v) Channel: The channel allows the modulated message for the transmission and works as a bridge between transmitter and receiver.

(vi) Digital Demodulator: At the receiver end, initially, the signal is demodulated by the digital demodulator and reconstructed into a digital bit stream from an analog signal.

(vii) Channel Decoder: The signal may be distorted during the transmission; therefore, some redundant bits are added as error correction bits for the complete recovery of the original message by the channel decoder.

(viii) Source Decoder: Original digital signal is recreated by the source decoder by sampling and quantization.

(ix) Output Transducer and Destination: The last block of the receiver is the output transducer which converts the electrical signal into physical quantity for the destination.

1.4.1 Advantages of Digital Communication

1. Digital signals are less affected by noise and distortion in comparison with analog signals.
2. Digital circuits are less expensive, easy to design and more reliable.
3. Error detection and correction is possible in digital communication due to channel encoding.
4. The bit error rate can be reduced by using error detecting and correcting codes.
5. A device can be used for a number of the process due to common encoding technique.
6. The spread spectrum technique is used to avoid signal jamming in digital communication.
7. The signal is unaltered as the pulse needs a high disturbance to alter its properties, which is very difficult.
8. Encryption and compression are possible in digital communication to keep the confidentiality of the information.
9. The probability of error occurrence is reduced by employing error detecting and error-correcting codes.
10. Digital signals can be retrieved more appropriately than analog signals.

1.4.2 Limitations of Digital Communication

1. High power consumption.
2. Need of synchronization.
3. It required more transmission bandwidth.
4. The occurrence of sampling error is possible.
5. It requires A/D conversion at a high rate.

1.5 Undesirable Effects During Transmission

Various undesirable effects occur during the transmission of the signal, which may attenuate or distort the signal. Attenuation decreases the strength of the signal but is not a severe problem because the use of an amplifier may increase the strength of the signal again. The more serious problems are distortion, interference and noise, which may alter the shape of the signal.

(a) Distortion: Imperfect response of the system is the cause of distortion of the signal. It disappears when the signal is turned off and may be corrected with the use of a special type of filter called equalizers if the channel is linear.

(b) Interference: It is contamination by unnecessary signals from other signal sources like power lines, other transmitters etc. Generally, interference occurs in radio systems where receiving antenna receives several signals at the same time.

(c) Noise: Undesired and random signals generated internally or externally to the system is termed noise. When noise is superimposed with the message signal, it may corrupt the message signal. Therefore, noise creates a limitation for the communication systems.

1.6 Concept of Bandwidth

Generally, the bandwidth is described as a measure of speed for signals and systems both, but it is defined as the amount of information received per second. In a communication system, bandwidth is defined as a difference between maximum and minimum frequencies in a continuous band range of frequencies. The spectrum of a multi-tone signal is shown in Fig. 1.4.

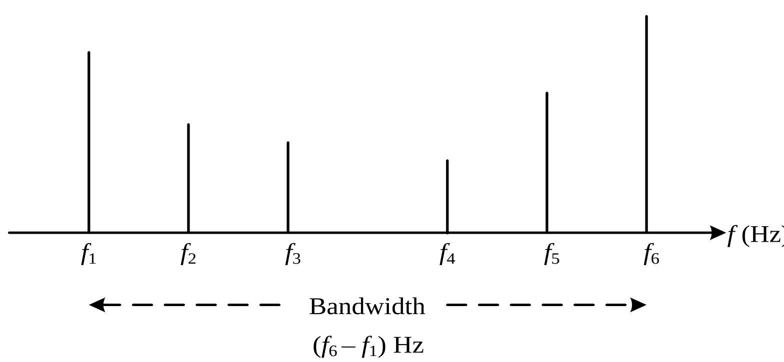


Fig. 1.4 Spectrum of the multi-tone signal

In the context of communication, bandwidth is referred to as baseband bandwidth and passband bandwidth. Baseband bandwidth is applied in the context of baseband signal or low pass filter where the upper cut off frequency is considered as baseband bandwidth. On the other hand, passband bandwidth is used for bandpass filter and defined as the difference between upper and lower cutoff frequencies (Fig. 1.5(a)-(b)).

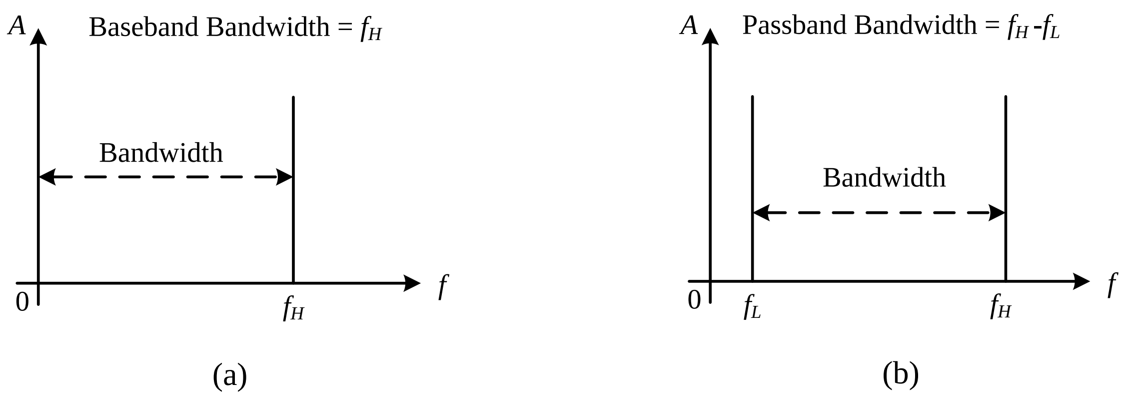


Fig. 1.5 (a) Baseband bandwidth (b) Passband bandwidth

1.7 Concept of Signal to Noise Ratio (*SNR* or *S/N*)

The ratio of signal power to noise power is defined as the signal-to-noise ratio (*SNR* or *S/N*). It is expressed in decibels (dB) and it represents a degree of signal power level to the noise power level. The value of SNR greater than 1 (or > 0dB) indicates more signal power level in comparison with the noise power level. The *S/N* is expressed as

$$S/N = \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{P_s}{P_n} \tag{1.1}$$

where, P_s and P_n are signal and noise average powers measured within the same system bandwidth and at the same equivalent points.

Calculation of SNR

(i) For power,

$$(S/N)_{dB} = 10\log_{10}\left(\frac{P_s}{P_n}\right) \tag{1.2}$$

For voltage,

$$(S/N)_{dB} = 20\log_{10}\left(\frac{V_s}{V_n}\right). \tag{1.3}$$

(ii) If the measured signal and noise powers are already in decibels form, then subtract signal power to noise power.

$$(S/N)_{dB} = (P_s)_{dB} - (P_n)_{dB} \tag{1.4}$$

ADDITIONAL SOLVED EXAMPLES

SE1.1 A signal with power 5 mW is distorted by noise with power 0.002 mW. What is the signal to noise ratio (SNR) in dB?

Sol: The signal to noise ratio is given as

$$(S/N)_{\text{dB}} = 10\log_{10}\left(\frac{P_s}{P_n}\right)$$

$$= 10\log_{10}\left(\frac{5\times10^{-3}}{0.002\times10^{-3}}\right) = 33.98 \text{ dB}$$

SE1.2 At the transmitter, the signal power is 30 mW. The input SNR is 40 dB. The channel offers 3 dB attenuation to the signal and the output noise is thrice the input noise level. Determine the SNR at the output.

Sol: The input signal to noise power is given as

$$(S_i/N_i)_{\text{dB}} = 10\log_{10}\left(\frac{P_s}{P_n}\right)_{\text{input}}$$

$$40 = 10\log\left(\frac{30\times10^{-3}}{P_n}\right)_{\text{input}}$$

$$(P_n)_{\text{input}} = \frac{30\times10^{-3}}{10000} = 3\ \mu\text{W}$$

At the output, there is 3 dB attenuation in signal power, i.e.

$$(S/N)_{\text{dB}} = 10\log_{10}(x)$$

$$-3 = 10\log_{10}(x) \Rightarrow x = 0.5$$

So, signal power becomes half. therefore, outut signal power becomes

$$(P_s)_{\text{output}} = 30/2 = 15 \text{ mW}$$

Since, output noise is thrice the input noise level, So, $(P_n)_{\text{output}} = 3\times3 = 9\ \mu\text{W}$

Therefore, the output signal to noise ratio is

$$(S_o/N_o)_{\text{dB}} = 10\log_{10}\left(\frac{(P_s)_{\text{output}}}{(P_n)_{\text{output}}}\right) = 10\log\left(\frac{15\times10^{-3}}{9\times10^{-6}}\right)$$

$$= 10\times3.222 = 32.22 \text{ dB}$$

SE1.3 A given signal has frequencies of 6125 MHz, 610 MHz, 2250 MHz and 7250 MHz. Determine the bandwidth of the signal?

Sol: The highest and lowest frequency components of the signal is 7250 MHz and 610 MHz, respectively. Therefore, the bandwidth of the signal is

$$\text{BW} = 7250 - 610 = 6640 \text{ MHz}$$

SE1.4 The available bandwidth for FM radio is 87.5 MHz - 108 MHz. When each channel is allocated a bandwidth of 200 kHz, what is the maximum number of channels that are available?

Sol: Available bandwidth of the FM radio is

$$(\text{BW})_{\text{FM}} = 108 - 87.5 = 20.5 \text{ MHz}$$

Therefore, the number of channels is

$$N = \frac{20.5 \times 10^6}{200 \times 10^3} = 102.5 \approx 102 \text{ channels}$$

SE1.5 A cable must carry 25 channels, each with a bandwidth of 10 kHz. What is the minimum bandwidth of the cable?

Sol: The bandwidth of the cable is

$$(\text{BW})_{\text{cable}} = 25 \times 10 = 250 \text{ kHz}$$

SE1.6 An amplifier has input and output signals of 40 mW and 10 W, respectively. Calculate the power gain in dB.

Sol: The power gain is defined as

$$G = \frac{\text{Output Power}}{\text{Input Power}}$$

Further, power gain in dB is given as

$$(G)_{dB} = 10\log(G)$$

So,

$$(G)_{dB} = 10\log\left(\frac{10}{40 \times 10^{-3}}\right) = 10\log 250 = 23.98 \text{ dB}$$

SE1.7 The signal and noise amplitudes in a transmission link are estimated as 4 V and 1 mV, respectively. Estimate the signal-to-noise ratio (SNR).

Sol: For voltage, signal to noise ratio is given as

$$(S/N)_{dB} = 20\log_{10}\left(\frac{V_s}{V_n}\right)$$

$$= 20\log_{10}\left(\frac{4}{1 \times 10^{-3}}\right) = 72.04 \text{ dB}$$

PROBLEMS

P1.1 Draw the block diagram of a communication system and explain the function of each block

P1.2 Explain the digital communication system with a suitable block diagram.

P1.3 Mention the advantages and limitations of the digital communication system.

P1.4 What are the various undesirable effects that occur during transmission? Explain in brief.

P1.5 Explain the concept of bandwidth and write about baseband bandwidth and passband bandwidth.

P1.6 Describe the importance of a higher signal to noise ratio in a communication system.

P1.7 Describe the fundamental limitations of a communication system.

NUMERICAL PROBLEMS

P1.8 At the transmitter, the signal and noise powers are 40 mW and 3 μW, respectively. Calculate the input SNR. If the channel offers 3 dB attenuation to the signal and the output noise is twice the input noise level, determine the SNR at the output.

P1.9 A given signal has frequencies of 2100 MHz, 205 MHz, 2250 MHz and 5300 MHz. Determine the bandwidth of the signal?

P1.10 A 3 km communication link has input and output signals of 200 mW and 40 mW, respectively. Calculate the power loss/km in dB for the link.

P1.11 Calculate the output power when a 25 mW signal is applied to a transmission path that has a loss of -5 dB.

P1.12 The amplitudes of the signal and noise in a transmission link are estimated at 10 V and 10 mV, respectively. Estimate the signal-to-noise ratio (SNR).

CHAPTER 2

SIGNALS AND FOURIER TRANSFORMATION

Definition

A signal is defined as a function of one or more independent variables that delivers some information or follow some pattern of variations.

Highlights

- ***2.1. Introduction***
- ***2.2. Signals***
- ***2.3. Classification of Continuous-Time and Discrete-Time Signals***
- ***2.4. Some Useful Functions***
- ***2.5. Fourier Transform***
- ***2.6. CTFT of Continuous-Time Periodic Signals***
- ***2.7. Duality of Fourier Transform***
- ***2.8. Properties of Fourier Transform***
- ***2.9. Discrete-Time Fourier Transform (DTFT)***
- ***2.10. Parseval's Relation***
- ***2.11. Properties of Discrete-Time Fourier Transform (DTFT)***
- ***2.12. DTFT of Discrete Time Periodic Signals***

Solved Examples

Expression

$$\text{CTFT}\left\{x(t)\right\} = X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

2.1 Introduction

Every communication starts with the processing of signals through the communication medium. In this chapter, the concept of different types of signals with frequency domain transformation method is discussed.

2.2 Signals

A signal is simply defined as a set of data (information). Mathematically, a signal is defined as a function of one or more independent variables that delivers some information or follows some pattern of variation. It is represented as $x(t)$ where, x is a function of independent variable time t as shown in Fig. 2.1.

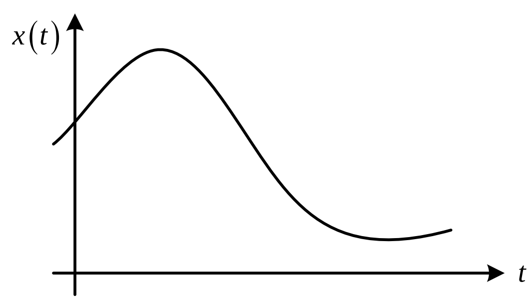


Fig. 2.1 A signal $x(t)$ as a function of t

Examples:

- Electrical signals: currents and voltages in a circuit
- Audio signal and video signals

2.2.1 Classifications of Signals

The classifications of the signals based on different constraints are shown in Fig. 2.2. The two most important classifications are as follows:

1. Based on The Condition of The Independent Variable of a One-Dimensional Signal

The signals are categorized as

(a) Continuous-Time Signals

A continuous-time signal or continuous signal is a function whose domain (generally time) has an uncountable or infinite set of numbers, i.e. function has a value at all points of time and varies continuously with time and is represented as $x(t)$ (Fig. 2.3(a)).

(b) Discrete-Time Signals

Unlike continuous times signal, discrete-time signal or discrete signal is a function whose domain (generally time) has a countable set of numbers, i.e. signals are defined at specific points (discrete values) of the independent variable (time) only. It is represented as $x[n]$, where, n is an integer value that varies discretely. When real-world signals are sampled, the signal becomes a discrete signal. Discrete signals are also defined as time series of sequences. (Fig. 2.3(b)).

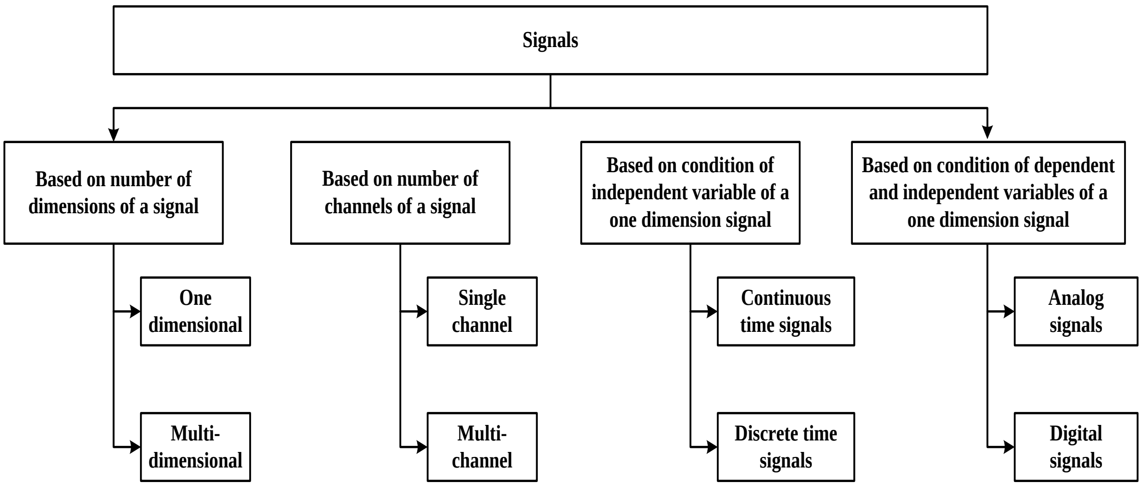
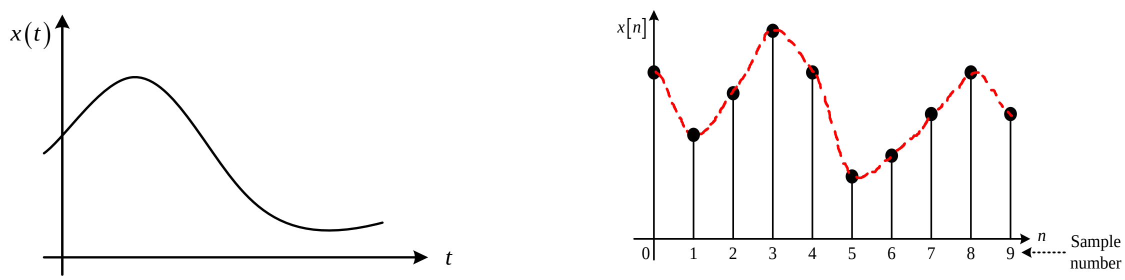


Fig. 2.2 Classification of signals



(a) (b)

Fig. 2.3 (a) Continuous time signal (b) Discrete time signal

SE2.1 A discrete signal is shown in Fig. 2.4. Find $x[0]$, $x[2]$, $x[3]$, $x[0.5]$ and $x[1.5]$.

Sol: Discrete signals are defined only at the integer value of time. Therefore,

$x[0] = 4;$ $x[2] = 2;$ $x[3] = 0.5$

$x[0.5] = x[1.5] =$ undefined.

$x[0.5] \neq x[1.5] \neq 0$; Since 0 is a defined value in itself.

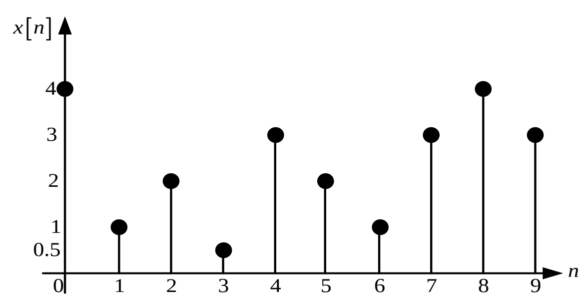


Fig. 2.4 Figure of SE2.1

2. Based on The Condition of Dependent Variable and Independent Variable of A Signal

The signals are categorized as

(a) Analog Signals

If the dependent and the independent both variables are continuous, i.e. signal can take any value in its amplitude, the signal is called analog signal. The difference between analog and digital signals is that the analog signal has a continuous dependent variable, whereas the digital signal has a discrete dependent variable.

(b) Digital Signals

The quantized signal with finite values of amplitudes is called digital signals. A digital signal whose amplitude can take only *M* different values is said to be *M*-ary. Binary signals are a special case for $M = 2$.

The waveforms of analog and digital signals are shown in Fig. 2.5 (a)-(b), respectively.

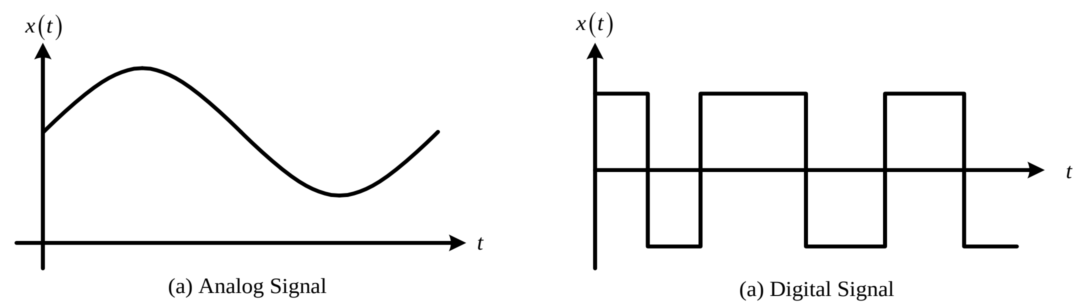


Fig. 2.5 (a) Analog signal (b) Digital signal

Now, Let take a view of the signals shown in Fig. 2.6 (a)-(d) and try to recognize them.

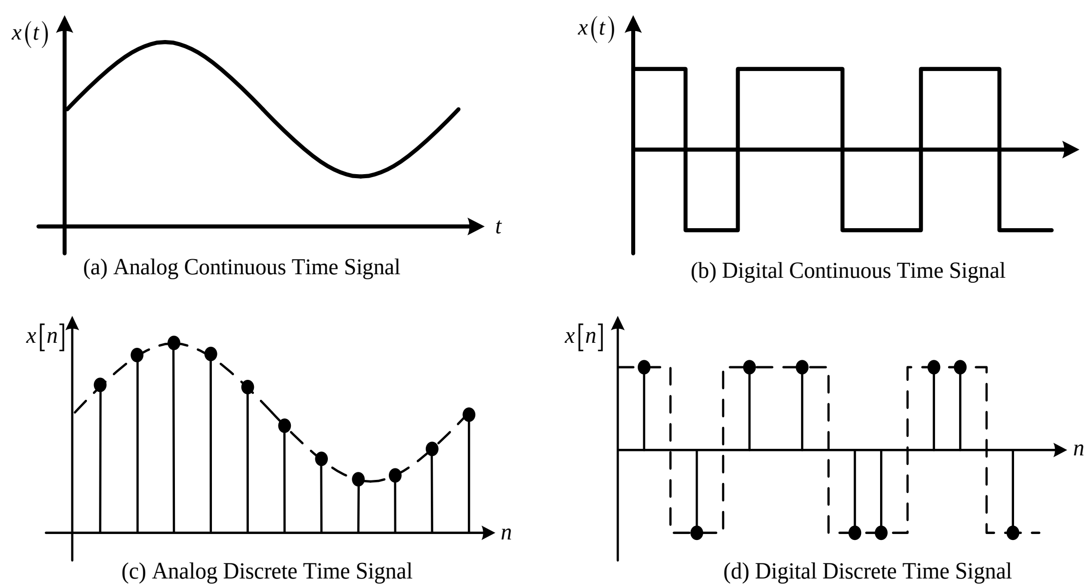


Fig. 2.6 (a) Analog continuous time signal (b) Digital continuous time signal (c) Analog discrete time signal (d) Digital discrete time signal

Further, the continuous and discrete-time signals are again classified into different categories.

2.3 Classification of Continuous-Time and Discrete-Time Signals

Based on signals properties, the categories of continuous-time and discrete-time signals are shown in Fig. 2.7.

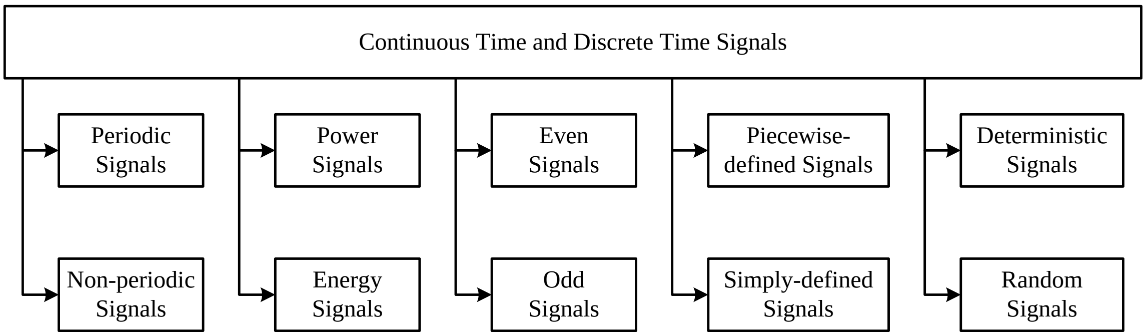


Fig. 2.7 Classification of continuous-time and discrete-time signals

A brief introduction of all of the above-mentioned signals are as follows

2.3.1 Periodic vs. Aperiodic

If a signal repeats itself after some period *T*, it is called a periodic signal. Mathematically, the continuous time-periodic signal is defined as

$$x(t) = x(t+T) \tag{2.1}$$

where, *T* is the time period of the signal.

On the other hand, aperiodic or non-periodic signals do not repeat themselves.

Similarly, a discrete-time signal is periodic if

$$x[n] = x[n+N] \text{ for all } n \tag{2.2}$$

Points to Remember

1. *The sum of N periodic continuous-time signals is not necessarily periodic.*
2. *The condition of periodicity for the sum of N periodic continuous-time signal signals is given by*

$$\frac{T_{01}}{T_{0i}} = \frac{n_1}{n_i} \qquad 2 \le i \le N \tag{2.3}$$

3. *The sum of the signal will be periodic if the above ratio is a rational number.*
4. *Since the ratio of sample periods of two discrete-time periodic signals are always rational, therefore, the sum of discrete-time periodic signals is always periodic*

$$\frac{N_{01}}{N_{0i}} = \frac{n_1}{n_i} \Rightarrow (\text{Always Rational}) \quad 2 \le i \le N \tag{2.4}$$

SE2.2 Let a signal $x[n]$ is the summation of two discrete periodic signals $x_1[n]$ and $x_2[n]$, with the period of 120 and 72 samples, respectively. Determine the period of the $x[n]$.

Sol: As we know, the sum of discrete-time signals are always periodic. Therefore $x[n]$ is a periodic signal. (This condition is only with discrete signal)

The period of $x_1[n]$ is N_1 = 120 samples and the period of $x_2[n]$ is N_2 = 72 samples. Therefore,

$$\frac{N_1}{N_2} = \frac{120}{72} = \frac{5}{3}$$

Hence, the period of $x[n]$ is $N = 120 \times 3 = 72 \times 5 = 360$ samples.

SE2.3 Find out which of the following signals are periodic:

(a) $x_1(t) = \sin 15\pi t$ (b) $x_2(t) = \sin 5\pi t + \sin 20\pi t$

Sol: The standard *sin* function is represented as:

$$x(t) = A\sin(\omega t + \varphi) = A\sin\left(\frac{2\pi}{T}t + \varphi\right)$$

Now compare all functions of problems with the standard functions

a) $x_1(t) = \sin 15\pi t$

$$\frac{2\pi}{T} = 15\pi$$

$$\Rightarrow T = \frac{2\pi}{15\pi} = \frac{2}{15}$$

b) $x_2(t) = \sin 5\pi t + \sin 20\pi t$

$x_{21}(t) = \sin 5\pi t$	$x_{22}(t) = \sin 20\pi t$	$x_2(t) = x_{21}(t) + x_{22}(t)$
$\frac{2\pi}{T_1} = 5\pi$ $\Rightarrow T_1 = \frac{2\pi}{5\pi}$ $\Rightarrow T_1 = \frac{2}{5}$	$\frac{2\pi}{T_2} = 20\pi$ $\Rightarrow T_2 = \frac{2\pi}{20\pi}$ $\Rightarrow T_2 = \frac{1}{10}$	$\frac{T_1}{T_2} = \frac{2/5}{1/10}$ $\Rightarrow \frac{T_1}{T_2} = \frac{4}{1}$

Since the ratio is rational, $x_2(t)$ is also a periodic signal with time period T as

$$T = T_1 \times 1 = T_2 \times 4 \Rightarrow \frac{2}{5} \times 1 = \frac{1}{10} \times 4$$

$$\Rightarrow T = \frac{2}{5}$$

2.3.2 Even and Odd Signals

A signal is defined as an even signal if it is symmetric about the *y*-axis, i.e. it is identical to its folded (time reversal) counterpart. On the other hand, if the signal is symmetric about the origin, the signal is called an odd signal, as shown in Fig. 2.8(a)-(b)

For continuous-time signal $x(t)$

$$\begin{aligned} &\text{If } x(-t) = x(t); && x(t) \text{ is an even signal} \\ &\text{If } x(-t) = -x(t); && x(t) \text{ is an odd signal} \end{aligned} \tag{2.5}$$

For discrete-time signal $x[n]$

$$\begin{aligned} &\text{If } x[-n] = x[n]; && x[n] \text{ is an even signal} \\ &\text{If } x[-n] = -x[n]; && x[n] \text{ is an odd signal} \end{aligned} \tag{2.6}$$

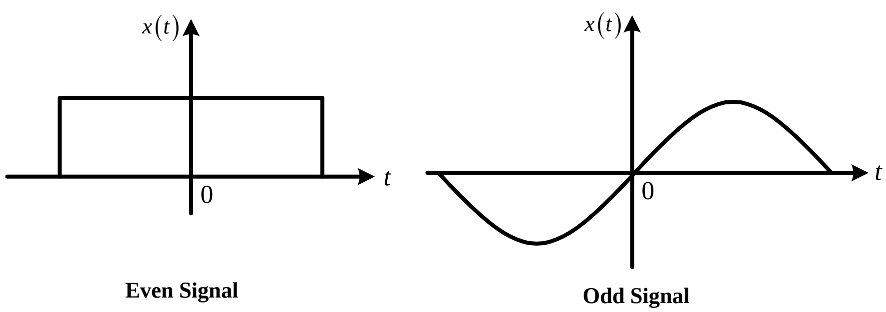


Fig. 2.8 (a) Even signal (b) Odd signal

The even and the odd parts of the continuous-time signal $x(t)$ are given as

$$\begin{aligned} \text{Even}\{x(t)\} &= \frac{1}{2}\{x(t) + x(-t)\} \\ \text{Odd}\{x(t)\} &= \frac{1}{2}\{x(t) - x(-t)\} \end{aligned} \tag{2.7}$$

Similarly, the even and the odd parts of the discrete-time signal $x[n]$ are given as

$$\begin{aligned} \text{Even}\{x[n]\} &= \frac{1}{2}\{x[n] + x[-n]\} \\ \text{Odd}\{x[n]\} &= \frac{1}{2}\{x[n] - x[-n]\} \end{aligned} \tag{2.8}$$

Points to remember

1. *An odd signal must be zero at t = 0 or n = 0.*
2. *Even and odd signals are also called symmetric and asymmetric signals, respectively.*
3. *Any signal can be decomposed into a sum of two signals, one of which is even (symmetric) and the other is odd (asymmetric).*

2.3.3 Power vs. Energy Signals

A signal is defined as an energy signal if it has finite total energy and zero average power, whereas a signal with finite average power and infinite energy is defined as a power signal.

Let a signal $x(t)$ has energy E_∞ and power P_∞.

(i) This signal is an energy signal if

$$0 < E_\infty < \infty \text{ and } P_\infty = 0 \tag{2.9}$$

(ii) This signal is a power signal if

$$0 < P_\infty < \infty \text{ and } E_\infty = \infty \tag{2.10}$$

Mathematically, the energy and the power for continuous-time signal are calculated by

$$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 \, dt = \int_{-\infty}^{\infty} |x(t)|^2 \, dt \tag{2.11}$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt \tag{2.12}$$

Similarly, the energy and the power for discrete-time signals are calculated by

$$E_\infty = \lim_{N\to\infty} \sum_{n=-N}^{N} |x[n]|^2 = \sum_{n=-\infty}^{\infty} |x[n]|^2 \tag{2.13}$$

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 \tag{2.14}$$

Points to remember

1. *Power signals and Energy signals are mutually exclusive, i.e. some signals have finite energy and some signals have finite power, but none of the signals has both finite energy and finite power.*
2. *If the power and the energy of any signal tends to be infinite, the signal is neither energy nor power signal.*
3. *The amplitude of an energy signal tends to zero as* $|t| \to \infty$.

2.4 Some Useful Functions

The useful functions are

(1) Impulse function
(2) Step function
(3) Ramp function
(4) Parabolic function
(5) Signum function
(6) Sinc function
(7) Rectangular function
(8) Triangular function

(1) Impulse function: The impulse function is shown in Fig. 2.9. The impulse function is zero for $t = 0^-$ and rises to infinity at $t = 0$ and immediately becomes 0 at $t = 0^+$. Therefore, the step function encloses an area A. If the area is unity, i.e. $A = 1$, the impulse function is called as unit impulse function and mathematically expressed as

$$\delta(t) = \lim_{a \to 0} \frac{1}{a}\left[u(t) - u(t-a)\right] \qquad (2.15)$$

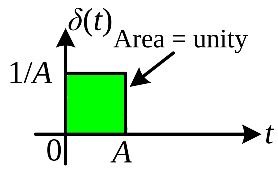


Fig. 2.9 Impulse function

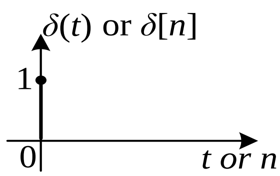
For numerical problems, expression for unit impulse function is simplified as

(i) In the continuous time domain

$$\delta(t) = \begin{cases} 1 & for \quad t = 0 \\ 0 & for \quad t \neq 0 \end{cases} \qquad (2.16)$$

(ii) In discrete domain,

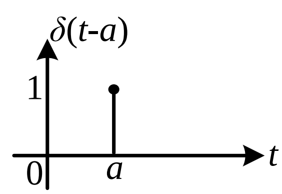
$$\delta[n] = \begin{cases} 1 & for \quad n = 0 \\ 0 & for \quad n \neq 0 \end{cases} \qquad (2.17)$$



$\delta[n]$ is called unit impulse sequence.

(iii) Unit impulse with shift a

$$\delta(t-a) = \begin{cases} 1 & for \quad t - a = 0 \Rightarrow t = a \\ 0 & for \quad t - a \neq 0 \Rightarrow t \neq a \end{cases}$$



Points to remember

Properties of unit impulse function are as follows:

(1) $\delta(-t) = \delta(t)$ (Time reversal)

(2) $x(t)\delta(t - t_o) = x(t_o)\delta(t - t_o)$

(3) $\int_{t_1}^{t_2} x(t)\delta(t - t_o) = \begin{cases} x(t_o) & t_1 < t_o < t_2 \\ 0 & otherwise \end{cases}$

(2) Step function: Step function is zero for $t < 0$ and rises to a value of A at $t = 0$ and remains at A for $t > 0$ (Fig. 2.10(a)-(b)). Mathematically, it is represented as

$$x(t) = Au(t) = \begin{cases} A & for \quad t > 0 \\ 0 & for \quad t < 0 \end{cases} \quad \text{(In continuous time domain)} \tag{2.18}$$

$$x[n] = Au[n] = \begin{cases} A & for \quad n \geq 0 \\ 0 & for \quad n < 0 \end{cases} \quad \text{(In discrete time domain)} \tag{2.19}$$

If $A = 1$; $x(t) = u(t)$ or $x[n] = u[n]$ is called unit step function.

(a) (b)

Fig. 2.10 Step function in (a) continuous domain (b) discrete domain

Points to remember

(1) $u(t)$ is discontinuous at time $t = 0$, whereas $u[n]$ is 1 at $n = 0$, i.e. $u[0] = 1$.

(2) $u(t)$ is the running integral of unit impulse function $\delta(t)$ whereas $u[n]$ is the running sum of $\delta[n]$.

$$u(t) = \int_{\tau=-\infty}^{\infty} \delta(\tau) d\tau \text{ and } u[n] = \sum_{k=-\infty}^{n} \delta[k] \tag{2.20}$$

(3) Unit step function with shift a (Fig. 2.11 (a)).

$$u(t-a) = \begin{cases} 1 & for \quad t-a > 0 \Rightarrow t > a \\ 0 & for \quad t-a < 0 \Rightarrow t < a \end{cases}$$

Unit step function with shift a (Fig. 2.11 (b)).

$$u[n-a] = \begin{cases} 1 & for \quad n-a \geq 0 \Rightarrow n \geq a \\ 0 & for \quad n-a < 0 \Rightarrow n < a \end{cases}$$

(a) (b)

Fig. 2.11 Shifted step function in (a) continuous domain (b) discrete domain

(3) Ramp Function: Ramp function is zero for $t < 0$ and increases with slope A for $t > 0$ as shown in Fig. 2.12. Mathematically, ramp function is represented as

$$x(t) = Ar(t) = \begin{cases} At & for \quad t > 0 \\ 0 & for \quad t < 0 \end{cases} \quad \text{(In continuous time domain)} \tag{2.21}$$

$$x[n] = Ar[n] = \begin{cases} An & for \quad n \geq 0 \\ 0 & for \quad n < 0 \end{cases} \quad \text{(In discrete time domain)} \tag{2.22}$$

If $A = 1$; $x(t) = r(t)$ or $x[n] = r[n]$ is called unit ramp function.

Fig. 2.12 Ramp function in (a) continuous domain (b) discrete domain

(4) Signum Function: Signum function is denoted by $\text{sgn}(t)$, and it is expressed as:

$$\text{sgn}(t) = \begin{cases} 1 & for \quad t > 0 \\ 0 & for \quad t = 0 \\ -1 & for \quad t < 0 \end{cases} \tag{2.23}$$

Graphically this function is shown in Fig. 2.13:

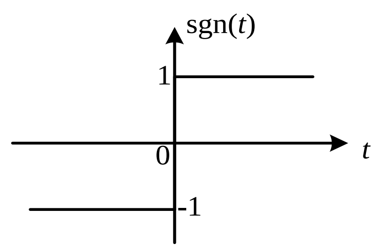


Fig. 2.13 Signum function

The relationship between signum function and unit step function is expressed as:

$$\text{sgn}(t) = 2u(t) - 1 \tag{2.24}$$

(5) Sinc Function

A *sinc* function or *Interpolation sinc* function is an even function with unity area and denoted as $\text{sinc}(t)$. It is represented in two forms:

(a) **Normalized sinc function:** Normalized sinc function (in mathematics) is defined for $t \neq 0$ and expressed as

$$\text{sinc}(t) = \frac{\sin(\pi t)}{t} \tag{2.25}$$

(b) **Unnormalized sinc function:** : Unnormalized sinc function (in digital signal processing) is defined for $t \neq 0$ and expressed as

$$\text{sinc}(t) = \frac{\sin(t)}{t} \tag{2.26}$$

Sinc function is shown in Fig 2.14. *sinc* pulse passes through zero at all positive and negative integers $(i.e.\ t = \pm 1, \pm 2, ...)$, but it has a maximum value of 1 at time $t = 0$.

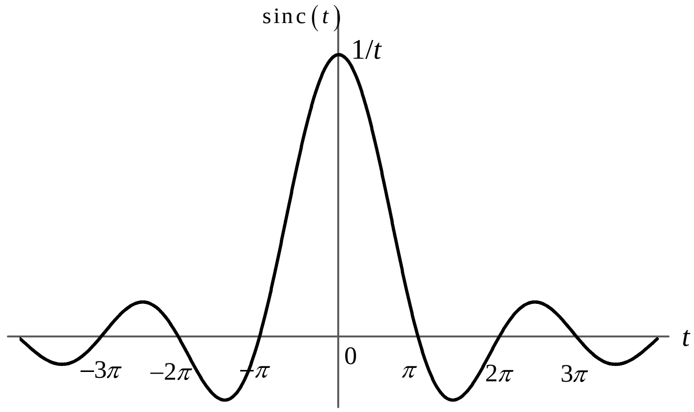


Fig. 2.14 Sinc function

(6) Unit Gate Function: It is defined as a gate pulse of unit height and unit width centered at the origin, as shown in Fig. 2.15(a). It is represented as $\text{rect}(t)$. The expanded rectangular function $\text{rect}(t/\tau)$ is shown in Fig. 2.15(b), where the denominator term τ is the width of the pulse. Unit gate function is also known as rectangular function, rect function, Pi function, unit pulse or the normalized boxcar function. Mathematically,

$$\text{rect}(t) = \begin{cases} 0 & |t| > \frac{1}{2} \\ \frac{1}{2} & |t| = \frac{1}{2} \\ 1 & |t| < \frac{1}{2} \end{cases}$$

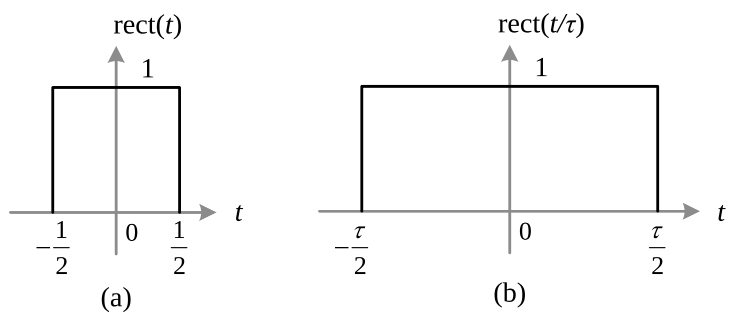


Fig. 2.15 (a) Rectangular function (b) Expanded rectangular function

(7) Unit Triangle Function: It is defined as a triangle pulse of unit height and unit width centered at the origin, as shown in Fig. 2.16(a). It is represented as $\Delta(t)$. The expanded rectangular function $\Delta(t/\tau)$ is shown in Fig. 2.16(b), where the denominator term τ is the width of the pulse. Mathematically,

$$\Delta(t) = \begin{cases} 0 & |t| > \frac{1}{2} \\ 1 - 2|t| & |t| < \frac{1}{2} \end{cases}$$

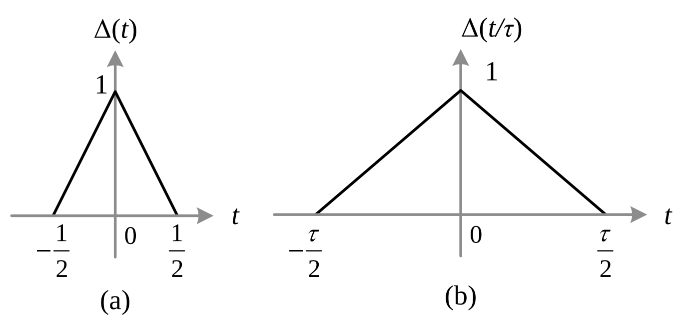


Fig. 2.16 (a) Triangular function (b) Expanded triangular function

2.5 Fourier Transform

The Fourier transform is used to get spectral representation for aperiodic signals. The Fourier transform of the continuous-time signal is called continuous-time Fourier transform (CTFT), whereas the Fourier transform of the discrete signal is called discrete-time Fourier transform (DTFT).

The CTFT of any continuous-time signal $x(t)$ is given

$$\text{CTFT}\{x(t)\} = X(\omega) = \int_{t=-\infty}^{\infty} x(t)e^{-j\omega t}dt \qquad (2.27)$$

The inverse Fourier transform is given as

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega \qquad (2.28a)$$

$$x(t) = \int_{-\infty}^{\infty} X(\omega)e^{j\omega t}df \qquad (\because \omega = 2\pi f) \qquad (2.28b)$$

Eq. (2.27) is the Fourier transform of any signal $x(t)$, whereas Eq. (2.28) is the inverse Fourier transform.

SE2.4 Determine the CTFT of a continuous-time signal given as:

$$x(t) = e^{-At}u(t), \qquad A > 0$$

where A is a scalar constant.

Sol: The CTFT of the signal $x(t)$ is given as

$$\text{CTFT}\{x(t)\} = X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

Substituting the value of $x(t)$

$$X(\omega) = \int_{-\infty}^{\infty} e^{-At}e^{-j\omega t}u(t)dt$$

$$= \int_{-\infty}^{\infty} e^{-(A+j\omega)t}u(t)dt \qquad u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

$$= \int_{0}^{\infty} e^{-(A+j\omega)t}dt$$

$$= \left[-\frac{1}{(A+j\omega)}e^{-(A+j\omega)t}\right]_0^{\infty}$$

$$X(\omega) = \frac{1}{(A+j\omega)} \qquad A > 0$$

The magnitude and the phase spectrum of $X(\omega)$ is

$$|X(\omega)| = \frac{1}{\sqrt{(A^2+\omega^2)}} \quad \text{and} \quad \angle X(\omega) = \tan^{-1}(\omega / A)$$

The magnitude and the phase spectrums are shown in Fig. 2.17(a) and Fig. 2.17(b), respectively.

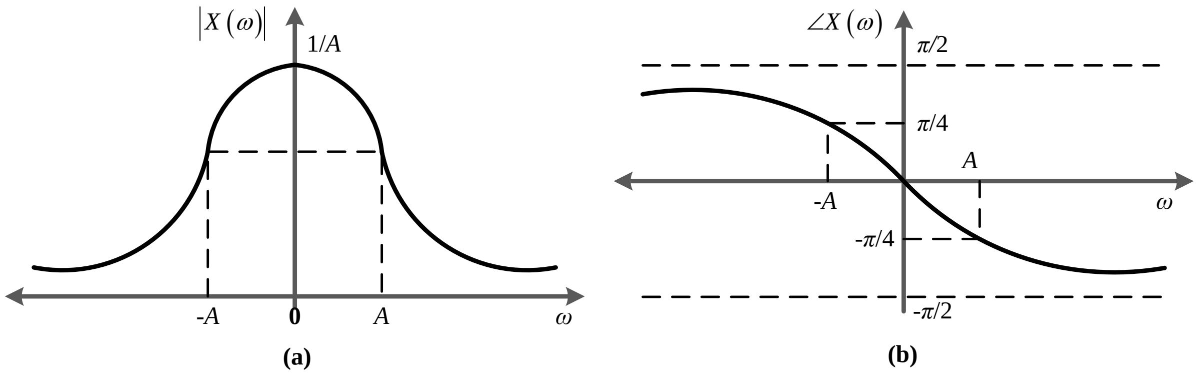


Fig. 2.17 (a) Magnitude spectrum of $X(\omega)$ (b) Phase spectrum of $X(\omega)$

2.6 CTFT of Continuous-Time Periodic Signals

The Fourier transform of a periodic signal with Fourier series coefficients A_k can be interpreted as a train of impulses occurring at harmonically related frequencies and given as

$$X(\omega) = \sum_{k=-\infty}^{\infty} 2\pi A_k \delta(\omega - k\omega_o) \tag{2.29}$$

2.7 Duality of Fourier Transform

Fourier Transform of any signal $x(t)$ is given as

$$\begin{aligned} X(\omega) &= \int_{u=-\infty}^{\infty} x(t) e^{-j\omega t} dt \\ x(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega \end{aligned} \tag{2.30}$$

By comparing the above two equations, we observe that these equations are similar but not identical in form. This symmetry leads to a property of a Fourier transform that is called **duality**. The application of the duality principle is explained by example SE2.5.

2.8 Properties of Fourier Transform

Let $x(t) \xleftrightarrow{\text{CTFT}} X(\omega)$; The different properties of Fourier transform is given in Table 2.1.

Table 2.1 Properties of Fourier Transform

S. No.	Properties	Non-Periodic Signal	CTFT of signals
1.	**Time Reversal**	$x(-t)$	$X(-\omega)$
2.	**Time Shifting**	$x(t-t_o)$	$e^{-j\omega t_o}X(\omega)$
3.	**Frequency Shifting**	$e^{j\omega_0 t}x(t)$	$X[(\omega-\omega_o)]$
4.	**Time Scaling**	$x(At)$	$\frac{1}{\lvert A\rvert}X\left(\frac{\omega}{A}\right)$
5.	**Complex Conjugation**	$x^*(t)$	$X^*(-\omega)$
6.	**Differentiation in Time**	$\frac{dx(t)}{dt}$	$j\omega X(\omega)$
7.	**Integration**	$\int_{-\infty}^{\infty} x(\tau)d\tau$	$\frac{1}{j\omega}X(\omega)+\pi X(0)\delta(\omega)$
8.	**Differentiation of CTFT of a signal in Frequency**	$tx(t)$	$j\frac{dX(\omega)}{d\omega}$
9.	**Convolution**	$x_1(t)*x_2(t)$	$X_1(\omega)X_2(\omega)$
10.	**Multiplication**	$x_1(t)\times x_2(t)$	$\frac{1}{2\pi}X_1(\omega)*X_2(\omega)$

SE2.5 Determine the CTFT $X(\omega)$ for the signal given below

$$x(t)=\frac{2}{1+t^2}$$

Sol: As we know if,

$$x(t)=e^{-2|t|} \Rightarrow X(\omega)=\frac{2}{1+\omega^2}$$

So, $$e^{-2|t|}=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{2}{1+\omega^2}e^{j\omega t}d\omega$$

Replace *t* = -*t*

$$2\pi e^{-2|t|}=\int_{-\infty}^{\infty}\frac{2}{1+\omega^2}e^{-j\omega t}d\omega$$

Interchange the variables

$$2\pi e^{-2|\omega|}=\int_{-\infty}^{\infty}\frac{2}{1+t^2}e^{-j\omega t}dt$$

Therefore, $$\text{CTFT}\left\{\frac{2}{1+t^2}\right\}=2\pi e^{-2|\omega|}$$

2.9 Discrete-Time Fourier Transform (DTFT)

The discrete-Time Fourier Transform (DTFT) is the member of the Fourier transform family that operates on aperiodic, discrete signals. The DTFT is often used to analyse samples of a continuous function. The term discrete-time refers to the fact that the transform operates on discrete data, often samples whose interval has units of time.

The discrete-time Fourier transform of a discrete set of real or complex numbers $x[n]$, for all integers *n*, is a Fourier series, which produces a periodic function of a frequency variable. When the frequency variable, ω, has normalized units of radians/sample, the periodicity is 2π.

With the use of the sampled version of a continuous-time signal, we can obtain the discrete-time Fourier transform (DTFT) or Fourier transform of discrete-time signals as follows.

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X\left(e^{j\omega}\right) \quad e^{j\omega n} d\omega \tag{2.31}$$

$$X\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \tag{2.32}$$

Eq. (2.31) is called the DTFT synthesis equation and Eq. (2.32) is called DTFT analysis. The significant differences between the DTFT and CTFT are:

1. DTFT is periodic in ω with period 2π, but CTFT is not periodic.
2. DTFT has a finite interval of integration in the synthesis equation, but CTFT has an infinite interval of integration in the synthesis equation.

2.10 Parseval's Relation: Parseval's relation addresses the energy of $x[n]$

$$\sum_{n=-\infty}^{\infty} \left|x[n]\right|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left|X\left(e^{j\omega}\right)\right|^2 d\omega \tag{2.33}$$

Proof:

$$\sum_{n=-\infty}^{\infty} \left|x[n]\right|^2 = \sum_{n=-\infty}^{\infty} x[n]^* x[n]$$

$$= \sum_{n=-\infty}^{\infty} x[n]\left(\frac{1}{2\pi}\int_{-\pi}^{\pi} X\left(e^{j\omega}\right)e^{j\omega n}d\omega\right)$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X^*\left(e^{j\omega}\right)\left(\sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}\right) d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \left|X\left(e^{j\omega}\right)\right|^2 d\omega$$

2.11 Properties of Discrete-Time Fourier Transform (DTFT)

Let $x[n] \xleftrightarrow{\text{DTFT}} X\left(e^{j\omega}\right)$; The different properties of Fourier transform is tabulated in the Table 2.2.

Table 2.2 Properties of discrete-time Fourier transform

S. No.	Properties	Non-Periodic Signal	DTFT of signals
1.	**Signal**	$x[n]$	$X\left(e^{j\omega}\right)$
2.	**Time Reversal**	$x[-n]$	$X\left(e^{-j\omega}\right)$
3.	**Time Shifting**	$x[n-n_o]$	$e^{-j\omega n_o} X\left(e^{j\omega}\right)$
4.	**Frequency Shifting**	$e^{j\omega_0 n} x[n]$	$X\left[e^{j(\omega-\omega_o)}\right]$
5.	**Complex Conjugation**	$x^*[n]$	$X^*\left(e^{-j\omega}\right)$
6.	**Differentiation of DTFT of a signal in Frequency**	$nx[n]$	$j\dfrac{dX\left(e^{j\omega}\right)}{d\omega}$
7.	**Convolution**	$x_1[n] * x_2[n]$	$X_1\left(e^{j\omega}\right) X_2\left(e^{j\omega}\right)$
8.	**Multiplication**	$x_1[n] \times x_2[n]$	$\dfrac{1}{2\pi} X_1\left(e^{j\omega}\right) * X_2\left(e^{j\omega}\right)$

SE2.6 Verify Parseval's theorem of the sequence $x[n] = \left(\frac{1}{4}\right)^n u[n]$.

Sol: Parseval's Relation: Parseval's relation addresses the energy of $x[n]$

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left|X\left(e^{j\omega}\right)\right|^2 d\omega$$

$$\text{LHS} = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^{2n} u[n]$$

$$\text{RHS} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left|X\left(e^{j\omega}\right)\right|^2 d\omega$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^{2n} = 1 + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^4 + \ldots\ldots + \infty \text{ terms} = \frac{1}{1-(1/4)^2} = \frac{16}{15}$$

Since,

$$X\left(e^{j\omega}\right) = \frac{1}{1-\frac{1}{4}e^{-j\omega}} = \frac{1}{1-\frac{1}{4}\cos\omega + j\frac{1}{4}\sin\omega}$$

$$X^*\left(e^{j\omega}\right)=\frac{1}{1-\frac{1}{4}\cos\omega-j\frac{1}{4}\sin\omega}$$

Therefore,

$$\left|X\left(e^{j\omega}\right)\right|^2=X\left(e^{j\omega}\right)X^*\left(e^{j\omega}\right)=\frac{1}{\left(1-\frac{1}{4}\cos\omega\right)^2+\left(\frac{1}{4}\sin\omega\right)^2}=\frac{1}{1.0625-0.5\cos\omega}$$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|X\left(e^{j\omega}\right)\right|^2 d\omega=\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1}{1.0625-0.5\cos\omega}d\omega=\frac{16}{15}$$

So, LHS = RHS

2.12 DTFT of Discrete Time Periodic Signals

The DTFT of the exponential function $x[n]=e^{j\omega_0 n}$ is impulse train which is represented as

$$X\left(e^{j\omega}\right)=\sum_{m=-\infty}^{\infty}2\pi\delta\left(\omega-\omega_o-2\pi m\right) \tag{2.34}$$

Fourier series coefficients A_k can be interpreted as a train of impulses occurring at harmonically related frequencies and given as

$$X\left(e^{j\omega}\right)=\sum_{m=-\infty}^{\infty}2\pi A_k\delta\left(\omega-\frac{2\pi k}{N_o}\right) \tag{2.35}$$

SE2.7 Determine the DTFT of a discrete-time periodic signal given as:

$x[n]=\cos\left(\omega_0 n\right)$, with fundamental frequency ω_0 = 2π/5.

Sol: $$x[n]=\cos\left(\omega_0 n\right)=\frac{e^{j\omega_0 n}+e^{-j\omega_0 n}}{2}$$

But we know that the DTFT of a period signal $x[n]=e^{j\omega_0 n}$ is given by

$$X\left(e^{j\omega}\right)=\text{DTFT}\left\{x[n]\right\}=\text{DTFT}\left\{e^{j\omega_0 n}\right\}=\sum_{m=-\infty}^{\infty}2\pi\delta\left(\omega-\omega_o-2\pi m\right)$$

Similarly, the DTFT of the above equation can be determined as

$$X\left(e^{j\omega}\right)=\text{DTFT}\left\{x[n]\right\}=\text{DTFT}\left\{\frac{1}{2}e^{j\omega_0 n}+\frac{1}{2}e^{-j\omega_0 n}\right\}=\frac{1}{2}\text{DTFT}\left\{e^{j\omega_0 n}\right\}+\frac{1}{2}\text{DTFT}\left\{e^{-j\omega_0 n}\right\}$$

$$=\frac{1}{2}\sum_{m=-\infty}^{\infty}2\pi\delta\left(\omega-\omega_o-2\pi m\right)+\frac{1}{2}\sum_{m=-\infty}^{\infty}2\pi\delta\left(\omega+\omega_o-2\pi m\right)$$

Substituting $\left(\omega_o=\frac{2\pi}{5}\right)$

$$X\left(e^{j\omega}\right) = \sum_{m=-\infty}^{\infty} \pi\delta\left(\omega - \frac{2\pi}{5} - 2\pi m\right) + \sum_{m=-\infty}^{\infty} \pi\delta\left(\omega + \frac{2\pi}{5} - 2\pi m\right)$$

$$= \left\{\ldots + \pi\delta\left(\omega - \frac{2\pi}{5} + 2\pi\right) + \pi\delta\left(\omega - \frac{2\pi}{5}\right) + \pi\delta\left(\omega - \frac{2\pi}{5} - 2\pi\right) + \ldots.\right\}$$

$$+ \left\{\ldots + \pi\delta\left(\omega - \frac{2\pi}{5} + 2\pi\right) + \pi\delta\left(\omega - \frac{2\pi}{5}\right) + \pi\delta\left(\omega - \frac{2\pi}{5} - 2\pi\right) + \ldots.\right\}$$

$$X\left(e^{j\omega}\right) = \pi\delta\left(\omega - \frac{2\pi}{5}\right) + \pi\delta\left(\omega + \frac{2\pi}{5}\right), \qquad -\pi \le \omega \le \pi$$

The spectrum of the above function is shown in Fig. 2.18.

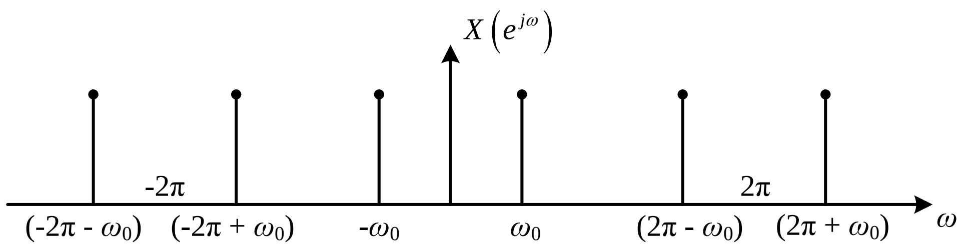


Fig. 2.18 Spectrum of SE2.7

ADDITIONAL SOLVED EXAMPLES

SE2.8 Find the Fourier transform of $\text{rect}(t/\tau)$.

Sol: The rectangular function is shown in Fig. 2.19(a).

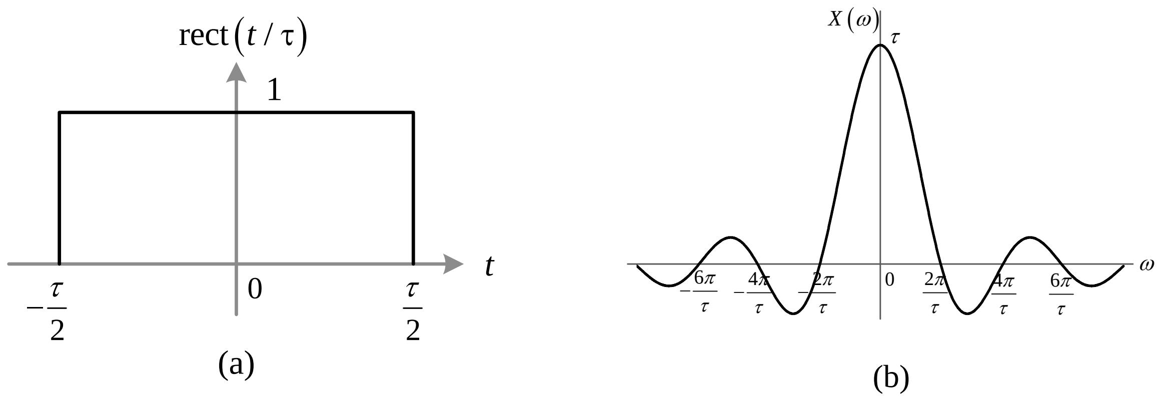


Fig. 2.19 (a) The rectangular function (b) Spectrum of $\text{sinc}\left(\frac{\omega\tau}{2}\right)$

The Fourier transform is obtained as

$$\text{CTFT}\{x(t)\} = X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt = \int_{-\infty}^{\infty} \text{rect}\left(\frac{t}{\tau}\right)e^{-j\omega t}dt$$

$$= \int_{-\tau/2}^{\tau/2} e^{-j\omega t}dt = -\frac{1}{j\omega}\left(e^{-j\omega t}\right)_{-\tau/2}^{\tau/2} = \frac{1}{j\omega}\left(e^{-j\omega\tau/2} - e^{-j\omega\tau/2}\right)$$

$$= \frac{2\sin(\omega\tau/2)}{\omega} = \tau\frac{\sin(\omega\tau/2)}{(\omega\tau/2)} = \tau\,\text{sinc}\left(\frac{\omega\tau}{2}\right)$$

Therefore, $\text{CTFT}\left\{\text{rect}\left(\frac{t}{\tau}\right)\right\} \Leftrightarrow \tau\,\text{sinc}\left(\frac{\omega\tau}{2}\right)$

Since, $\text{sinc}(t) = 0$ for $t = \pm n\pi$

Therefore, $\text{sinc}\left(\frac{\omega\tau}{2}\right) = 0$ for $\frac{\omega\tau}{2} = \pm n\pi \Rightarrow \omega = \pm\frac{2n\pi}{\tau}$

The Fourier transform is shown in Fig. 2.19(b).

SE2.9 Determine the values of P_∞ and E_∞ for the following signals:

(a) $x[n] = \left(\frac{1}{2}\right)^n u[n]$

(b) $x(t) = e^{-\alpha t}u(t)$

Sol: (a) Energy of the discrete signal is given by

$$E_\infty = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$= \sum_{n=-\infty}^{\infty} \left| \left(\frac{1}{2}\right)^n u[n] \right|^2$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{2n} \qquad \left(\because u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases} \right)$$

$$= \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$$

Similarly, $P_\infty = 0$

(b) Energy of the discrete signal is given by

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} \left|e^{-\alpha t} u(t)\right|^2 dt$$

$$= \int_0^{\infty} e^{-2\alpha t} dt \qquad \left(\because u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases} \right)$$

$$= -\frac{1}{2\alpha}\left(e^{-2\alpha t}\right)_0^{\infty} = \frac{1}{2\alpha}$$

Similarly, $P_\infty = 0$

SE2.10 Find out the fundamental period of the signal

$$x(t) = 15\cos(20t+1) - 10\sin(40t-1)$$

Sol: The given signal can be written as a combination of two signals

$$x(t) = x_1(t) + x_2(t)$$

where, $x_1(t) = 15\cos(20t+1)$ and $x_2(t) = -10\sin(40t-1)$

The fundamental periods of $x_1(t)$ and $x_2(t)$ are T_1 and T_2 respectively.

$$T_1 = \frac{2\pi}{20} = \frac{\pi}{10} \text{ and } T_2 = \frac{2\pi}{40} = \frac{\pi}{20}$$

The ratio of fundamental periods is

$$\frac{T_1}{T_2} = \frac{\pi/10}{\pi/20} = \frac{2}{1}$$

The ratio is a rational number, so the signal $x(t)$ is also periodic with

$$T = T_1(t) \times 1 = T_2(t) \times 2$$

$$= \frac{\pi}{10} \times 1 = \frac{\pi}{20} \times 2 = \frac{\pi}{10}$$

PROBLEMS

P2.1 Define and give the classification of continuous-time signals and discrete-time signals.

P2.2 Define various standard continuous-time signals graphically.

P2.3 Explain unit impulse and unit ramp functions.

P2.4 Define periodic and non-periodic signals.

P2.5 Explain the power and the energy signals.

P2.6 State and prove duality property of the Fourier transform.

P2.7 Define DTFT. What are the different properties of DTFT?

NUMERICAL PROBLEMS

P2.8 Determine whether or not each of the following signals is periodic. If a signal is periodic, determine the fundamental period:

(a) $x(t) = \left[2\cos(2\pi t)\right]u(t)$

(b) $x[n] = \cos\left(\pi n^2 / 8\right)$

(c) $x(t) = \sum_{n=-\infty}^{+\infty} e^{-(t-3n)^2}$

P2.9 Find the even and odd parts of each signal:

(a) $x(t) = \left(7+3t^2\right)\sin(24\pi t)$

(b) $x(t) = (7+3t)\cos(24\pi t)$

(c) $x(t) = (\pi t)\sin(24\pi t)$

(d) $x(t) = 4 + \dfrac{\sin(3\pi t)}{\pi t}$

(e) $x(t) = 1 + t\sin(t) + t^2\cos(t) + t^3\cos(t)\sin(t)$

(f) $x(t) = \sin(t) + t^2\cos(t) + \cos(t)\sin(t)$

P2.10 Verify whether the signal $x(t) = Au(t)$, is a power signal or not.

P2.11 Show that the signal $x(t) = t^{-1/4}u(t-1)$ is neither energy nor a power signal.

P2.12 Determine the values of P_∞ and E_∞ for the following signals:

(c) $x(t) = e^{-2t}u(t)$

(d) $x(t) = \cos(t)$

(e) $x[n] = \cos\left[\dfrac{\pi n}{4}\right]$

P2.13 Determine the CTFT of a continuous-time signal given as:

$$x(t) = e^{-A|t|}, \qquad A > 0$$

where, A is a scalar constant.

P2.14 Draw the waveform of the signal represented by the equation given below and determine the CTFT of the signal

$$x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & |t| > T_1 \end{cases}$$

P2.15 Draw the waveform of the signal represented by the equation given below and determine the CTFT of the signal using the duality property

$$X(\omega) = \begin{cases} 1 & |\omega| < W \\ 0 & |\omega| > W \end{cases}$$

P2.16 Determine the CTFT of the signal shown in Fig. 2.20.

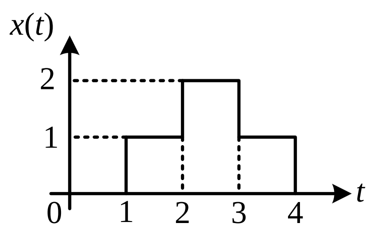


Fig. 2.20 Figure of problem P2.16

P2.17 Determine the DTFT of a continuous-time signal given as:

$$x[n] = A^n u[n], \qquad |A| < 1$$

where, A is a scalar constant.

MULTIPLE-CHOICE QUESTIONS

MCQ2.1 What is the period of the following signal $x[n]=\sin\left[\frac{n}{12}\right]$?

(a) 12 (c) 6π

(b) 24π (d) Non-periodic

MCQ2.2 The period of the signal $x(t)=20\sin(t+\phi)$ is

(a) 2π (c) $2\pi+\phi$

(b) 4π (d) $4\pi+\phi$

MCQ2.3 What is the period of the following signal

$x[n]=4e^{j(7\pi n/8)}+12e^{j(3\pi n/5)}$?

(a) 40 (c) 21

(b) 80 (d) 100

MCQ2.4 What is the period of the following signal?

$$x(t)=\begin{cases}\sin t & if \quad t>0\\ \cos t & if \quad t\le 0\end{cases}$$

(a) Non-periodic (c) 2π

(b) π (d) π/2

MCQ2.5 The power of signal $x(t)=15\sin(20\pi t+\pi/3)+20\cos(40\pi t+\pi/4)$ within the frequency band of 15 Hz to 30 Hz is

(a) 15 W (c) 112.5 W

(b) 20 W (d) 200 W

MCQ2.6 The power of the signal $x(t)=5+10\sin t$ is

(a) 25 W (c) 75 W

(b) 50 W (d) 100 W

MCQ2.7 The signal $x[n]=u[n]$ is

(a) Energy signal with $E=0.5$

(b) Power signal with $P=0.5$

(c) Energy signal with $E=1$

(d) Power signal with $P=1$

MCQ2.8 If $X(\omega)=\delta(\omega-\omega_0)$, then $x(t)$ is

(a) $\delta(t)$ (c) 1

(b) $e^{-j\omega_0 t}$ (d) $\frac{1}{2\pi}e^{j\omega_0 t}$

MCQ2.9 Fourier transform of a real and odd function is

(a) Real and odd

(b) Imaginary and odd

(c) Real and even

(d) Imaginary and even

MCQ2.10 CTFT$[\delta(t)]$ is

(a) 1 (c) 0

(b) $\delta(\omega)$ (d) π

MCQ ANSWERS

MCQ2.1	(d)	MCQ2.6	(c)
MCQ2.2	(a)	MCQ2.7	(b)
MCQ2.3	(b)	MCQ2.8	(d)
MCQ2.4	(a)	MCQ2.9	(b)
MCQ2.5	(d)	MCQ2.10	(a)

CHAPTER 3

AMPLITUDE MODULATION

Definition

The amplitude of the carrier signal is varied in accordance with the instantaneous value of modulating signal.

Highlights

Expression

$$\phi_{\text{AM}}(t) = \left(A_c + m(t)\right)\cos\omega_c t$$

3.1 Introduction

Baseband communication is a communication without modulation, whereas carrier communication is defined as communication with modulation. There is no frequency shift in the spectrum of the baseband communication.

3.2 Modulation

Modulation is the process in which specific properties of a carrier signal (a high-frequency signal) are varied in accordance with the instantaneous value of baseband signal (a low-frequency signal). The baseband signal is also termed as an information signal or modulating signal.

Many a ways are used to categories the modulation techniques. The nature of the carrier signal is used to categories the modulation techniques into the following two categories:

1. Analog wave or continuous wave modulation
2. Pulse modulation

Another type of categorization of the modulation process is shown in Fig. 3.1.

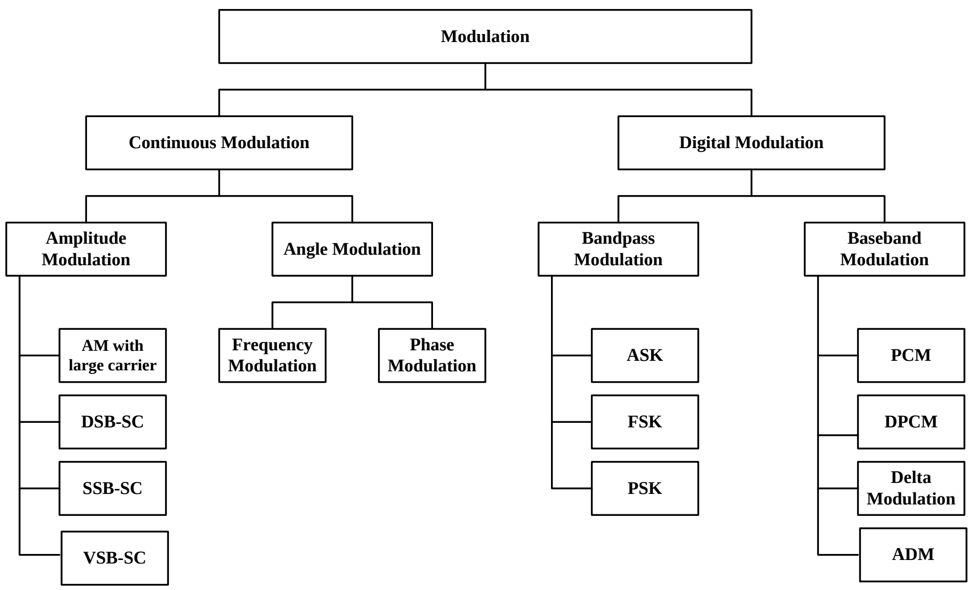


Fig. 3.1 Classification of modulation techniques

3.2.1 Analog Modulation

If the carrier wave used in the modulation process is continuous in nature, the modulation process is called analog or continuous wave modulation. Let the carrier wave is expressed as

$$c(t) = A_c \cos(\omega_c t + \varphi_c) \tag{3.1}$$

Above mentioned carrier signal has the following three parameters:

(i) Amplitude A_c

(ii) Frequency ω_c

(iii) Phase angle $\psi = (\omega_c t + \varphi_c)$

Change in any of the above properties according to the instantaneous value of modulating signal is defined as modulation. On this basis of the above parameters, the classification and the equations of corresponding modulations are shown in Fig. 3.2. Further, the waveforms of different modulation techniques are shown in Fig. 3.3.

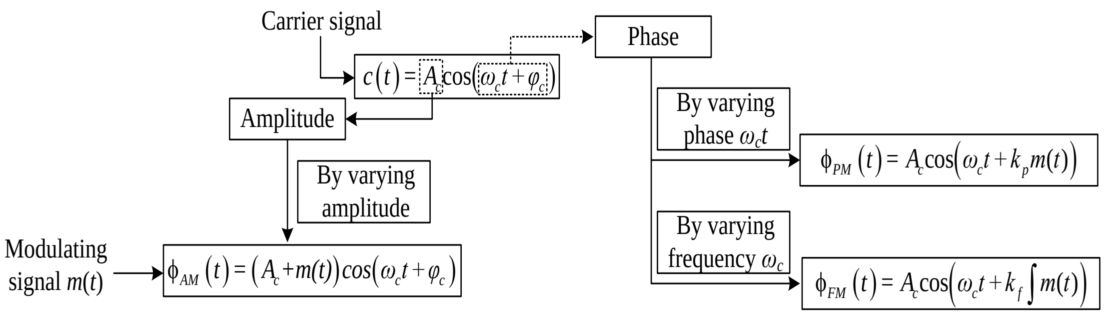


Fig. 3.2 Classification of analog modulation techniques

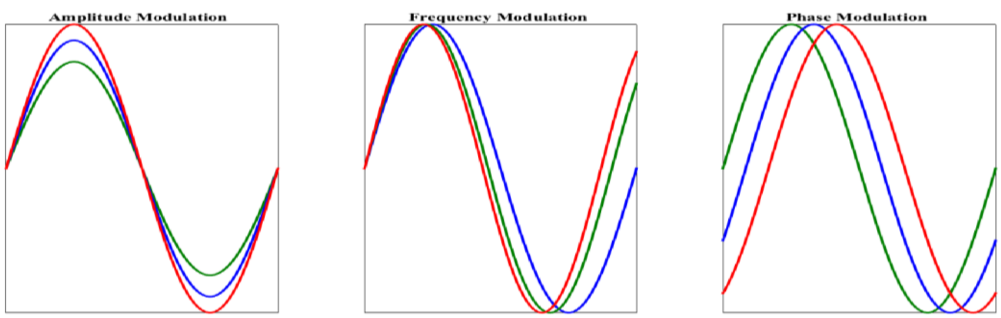


Fig. 3.3 Waveforms of different types of modulations

3.2.2 Pulse Modulation

If the nature of the carrier wave is pulse type, the modulation technique is termed as pulse modulation. In pulse modulation, the modulated signal is transmitted over the channel in the form of pulses. Examples of pulse modulation are PAM, PWM, PPM etc.

3.3 Need for Modulation

The baseband communication is not compatible with direct transmission for long-distance communication. Therefore, the signal strength has to increase for the long-distance transmission in such a way so that the parameters of the modulating signal cannot be affected. Hence, a high-frequency carrier signal is used whose properties (like amplitude, frequency or phase) changes with the instantaneous value of modulating signal to make the long-distance transmission possible. Such types of approaches are called modulation techniques. The needs of the modulation process in the communication system are as follows:

3.3.1 Practicability of Antenna Height

To receive a signal at the receiver end, the antenna height must be the order of the wavelength of that signal (minimum height of antenna must be $\lambda/4$). For example, if a signal having a frequency of 20 kHz is transmitted, then for faithful receiving of that signal, the minimum height of the antenna must be

$$\frac{\lambda}{4}=\frac{c}{4f}=\frac{3\times10^{8}}{4\times20\times10^{3}}=3750 \text{ meter} = 3.75 \text{ km}.$$

So, the height of the antenna has to be very large in baseband communication which is practically not possible.

Now, if this signal is transmitted by the carrier signal of frequency 1 MHz, then the antenna height must be

$$\frac{\lambda}{4}=\frac{c}{4f}=\frac{3\times10^{8}}{4\times1\times10^{6}}=75 \text{ meter}$$

Therefore, the antenna height is reduced to 75 meters with the use of a modulation. Hence, it can be concluded that the height of the antenna is reduced by the modulation.

3.3.2 Multiplexing

Multiplexing is the process of simultaneous transmission of multiple messages over a single channel. It reduces the number of channels as well as the installation and maintenance cost of the overall communication system. The multiplexing techniques are categorized as:

1. Time-division multiplexing (TDM) and
2. Frequency division multiplexing (FDM).

TDM uses pulse modulation system, whereas FDM uses analog modulation system.

3.3.3 Narrowbanding

Assume that the baseband communication system radiates message signals within the frequency range of 100 Hz to 20 kHz. The antenna height must be $c/(4f)=3\times10^{8}/4\times100=750 \text{ km}$ for 100 Hz radiation and 3.75 km for 20 kHz radiation. Therefore, the antenna designed for 100 Hz will be too large for 20 kHz.

Now the signal is modulated with a 2 MHz carrier signal. Then the ratio of highest to lowest frequency components would be

$$\text{Ratio} = \frac{20\times10^{3}+2\times10^{6}}{100+2\times10^{6}}=\frac{2020\times10^{3}}{2000.1\times10^{3}}=1.0099$$

This technique changes the wideband frequency range to narrowband. Therefore, it is called narrowbanding. Some other needs for the modulation are as follows:

- To increase the communication range
- To improve the received signal.
- To allow the bandwidth adjustment.

3.4 Different Types of Signals in Modulation Process

In modulation techniques, the following three types of signals are used:

1. **Modulating (Message) signal**

   The message (information) carrying signal which has to be transmitted, is called message signal. Sometimes this signal is also termed as information signal or baseband signal. The property of the message signal remains unaltered during the modulation process. Usually, the modulating signal is considered as $m(t) = A_m \cos(\omega_m t)$.

2. **Carrier signal**

   The high-frequency signal which changes any of its properties (amplitude, frequency or phase) in accordance with the instantaneous value of modulating signal is called a carrier signal. Usually, the carrier signal is considered as $c(t) = A_c \cos(\omega_c t)$.

3. **Modulated signal**

   The resultant signal obtained by combining the modulating signal and carrier signal is called modulated signal.

In this chapter, different types of amplitude modulations are discussed. The amplitude modulation is categorized as

1. Amplitude modulation with a large carrier
2. Amplitude modulation with the suppressed carrier (AM-SC)
   (i) Double side band suppressed carrier (DSB-SC)
   (ii) Single side band suppressed carrier (SSB-SC)
   (iii)Vestigial side band suppressed carrier (VSB-SC)

3.5 Amplitude Modulation with a Large Carrier

This technique is simply called amplitude modulation (AM). In this chapter, the term AM is referred as amplitude modulation with a large carrier. As the name indicates the term “Amplitude”, in this approach, the amplitude of the carrier signal is varied in accordance with the instantaneous value of modulating signal. Therefore, the modulating signal *m*(*t*) is added with carrier amplitude A_c to generate the AM signal $\phi_{\text{AM}}(t)$, which is represented in the time domain as:

$$\phi_{\text{AM}}(t) = (A_c + m(t))\cos\omega_c t = A_c \cos\omega_c t + m(t)\cos\omega_c t \tag{3.2}$$

where, $A_c \geq |m(t)|_{\max}$

Further, frequency domain representation (or spectrum) of AM signal is obtained by the Fourier transform of the above signal.

The Fourier transform of the first term in AM wave expression of Eq. (3.2) is given by

$$A_c \cos \omega_c t \xleftrightarrow{\text{FT}} \pi A_c \left[\delta(\omega+\omega_c) + \delta(\omega-\omega_c) \right] \tag{3.3}$$

According to the multiplication property of Fourier transform

$$c(t)\times m(t) \xleftrightarrow{\text{FT}} \frac{1}{2\pi}\left[C(\omega)M(\omega)\right]$$

where, $F\left[c(t)\right]\to C(\omega)$ and $F\left[m(t)\right]\to M(\omega)$

So, $$m(t)\times\cos\omega_c t \xrightarrow{\text{FT}} \frac{1}{2\pi}\left[M(\omega)\times\pi\left(\delta(\omega+\omega_c)+\delta(\omega-\omega_c)\right)\right]$$

$$\xrightarrow{\text{FT}} \frac{\pi}{2\pi}\left[M(\omega)\times\left(\delta(\omega+\omega_c)+\delta(\omega-\omega_c)\right)\right]$$

$$\xrightarrow{\text{FT}} \frac{1}{2}\left[M(\omega+\omega_c)+M(\omega-\omega_c)\right] \tag{3.4}$$

The Fourier transform of Eq. (3.2) is obtained by the addition of Eq. (3.3) and Eq. (3.4)

$$\phi_{\text{AM}}(\omega) = \pi A_c\left[\delta(\omega+\omega_c)+\delta(\omega-\omega_c)\right]+\frac{1}{2}\left[M(\omega+\omega_c)+M(\omega-\omega_c)\right] \tag{3.5}$$

The spectrum of conventional AM waves consists of the spectrums of message signal (lower side band and upper side band) as well as a carrier signal, as shown in Fig. 3.4.

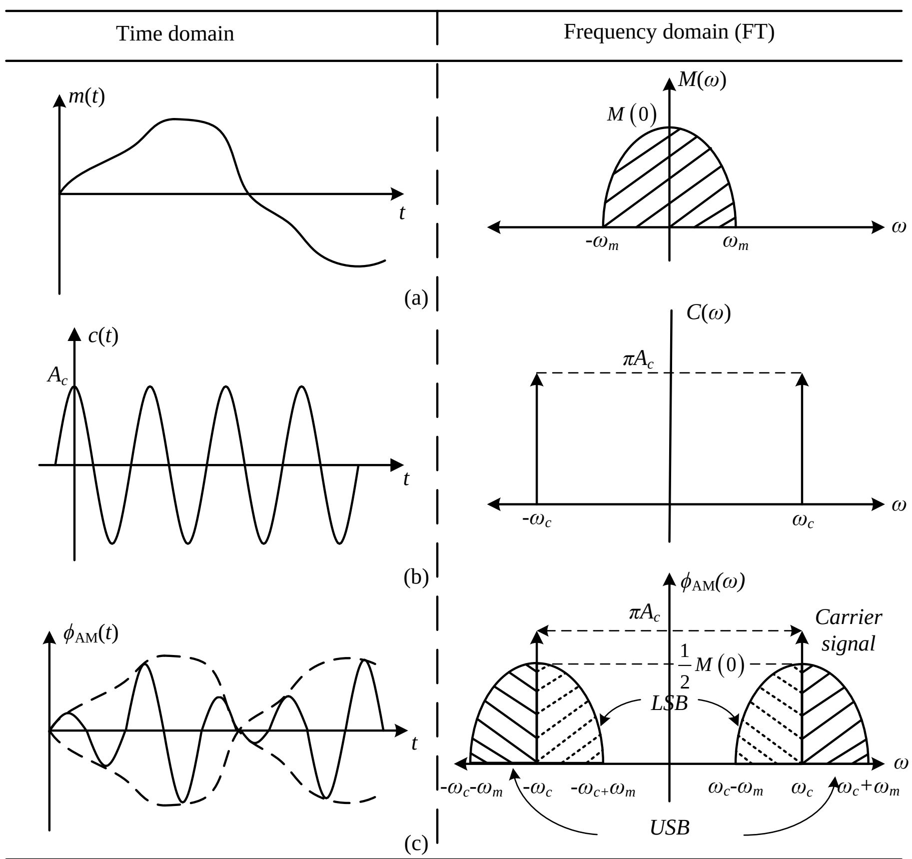


Fig. 3.4 Time domain and corresponding Fourier spectrum of (a) Modulating signal (b) Carrier signal (c) AM signal

3.5.1 Bandwidth of the AM Wave

The spectrum shows that the frequency range of AM wave is from $(\omega_c - \omega_m)$ to $(\omega_c + \omega_m)$. So, the bandwidth of the AM wave is given by

$$(\text{BW})_{\text{AM}} = (\omega_c + \omega_m) - (\omega_c - \omega_m) = 2\omega_m \tag{3.6}$$

where, ω_m is the highest frequency component present in the modulating signal. It is concluded that the bandwidth of the AM wave is twice the bandwidth of modulating signal.

3.5.2 Modulation Index

AM signal $\phi_{\text{AM}}(t)$, which is represented in the time domain as:

$$\begin{aligned}\phi_{\text{AM}}(t) &= (A_c + A_m \cos\omega_m t)\cos\omega_c t \\ &= A_c\left(1 + \frac{A_m}{A_c}\cos\omega_m t\right)\cos\omega_c t \\ &= A_c(1 + m_a \cos\omega_m t)\cos\omega_c t\end{aligned} \tag{3.7}$$

where, m_a is called modulation index.

Therefore, the modulation index m_a is defined as the ratio of maximum amplitude of the modulating signal to the amplitude of the carrier signal and given as

$$m_a = \frac{|m(t)|_{\max}}{A_c} \tag{3.8}$$

The modulation index m_a for experimental output, as shown in Fig. 3.4(d), is

$$m_a = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}} \tag{3.9}$$

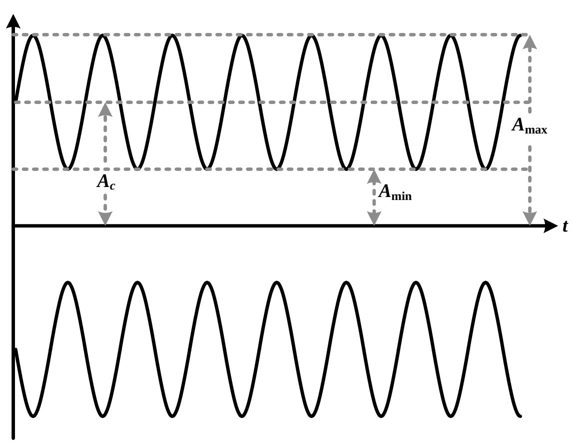


Fig. 3.4 (d) Experimental AM waveform

The modulation index is the extent of amplitude variation in AM wave about a carrier wave. The modulation index is also known as modulation factor or degree of modulation or depth of modulation.

Normally, $\left|m(t)\right|_{\max} \le A_c$, so, m_a is always less than or equal to unity. However, if $m_a > 1$, the envelope is distorted and the baseband signal is no longer preserved in the envelope of the modulated signal. Such type of distortion is called envelope distortion, which occurs with an AM signal with $m_a > 1\left(m_a > 100\%\right)$ as shown in Fig. 3.5.

3.5.3 Power Contents of AM Wave

The AM wave is given as

$$\phi_{\text{AM}}(t) = \underbrace{A_c \cos\omega_c t}_{\text{Carrier part}} + \underbrace{m(t)\cos\omega_c t}_{\text{Message part}} \tag{3.10}$$

Therefore, total power P_T in AM wave consists of carrier signal power P_c (first term in the above expression) and sidebands power P_s (second term in the expression), i.e. total power P_T is

$$P_T = P_c + P_s \tag{3.11}$$

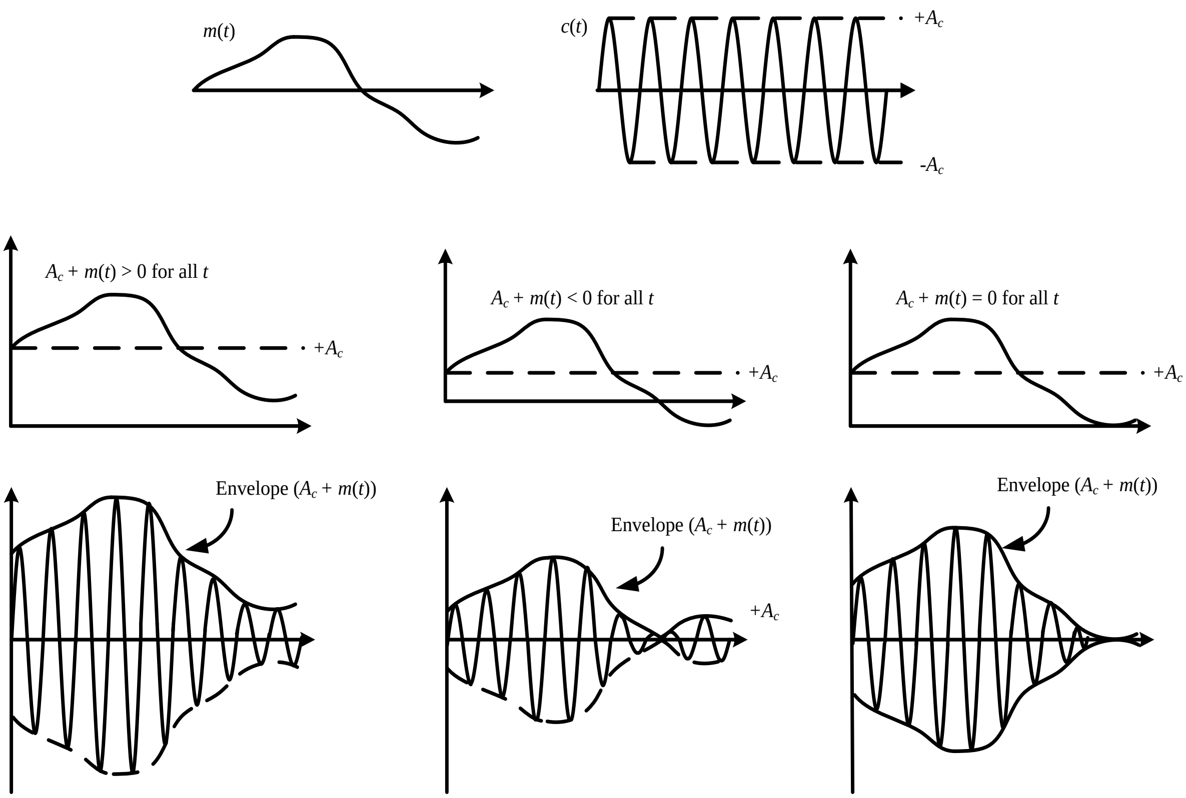


Fig. 3.5 Envelope distortion

3.5.3.1 Carrier Power

The power of any signal is the mean square value of that signal. So, carrier power P_c in AM wave is given by

$$P_c = \overline{\left[A_c \cos\omega_c t\right]^2} = \frac{A_c^2}{2} \tag{3.12}$$

3.5.3.2 Sidebands' Power

Like carrier power, the power contents in the sideband P_s is given by

$$P_s = \overline{\left[m(t)\cos\omega_c t\right]^2} = \frac{\overline{m^2(t)}}{2} \tag{3.13}$$

Total power P_T is

$$P_T = \frac{A_c^2}{2} + \frac{\overline{m^2(t)}}{2} \tag{3.14}$$

If the message signal is single tone as $m(t) = A_m \cos\omega_m t$, then $\overline{m^2(t)}$ is given as

$$\overline{m^2(t)} = \overline{(A_m \cos\omega_m t)^2} = \frac{A_m^2}{2} \tag{3.15}$$

As we know, the complete sideband powers in AM wave is the sum of lower sideband (LSB) and upper sideband (USB) powers. Therefore, individual sideband's power is expressed as

$$P_{USB} = P_{LSB} = \frac{1}{2}P_s = \frac{\overline{m^2(t)}}{4} \tag{3.16}$$

where, P_{USB} and P_{LSB} are upper sideband power and lower side band power, respectively.

Substitute the value from Eq. (3.15) into Eq. (3.13), the total sideband power is

$$P_s = \frac{A_m^2}{4} \tag{3.17}$$

Therefore, USB and LSB powers are

$$P_{USB} = P_{LSB} = \frac{1}{2}P_s = \frac{A_m^2}{8} \tag{3.18}$$

Substituting the value of $\overline{m^2(t)}$ from Eq. (3.15) into Eq. (3.14), total power P_T is

$$P_T = \frac{A_c^2}{2} + \frac{A_m^2}{4} = \frac{A_c^2}{2}\left(1 + \frac{A_m^2}{2A_c^2}\right)$$

$$P_T = P_c\left(1 + \frac{m_a^2}{2}\right) \tag{3.19}$$

From Eq. (3.11) and Eq. (3.19), sidebands power P_s is

$$P_s = P_c \frac{m_a^2}{2} \tag{3.20}$$

Therefore, USB and LSB powers are

$$P_{USB} = P_{LSB} = \frac{1}{2}P_s = P_c \frac{m_a^2}{4} \tag{3.21}$$

3.5.3.3 Power Contents in Multiple Tone AM Wave

If the message signal consists of more than one frequency component, then the modulation is called multiple tone modulation.

Let the expression of multiple tones modulating signal is

$$m(t) = A_{m1}\cos\omega_{m1}t + A_{m2}\cos\omega_{m2}t + A_{m3}\cos\omega_{m3}t \tag{3.22}$$

The expression for multiple tone AM wave is

$$\phi_{\text{AM}}(t) = \left(A_c + m(t)\right)\cos\omega_c t$$

$$= \left(A_c + A_{m1}\cos\omega_{m1}t + A_m\cos\omega_{m2}t + A_{m3}\cos\omega_{m3}t\right)\cos\omega_c t \tag{3.23}$$

$$= A_c\left(1 + \frac{A_{m1}}{A_c}\cos\omega_{m1}t + \frac{A_{m2}}{A_c}\cos\omega_{m2}t + \frac{A_{m3}}{A_c}\cos\omega_{m3}t\right)\cos\omega_c t$$

$$= A_c\left(1 + m_{a1}\cos\omega_{m1}t + m_{a2}\cos\omega_{m2}t + m_{a3}\cos\omega_{m3}t\right)\cos\omega_c t \tag{3.24}$$

where, m_{a1}, m_{a2} and m_{a3} are modulation indices of corresponding frequency components signals.

Total power P_T is

$$P_T = P_c + P_s \tag{3.25}$$

The carrier power $P_c = \overline{\left[A_c\cos\omega_c t\right]^2} = \frac{A_c^2}{2}$

Sideband power P_s is given by

$$P_s = \overline{\left[m(t)\cos\omega_c t\right]^2} = \frac{\overline{m^2(t)}}{2} = \frac{1}{2}\left[\overline{\left(A_{m1}\cos\omega_{m1}t\right)^2} + \overline{\left(A_{m2}\cos\omega_{m2}t\right)^2} + \overline{\left(A_{m3}\cos\omega_{m3}t\right)^2}\right] \tag{3.26}$$

In other way, from Eq. (3.24)

$$P_s = \frac{1}{2}\left[\overline{\left(m_{a1}A_c\cos\omega_{m1}t\right)^2} + \overline{\left(m_{a2}A_c\cos\omega_{m2}t\right)^2} + \overline{\left(m_{a3}A_c\cos\omega_{m3}t\right)^2}\right]$$

$$\Rightarrow P_s = \frac{\left(m_{a1}A_c\right)^2}{4} + \frac{\left(m_{a2}A_c\right)^2}{4} + \frac{\left(m_{a2}A_c\right)^2}{4} = \frac{1}{4}A_c^2\left[m_{a1}^2 + m_{a2}^2 + m_{a3}^2\right] \tag{3.27}$$

Total power P_T is

$$P_T = P_c + P_s = \frac{A_c^2}{2} + \frac{1}{4}A_c^2\left[m_{a1}^2 + m_{a2}^2 + m_{a3}^2\right] = \frac{A_c^2}{2}\left[1 + \frac{m_{a1}^2 + m_{a2}^2 + m_{a3}^2}{2}\right] \tag{3.28}$$

Therefore, generalized power equation for modulating signal with *n*-frequency components

$$P_T = P_c\left[1 + \frac{m_{a1}^2 + m_{a2}^2 + m_{a3}^2 + \text{........}m_{an}^2}{2}\right] \tag{3.29}$$

Compare Eq. (3.29) with Eq. (3.19); the net modulation index m_{aT} is given as

$$m_{aT}^2 = m_{a1}^2 + m_{a2}^2 + m_{a3}^2 + ...m_{an}^2$$

$$\Rightarrow m_{aT} = \sqrt{m_{a1}^2 + m_{a2}^2 + m_{a3}^2 + ...m_{an}^2} \tag{3.30}$$

3.5.3.4 Current Calculation in AM wave

Let R is the resistance of transmitting antenna and I_T and I_c are RMS values of modulated and unmodulated currents, respectively. Then

$$P_T = I_T^2 R \quad \text{and} \quad P_c = I_c^2 R \tag{3.31}$$

Substituting the values from Eq. (3.31) into Eq. (3.19)

$$I_T^2 R = I_c^2 R \left(1 + \frac{m_a^2}{2}\right)$$

$$I_T = I_c \left(1 + \frac{m_a^2}{2}\right)^{1/2} \tag{3.32}$$

3.5.4 Transmission Efficiency

As previously explained, the AM signal consists of two parts 1) carrier signal part and 2) modulating signal part. However, the information is carried only by the modulating signal part. Therefore, the useful power in the AM wave is only with that part which carried the message term. The carrier power in the transmitted message is waste from the transmission point of view. So, the transmission efficiency η of the AM is given as

$$\eta = \frac{P_s}{P_T} = \frac{\dfrac{\overline{m^2(t)}}{2}}{\dfrac{A_c^2}{2} + \dfrac{\overline{m^2(t)}}{2}} = \frac{\overline{m^2(t)}}{A_c^2 + \overline{m^2(t)}} \tag{3.33}$$

If $m(t) = A_m \cos \omega_m t$, then $\overline{m^2(t)} = \frac{A_m^2}{2}$. Then efficiency η is

$$\eta = \frac{\dfrac{A_m^2}{2}}{A_c^2 + \dfrac{A_m^2}{2}}$$

$$= \frac{A_m^2 / A_c^2}{2 + A_m^2 / A_c^2} \qquad \left(\because \quad \frac{A_m}{A_c} = m_a\right)$$

$$\Rightarrow \eta = \frac{m_a^2}{2 + m_a^2} \tag{3.34}$$

Since the maximum value of modulation index $m_a = 1$

Hence, the maximum efficiency $\eta_{\max}$ is

$$\eta_{\max} = \frac{1}{2+1} = \frac{1}{3}$$

Or

$$\eta_{\max}(\%) = \frac{1}{3} \times 100 = 33.33\%$$

Therefore, the maximum transmission efficiency of the AM is only 33.33%.

SE3.1 For an AM, the amplitude of modulating signal is 0.75 V, and carrier amplitude is 1.5 V. Find modulation index.

Sol: Given data:

The amplitude of modulating wave $A_m = 0.75\ \text{V}$

The amplitude of carrier wave $A_c = 1.5\ \text{V}$

Modulation index $m_a = ?$

Since, $m_a = \frac{A_m}{A_c}$

So, $m_a = \frac{0.75}{1.5} = 0.5$

SE3.2 A 400 W(P_c) carrier is modulated to a depth of 75%. Calculate the total power (P_T) in the modulated wave? **(IES: 2004)**

Sol: Given data:

Carrier power $P_c = 400\ \text{W}$

Modulation percentage $m_a(\%) = 75\% \Rightarrow m_a = 0.75$

To calculate:

Total power content $P_T = ?$

Total power of AM wave is given by

$$P_T = P_c\left(1 + \frac{m_a^2}{2}\right)$$

$$= 400\left(1 + \frac{0.75^2}{2}\right)$$

$$= 400 \times \frac{2.5625}{2} = 512.5\ \text{W}$$

SE3.3 The total power content of an AM signal is 1000 W. Determine the power being transmitted at the carrier frequency and at each sideband when modulation percentage is 100%.

Sol: Given data: Total power content $P_T = 1000 \text{ W}$

Modulation percentage $m_a(\%) = 100\% \Rightarrow m_a = 1$

To calculate:

Carrier power $P_c = ?$

Each sidebands power $P_{USB}, P_{LSB} = ?$

Since total power of AM wave is given by

$$P_T = P_c\left(1+\frac{m_a^2}{2}\right) = \underbrace{P_c}_{\text{carrier power}} + \underbrace{\frac{m_a^2}{2}P_c}_{\text{Total sidebands power}}$$

Putting the value of P_c and m_a in the above equation

$$1000 = P_c\left(1+\frac{1^2}{2}\right) = \frac{3}{2}P_c$$

$$\Rightarrow P_c = \frac{2}{3}\times 1000 = \frac{2000}{3} = 666.67 \text{ W}$$

The total sidebands power is given by

$$P_s = \frac{m_a^2}{2}P_c = \frac{1}{2}\times\frac{2000}{3} = \frac{1000}{3} \text{ W}$$

Each sidebands power is given by

$$P_{USB} = P_{LSB} = \frac{1}{2}P_s = \frac{1}{2}\times\frac{1000}{3} = \frac{500}{3} = 166.67 \text{ W}$$

SE3.4 A 500 W, 100 kHz carrier is modulated to a depth of 60% by modulating a frequency of 1 kHz. Calculate the total power transmitted. What are the sideband components of AM wave?

Sol: Given data: Carrier power $P_c = 500 \text{ W}$

Modulation percentage $m_a(\%) = 60\% \Rightarrow m_a = 0.6$

Carrier frequency $f_c = 100 \text{ kHz}$

Modulating frequency $f_m = 1 \text{ kHz}$

To calculate:

Total power content $P_T = ?$

Each sidebands components $f_{USB}, f_{LSB} = ?$

Since total power of AM wave is given by

$$P_T = P_c\left(1+\frac{m_a^2}{2}\right)$$

Putting the value of P_c and m_a in the above equation

$$P_T = 500\left(1+\frac{0.6^2}{2}\right) = 500\times 1.18 = 590\text{ W}$$

The upper and lower sidebands are obtained at $\left(f_c \pm f_m\right)$

So, $f_{USB} = f_c + f_m = 100+1 = 101\text{ kHz}$

$$f_{LSB} = f_c - f_m = 100-1 = 99\text{ kHz}$$

SE3.5 The antenna current of an AM transmitter is 8 A if only the carrier is sent, but it increases to 8.93A if the carrier is modulated by a single sinusoidal wave. Determine the percentage modulation. Also, find the antenna current if the percent of modulation changes to 0.8.

(UPTU: 2004-05)

Sol: Given data:

Unmodulated carrier current	$I_c = 8\text{A}$
Modulated carrier current	$I_T = 8.93\text{A}$

To calculate:

Modulation index	$m_a = ?$

Since, $I_T = I_c\left(1+\frac{m_a^2}{2}\right)^{1/2}$

So, $8.93 = 8\left(1+\frac{m_a^2}{2}\right)^{1/2}$

$$m_a^2 = \left[\left(\frac{8.93}{8}\right)^2 - 1\right]\times 2 = 0.492$$

$$\Rightarrow m_a = \sqrt{0.492} = 0.7014$$

$$\Rightarrow m_a\left(\%\right) = 70.14\%$$

Now, m_a changes to 0.8. So, modulated carrier current is

$$I_T = I_c\left(1+\frac{m_a^2}{2}\right)^{1/2} = 8\times\left(1+\frac{0.8^2}{2}\right)^{1/2}$$

$$\Rightarrow I_T = 9.191\text{A}$$

3.5.5 Generation of the AM Wave

The carrier signal is modulated with the help of a modulator. Simply we can say that the modulator is a circuit configuration which is used to modulate the carrier wave by the input of modulating signal (Fig. 3.6(a)).

The demodulator performs the inverse operation of the modulator. Modulated signal acts as input to the demodulator, and the demodulated signal or message signal is recovered at the output of the demodulator (Fig. 3.6(b)).

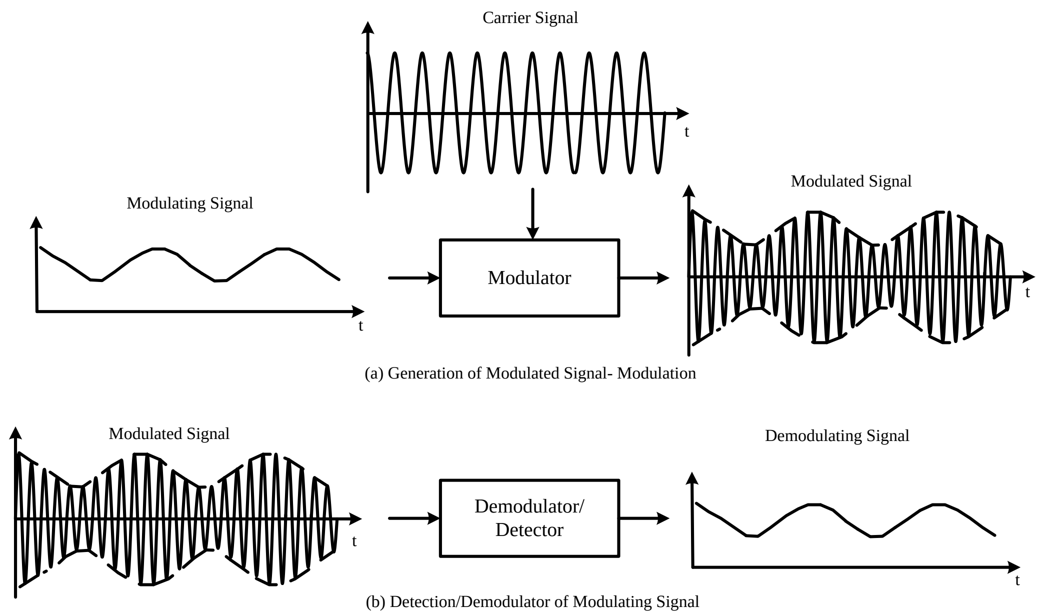


Fig. 3.6 Function of (a) Modulator (b) Demodulator/Detector

The AM wave is generated by the following two types of modulators

1. Square law modulator
2. Switching modulator

3.5.5.1 Square Law Modulator

The devices used in the square law modulator shows the non-linear transfer characteristics and follows the relation as

$$e_o(t) = a_1 e_i(t) + a_2 e_i^2(t) \tag{3.35}$$

where, $e_i(t)$ and $e_o(t)$ are input and output of the device, respectively.

The block diagram of the square-law modulator is presented in Fig. 3.7.

where, $e_i(t) = A_c \cos\omega_c t + m(t)$

The output of the square-law modulator is given as

$$e_o(t) = a_1\left[m(t) + A_c\cos\omega_c t\right] + a_2\left[m(t) + A_c\cos\omega_c t\right]^2$$

$$= a_1 m(t) + a_1 A_c\cos\omega_c t + a_2 m^2(t) + 2a_2 m(t) A_c\cos\omega_c t + a_2 A_c^2\cos^2\omega_c t \quad (3.36)$$

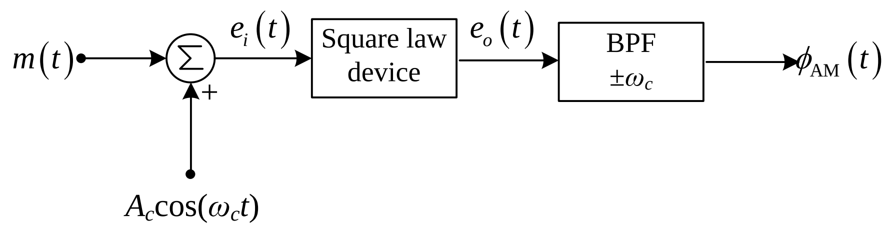


Fig. 3.7 Block diagram of square-law modulator for modulation

Rearrange the equation

$$e_o(t) = a_1 m(t) + a_2 A_c^2\cos^2\omega_c t + a_2 m^2(t) + a_1 A_c\cos\omega_c t + 2a_2 m(t) A_c\cos\omega_c t$$

$$= \underbrace{a_1 m(t) + a_2 A_c^2\cos^2\omega_c t + a_2 m^2(t)}_{\text{Unwanted terms}} + \underbrace{a_1 A_c\left[1 + \frac{2a_2}{a_1}m(t)\right]\cos\omega_c t}_{\text{Amplitude modulated wave}} \quad (3.37)$$

The first three terms are unwanted signals which are filtered out by a BPF centred around $\pm\omega_c$. Therefore, the output of BPF is given as

$$\phi_{AM}(t) = a_1 A_c\left[1 + \frac{2a_2}{a_1}m(t)\right]\cos\omega_c t \quad (3.38)$$

The Eq. (3.38) is similar to the expression of standard AM wave given in Eq. (3.7) with $m_a = \frac{2a_2}{a_1}$.

3.5.5.2 Switching Modulator

The diagram of the switching modulator is presented in Fig. 3.8. Instead of modulating signal $m(t)$ alone, the summation of carrier and modulating signal $A_c\cos\omega_c t + m(t)$ is considered as the input signal to the modulator. The switching action of the diode is considered as the pulse waveform $p_o(t)$. So, the output of the diode is given as

$$e_o(t) = \left[A_c\cos\omega_c t + m(t)\right]p_o(t)$$

$$= \left[A_c\cos\omega_c t + m(t)\right]\left[\frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - ....\right)\right]$$

$$= \underbrace{\frac{1}{2}A_c\cos\omega_c t + \frac{2}{\pi}m(t)\cos\omega_c t}_{\text{AM wave}} + \underbrace{\text{other terms}}_{\text{Suppressed by BPF}} \quad (3.39)$$

So, the output of BPF is given by

$$\phi_{\text{AM}}(t) = \frac{1}{2}A_c\left[1+\frac{4}{\pi A_c}m(t)\right]\cos\omega_c t \tag{3.40}$$

The Eq. (3.40) is similar to the expression of the standard AM wave given in Eq. (3.7) with

$$m_a = \frac{4}{\pi A_c}$$

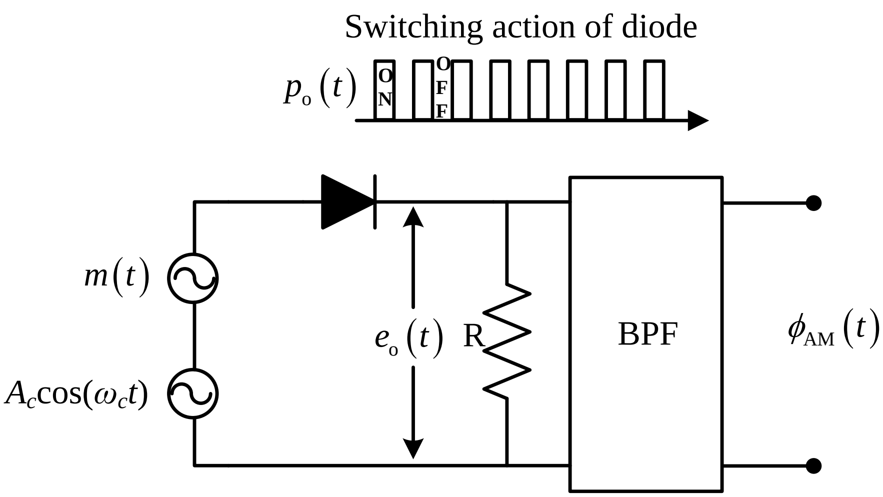


Fig. 3.8 Switching modulator for AM wave generation

3.5.6 Detection/Demodulation of AM Wave

Demodulation is a method of baseband signal extraction from the modulated signal. Two important methods of AM demodulation are as follows

1. Non-coherent or Envelope demodulation/detection.
2. Coherent or Synchronised demodulation/detection.

3.5.6.1 Envelope or Non-Coherent Detection/Demodulation

As shown in Fig. 3.9, the envelope, either in the positive or negative direction of the AM wave, is modulating signal. The linear region characteristics of the diode are used to extract the envelope of an AM wave. Such configuration of the detector is called envelope detector (Fig. 3.9). The working operation of the envelope detector is as follows:

The diode conducts only for the positive direction of the AM wave. Therefore, the negative part of the AM wave is clipped off. The negative clipped waveform of the AM wave is obtained at point *a*. This waveform still consists of modulating signals in its envelope. Further, for one cycle of the waveform, capacitor *C* charges towards the peak of the modulated waveform. Now, the AM signal starts decaying below the peak value. Since the diode is reverse biased, therefore, the capacitor discharges slowly through the resistor with the time constant *RC*. The capacitor repeats the same process of charging and discharging for each positive cycle, as shown in Fig. 3.9.

The capacitor discharges between the positive peaks of the modulated waveform, which results in a ripple signal at the output with frequency ω_c (carrier frequency). However, the ripple content may be minimized by keeping the value of time constant *RC* high enough to make the discharging rate of the capacitor slows $\left(\text{i.e. } RC >> \frac{1}{\omega_c}\right)$. On the other hand, if the value of *RC* is kept very high, it is impossible for the detector to follows the envelope of the signal. Therefore, the value of *RC* must be greater than $1/\omega_c$ but should be less than $1/\omega_m$ where ω_m is the highest frequency component present in modulating signal.

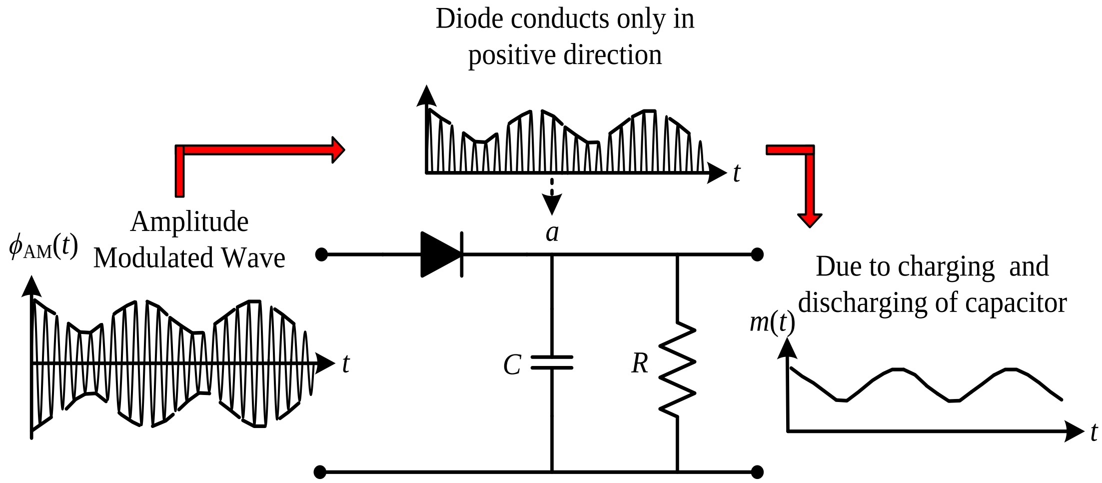


Fig. 3.9 Envelope detector

Therefore, the value of time constant *RC* for proper envelope detection should follow the condition given by Eq. (3.41)

$$\frac{1}{\omega_c} << RC << \frac{1}{\omega_m} \tag{3.41}$$

Envelope detector output may be suffered by two types of distortions:

1. **Diagonal Clipping**

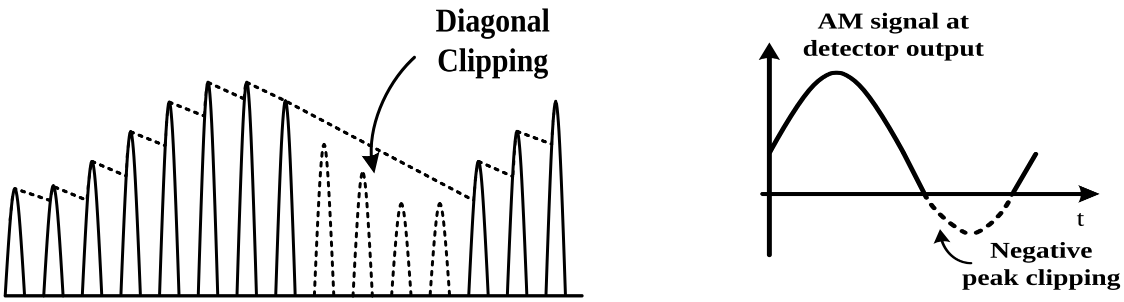
If the discharging rate of the capacitor is less than the decay rate of the slope of the modulating signal, the recovered signal will miss some peaks. Such a condition is called diagonal clipping, as shown in Fig. 3.10(a).

2. **Negative Peak Clipping**

Negative peak clipping happens due to overmodulation, as shown in Fig. 3.10(b).

To avoid the diagonal clipping and to recover the modulating signal faithfully from the modulated signal, the following condition must be satisfied

The rate of discharge of capacitor ≥ rate of decrease of the modulation envelope



(a) (b)

Fig. 3.10 (a) Diagonal clipping (b) Negative peak clipping

Let the AM wave is given as

$$\phi_{\text{AM}}(t) = A_c\left(1+m_a\cos\omega_m t\right)\cos\omega_c t \tag{3.42}$$

Therefore, the envelope A_{en} of the above signal is

$$A_{en} = A_c\left(1+m_a\cos\omega_m t\right) \tag{3.43}$$

So, the rate of change of slope of the envelope is given as

$$-\frac{dA_{en}}{dt} = A_c m_a \omega_m \sin\omega_m t \tag{3.44}$$

The slope at any time instant $t = t_o$ is given as

$$-\left(\frac{dA_{en}}{dt}\right)_{t=t_o} = A_c m_a \omega_m \sin\omega_m t_o \tag{3.45}$$

At any time $t = t_o$, the value of the envelope is obtained by substituting $t = t_o$ in Eq. (3.43)

$$A_{en,o} = A_c\left(1+m_a\cos\omega_m t_o\right) \tag{3.46}$$

The capacitor starts discharging at time $t = t_o$. So, the capacitor voltage at $t = t_o$

$$A_{en,o} = (A_{en,o})_{t=t_o} = A_{en,o}e^{-\frac{t-t_o}{RC}} \tag{3.47}$$

Let the capacitor starts discharging from time $t = t_o$, the rate of discharge of the capacitor is given by

$$\begin{aligned} -\frac{dA_{cap}}{dt} &= -\frac{d}{dt}\left[A_{en,o}e^{-\frac{t-t_o}{RC}}\right] \\ &= \frac{A_{en,0}}{RC}e^{-\frac{t-t_0}{RC}} \end{aligned} \tag{3.48}$$

The rate of change at any time $t = t_o$

$$-\left(\frac{dA_{cap}}{dt}\right)_{t=t_0} = \frac{A_{en,o}}{RC} \tag{3.49}$$

Substituting the value of $A_{en,o}$ from Eq. (3.46) into Eq. (3.49)

$$-\left(\frac{dA_{cap}}{dt}\right)_{t=t_0} = \frac{A_c}{RC}\left[1+m_a\cos\omega_m t_o\right] \tag{3.50}$$

For faithful recovery

The rate of discharge of capacitor ≥ rate of decrease of the modulation envelope. So,

$$\begin{aligned}&\frac{A_c}{RC}\left[1+m_a\cos\omega_m t_o\right]\ge A_c m_a\omega_m\sin\omega_m t_o\\&\frac{1}{RC}\ge\frac{m_a\omega_m\sin\omega_m t_o}{1+m_a\cos\omega_m t_o}\end{aligned} \tag{3.51}$$

For maximization of *RC*, differentiate RHS and equate it to zero.

$$\frac{d}{dt}\left(\frac{m_a\omega_m\sin\omega_m t}{1+m_a\cos\omega_m t}\right)=0 \tag{3.52}$$

After solving the above expression

$$\Rightarrow\begin{cases}\cos\omega_m t=-m_a\\ \sin\omega_m t=\sqrt{1-m_a^2}\end{cases} \tag{3.53}$$

Substituting the values from Eq. (3.53) into Eq. (3.51)

$$\frac{1}{RC}\ge\omega_m\frac{m_a\sqrt{1-m_a^2}}{1-m_a^2} \tag{3.54}$$

$$\frac{1}{RC}\ge\frac{\omega_m m_a}{\sqrt{1-m_a^2}}$$

Or (3.55)

$$RC\le\frac{1}{\omega_m}\left(\frac{\sqrt{1-m_a^2}}{m_a}\right)$$

The above expression shows the upper limit of time constant *RC* for faithful recovery of modulating signal.

3.5.6.2 Square Law Detector

Square law detectors are used for low level modulating signals. Like square law modulator, the detector also uses the same devices of nonlinear transfer characteristics with input and output relation as

$$e_o(t)=a_1e_i(t)+a_2e_i^2(t) \tag{3.56}$$

where, $e_i(t)$ and $e_o(t)$ are input and output of the device, respectively.

The block diagram of the square law detector is shown in Fig. 3.11.

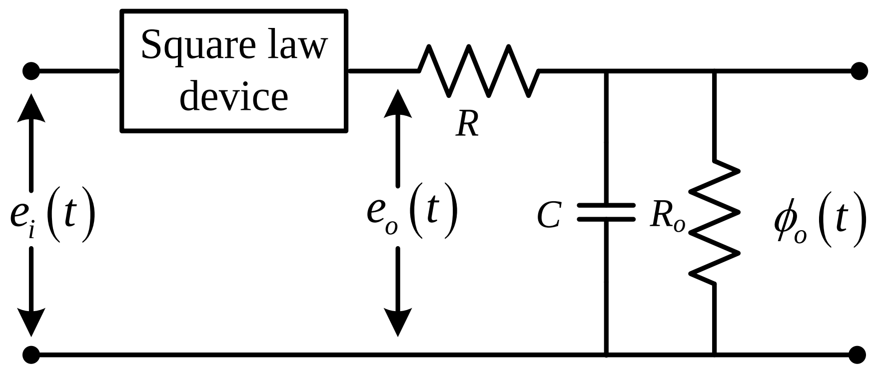


Fig. 3.11 Square law detector

The input of the square law detector is AM wave which is given as

$$e_i(t) = A_c[1 + m_a \cos\omega_m t]\cos\omega_c t \tag{3.57}$$

Substituting the value of $e_i(t)$ from Eq. (3.57) into output expression of Eq. (3.56)

$$\begin{aligned} e_o(t) &= a_1\left[A_c(1+m_a\cos\omega_m t)\cos\omega_c t\right] + a_2\left[A_c(1+m_a\cos\omega_m t)\cos\omega_c t\right]^2 \\ &= a_1\left[A_c(1+m_a\cos\omega_m t)\cos\omega_c t\right] + a_2A_c^2(1+m_a\cos\omega_m t)^2\cos^2\omega_c t \\ &= a_1\left[A_c(1+m_a\cos\omega_m t)\cos\omega_c t\right] + \frac{a_2A_c^2}{2}\left(1+\underbrace{2m_a\cos\omega_m t}_{\text{Desired term}} + m_a^2\cos^2\omega_m t\right)(1+\cos 2\omega_c t) \end{aligned} \tag{3.58}$$

The desired term is the low-frequency component of modulating frequency, i.e. $a_2A_c^2m_a\cos\omega_m t$ term. This term is obtained by passing the signal through LPF. The undesired term $\frac{a_2A_c^2}{2}m_a^2\cos^2\omega_m t$ is suppressed by the LPF. So, the output after the LPF is message signal and given as

$$\phi_o(t) = m(t) = a_2A_c^2m_a\cos\omega_m t \tag{3.59}$$

The ratio of the desired signal to the undesired signal is

$$\text{Ratio} = \frac{\text{Desired term}}{\text{Undesired term}} = \frac{a_2A_c^2m_a\cos\omega_m t}{\frac{a_2A_c^2}{2}m_a^2\cos^2\omega_m t} = \frac{2}{m_a\cos\omega_m t} \tag{3.60}$$

The ratio of Eq. (3.60) should be maximum to achieve minimum distortion. This objective is achieved by the minimum value of m_a (i.e. weak AM signal). Therefore, a square law detector is used for low level modulating signals.

3.6 Double Sidebands Suppressed Carrier (DSB-SC)

The transmitted signal in conventional AM wave consists of spectrums of the carrier and the message signals both. For the purpose of transmission, the carrier power is waste as it does not consist of the message part. Therefore, carrier power is suppressed and only sidebands powers are transmitted. This modulation approach is called double sideband suppressed carrier (DSB-SC) modulation.

Since the AM signal is expressed as

$$\begin{aligned}\phi_{\mathrm{AM}}(t) &= \left(A_c + m(t)\right)\cos\omega_c t \\ &= \underbrace{A_c\cos\omega_c t}_{\text{carrier term}} + \underbrace{m(t)\cos\omega_c t}_{\text{sidebands term}}\end{aligned} \qquad (3.61)$$

Therefore, after the suppression of carrier term from Eq. (3.61), the obtained expression is

$$\phi_{\mathrm{DSB}}(t) = m(t)\cos\omega_c t \qquad (3.62)$$

Eq. (3.62) represents the DSB-SC modulated signal. The block diagram of the DSB-SC wave generation is presented in Fig. 3.12. The message (modulating) signal and the carrier signal are multiplied in the product modulator to generate the DSB-SC signal.

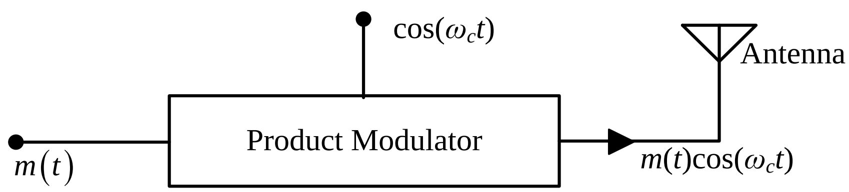


Fig. 3.12 DSB-SC modulator

3.6.1 DSB-SC Modulators

There are three important categories of DSB-SC modulators which are as follows:

1. Multiplier Modulators
2. Nonlinear Modulators
3. Switching Modulators

3.6.1.1 Multiplier Modulator

The DSB-SC signal can be generated by the direct multiplication of the $m(t)$ and $\cos\omega_c t$ using an analog multiplier. But this process is very expensive and does not keep linearity. Therefore, this method is not used for DSB-SC generation.

3.6.1.2 Nonlinear Modulators

The DSC-SC signal is generated by nonlinear devices like diodes or transistors. The input-output characteristics of these devices are approximated by the square law which is given as

$$y(t) = ax(t) + bx^2(t) \qquad (3.63)$$

where, $y(t)$ and $x(t)$ are output and input of the nonlinear device, respectively.

The nonlinear DSB-SC modulator, also called a balanced modulator, is shown in Fig. 3.13. The inputs of the two nonlinear devices are given as

$$x_1(t) = \cos\omega_c t + m(t) \tag{3.64}$$

$$x_2(t) = \cos\omega_c t - m(t) \tag{3.65}$$

The outputs of the nonlinear devices are

$$\begin{aligned} y_1(t) &= a\left[\cos\omega_c t + m(t)\right] + b\left[\cos\omega_c t + m(t)\right]^2 \\ &= a\left[\cos\omega_c t + m(t)\right] + b\left[\cos^2\omega_c t + 2m(t)\cos\omega_c t + m^2(t)\right] \end{aligned} \tag{3.66}$$

$$\begin{aligned} y_2(t) &= a\left[\cos\omega_c t - m(t)\right] + b\left[\cos\omega_c t - m(t)\right]^2 \\ &= a\left[\cos\omega_c t - m(t)\right] + b\left[\cos^2\omega_c t - 2m(t)\cos\omega_c t + m^2(t)\right] \end{aligned} \tag{3.67}$$

The output of the summer is

$$z(t) = y_1(t) - y_2(t) = \underbrace{2am(t)}_{\text{DC term}} + \underbrace{4bm(t)\cos\omega_c t}_{\text{DSB-SC term}} \tag{3.68}$$

The first term of Eq. (3.68) is filtered out by a bandpass filter with center frequency ω_c. The output of BPF is a DSB-SC signal given as

$$\phi_{\text{DSB}}(t) = 4bm(t)\cos\omega_c t \tag{3.69}$$

Although the modulator shown in Fig. 3.13 has two inputs, $m(t)$ and $\cos\omega_c t$ but the output of the summer circuit consists of only a single input $m(t)$. The second input $\cos\omega_c t$ is suppressed by the circuit arrangement. Since this arrangement is balanced for one of the inputs, this modulator is also called a **single balanced modulator**. If both inputs do not appear at the summer output, then it is called **double balanced modulator** (as ring modulator).

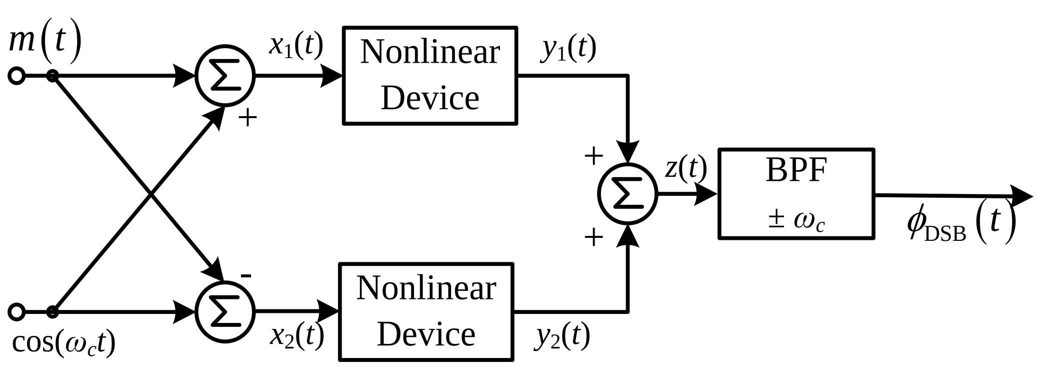


Fig. 3.13 Nonlinear DSB-SC modulator

3.6.1.3 Switching Modulators

The multiplication operation of the DSB-SC generation can be replaced by the switching operation. The ON/OFF or switching operation is represented by a periodic pulse train, as shown in Fig. 3.14. The multiplication of message signal with this periodic signal comes into a form of shifted version of $M(\omega)$ at $\pm\omega_c, \pm 2\omega_c, \pm 3\omega_c, ...$ and so on. Hence, the DSB-SC signal is obtained by the use of a BPF centred around $\pm\omega_c$. There are two types of popular switching modulators

1. Diode bridge modulator
2. Ring modulator

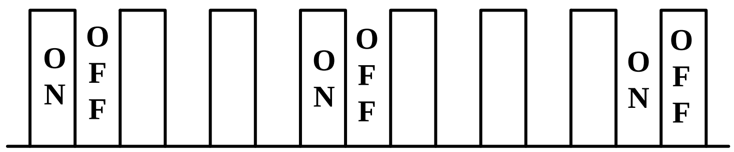


Fig. 3.14 Periodic pulse as the on-off switching operation

Diode Bridge Modulator

The circuit diagram of the diode bridge modulator and its connection arrangement with BPF is shown in Fig. 3.15. According to connection, the diode bridge modulator is categorised into two categories:

1. Shunt bridge diode modulator
2. Series bridge diode modulator

The Fourier series of the pulse waveform is given by

$$p_o(t) = \frac{1}{2} + \frac{2}{\pi}\left[\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - .....\right] \quad (3.70)$$

The multiplied signal is given by

$$\begin{aligned} e_o(t) &= m(t) \times p_o(t) \\ &= \frac{1}{2}m(t) + \frac{2}{\pi}\left[m(t)\cos\omega_c t - \frac{1}{3}m(t)\cos 3\omega_c t + \frac{1}{5}m(t)\cos 5\omega_c t - .....\right] \end{aligned} \quad (3.71)$$

The high-frequency terms and the DC term are filtered out by a BPF centered around $\pm\omega_c$, and hence the obtained output is given by

$$\phi_{\text{DSB}}(t) = \frac{2}{\pi}m(t)\cos\omega_c t \quad (3.72)$$

Since $e_o(t)$ acts as the input of the BPF and consists of only one input signal term; therefore, this type of modulator is an example of a single balanced modulator.

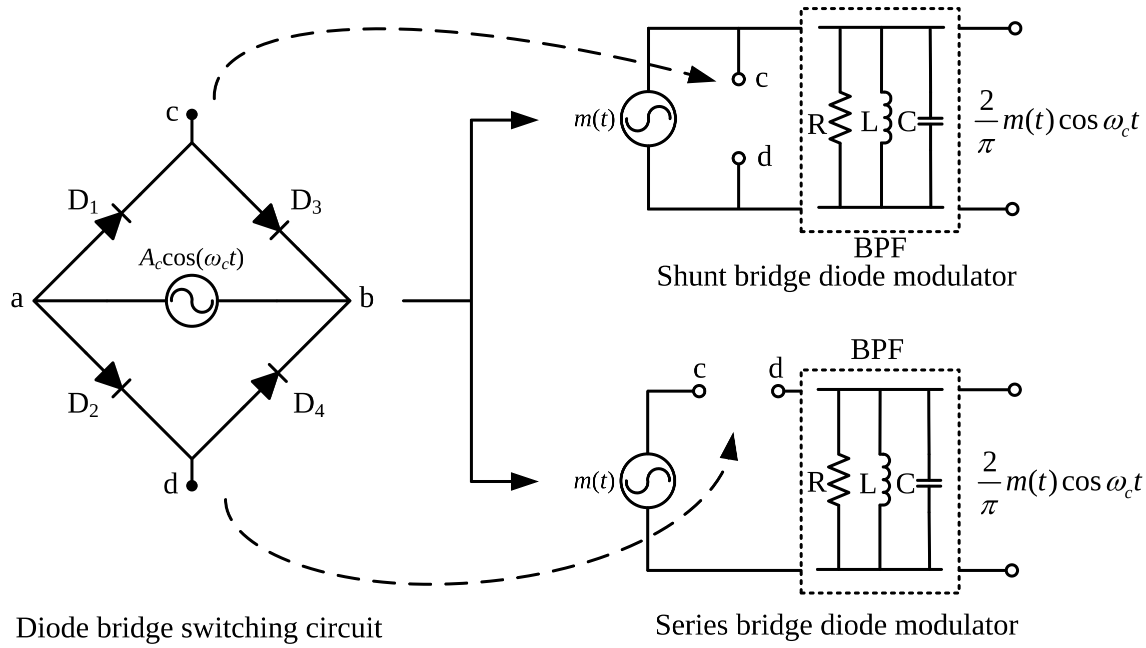


Fig. 3.15 (a) Diode bridge switching circuit (b) Shunt bridge configuration (c) Series bridge configuration

In Fig. 3.15, the diode bridge modulator is driven by the sinusoidal carrier signal. The positive polarity of the carrier makes the terminal '*a*' positive with respect to '*b*'. Therefore, all four diodes conduct. Since diodes D_1 and D_2 are matched, they act as a short circuit for modulating signals. If this bridge is used in shunt bridge mode, the input to the BPF is zero. If the same is used in a series bridge configuration, the input to BPF is $m(t)$. Similarly, for the negative polarity of the carrier, all diodes are in OFF condition. Shunt configuration provides $m(t)$ and series configuration provides '0' input to the BPF. The spectrum of different signals for the diode bridge DSB-SC modulator is shown in Fig. 3.16(a)-(c).

Ring Modulator

A ring modulator is another type of switching modulator. The circuit configuration of the ring modulator is shown in Fig. 3.17. The operation of the ring modulator is as follows:

Carrier signal	Diodes	Terminals	Output
For positive polarity	D_1 & $D_2 \Rightarrow$ ON D_3 & $D_4 \Rightarrow$ OFF	a $\xrightarrow{\text{Connected to}}$ c b $\xrightarrow{\text{Connected to}}$ d	$m(t)$
For negative polarity	D_1 & $D_2 \Rightarrow$ OFF D_3 & $D_4 \Rightarrow$ ON	a $\xrightarrow{\text{Connected to}}$ d b $\xrightarrow{\text{Connected to}}$ c	$-m(t)$

Thus the chopped version of the modulating signal appears at the output. The switching operation is considered as a square wave, as shown in Fig. 3.18.

The Fourier series of the square waveform is given by

$$p_o(t) = \frac{4}{\pi}\left[\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - .....\right] \tag{3.73}$$

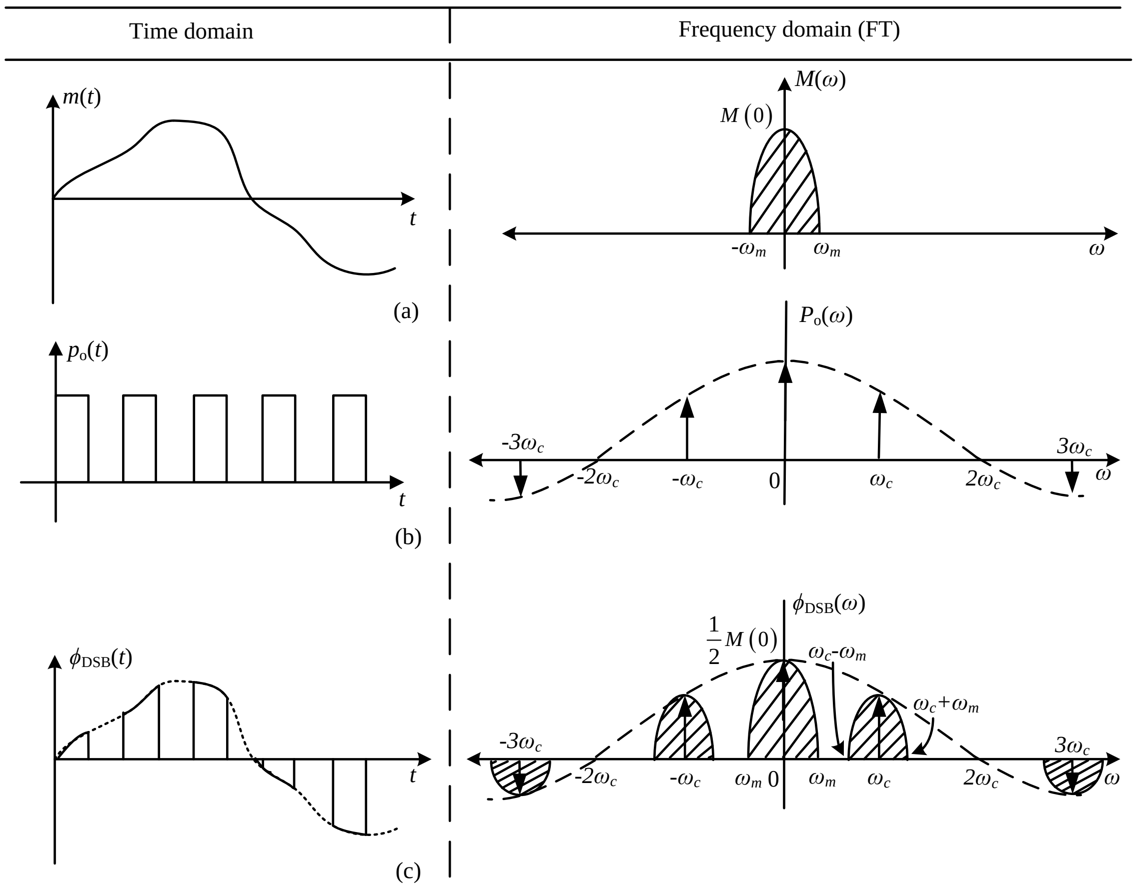


Fig. 3.16 Time domain and corresponding Fourier spectrum of (a) Modulating signal (b) Carrier signal (c) DSB-SC signal for diode bridge modulator

The modulating signal is multiplied with the square waveform and the output after the multiplication is given by

$$e_o(t) = m(t) * p_o(t) = \frac{4}{\pi}\left[m(t)\cos\omega_c t - \frac{1}{3}m(t)\cos 3\omega_c t + \frac{1}{5}m(t)\cos 5\omega_c t - .....\right] \tag{3.74}$$

Above mentioned signal works as the input of the BPF. The input of the BPF does not consist of any of the inputs of the circuit. Therefore, the **ring modulator is an example of a double balanced modulator**.

The above signal is passed through the BPF $(\pm\omega_c)$ to get the DSB-SC signal as

$$\phi_{DSB}(t) = \frac{4}{\pi}m(t)\cos\omega_c t \tag{3.75}$$

The different waveform of DSB-SC generation using ring modulator is shown in Fig. 3.18(a)-(c).

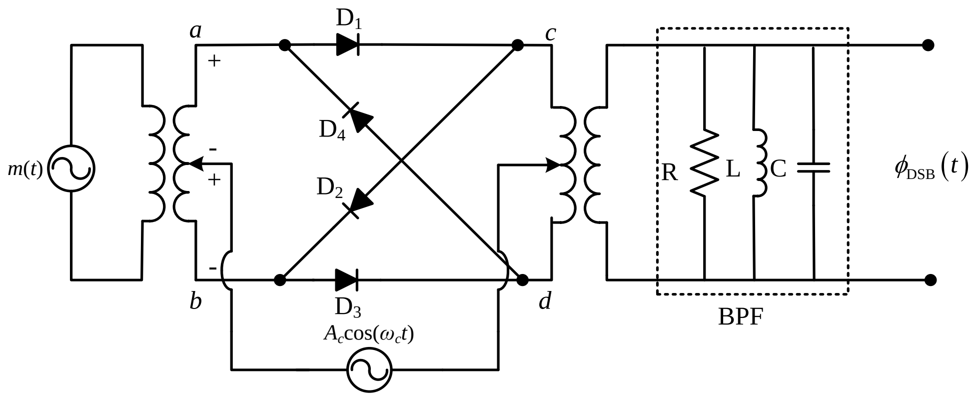


Fig. 3.17 Ring modulator

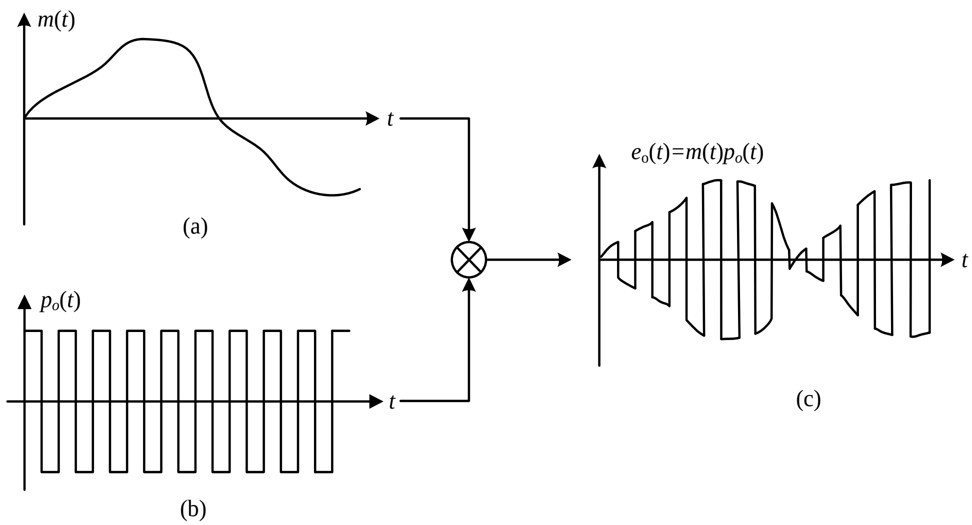


Fig. 3.18 Different waveforms of DSB-SC modulation using ring modulator (a) Modulating signal (b) Switching pattern (c) DSB-SC modulated signal

3.6.2 Demodulation/Detection of the DSB-SC Signal

Synchronous detection or coherent demodulation method is used to recover the message or baseband signal from DSB-SC modulated signal. The block diagram of coherent detection is shown in Fig. 3.19.

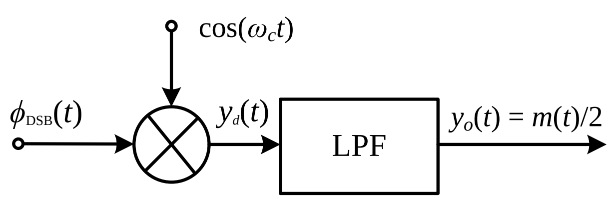


Fig. 3.19 Demodulation of DSB-SC by synchronous (coherent) detection

3.6.2.1 Coherent Demodulation

The locally generated carrier signal is multiplied with the modulated signal to retranslate the spectrum of the DSB-SC signal from $\omega = \pm\omega_c$ to $\omega = 0$. Further, a low pass filter is used to recover the baseband (message) signal.

The output signal $y_d(t)$ of the multiplier network is given as

$$y_d(t) = \phi_{\text{DSB}}(t) * \cos(\omega_c t) \tag{3.76}$$

Substituting the value of $\phi_{\text{DSB}}(t)$

$$y_d(t) = \left[m(t)\cos(\omega_c t)\right]\cos(\omega_c t) = m(t)\cos^2(\omega_c t)$$

$$= \frac{1}{2}m(t)\left[1 + \cos(2\omega_c t)\right]$$

$$= \frac{1}{2}m(t) + \frac{1}{2}m(t)\cos(2\omega_c t) \tag{3.77}$$

Low pass filter allows only low-frequency component signal, i.e., $\frac{1}{2}m(t)$. Therefore, the output of the demodulator is expressed as:

$$y_o(t) = \frac{1}{2}m(t) \tag{3.78}$$

The above expression shows the detected output is message signal with attenuation, but there is no distortion.

3.6.2.2 Effect of Phase and Frequency Errors in Synchronous Detection

The phase and the frequency of the local oscillator generated signal must be identical to the that of transmitted carrier.

Let a modulated DSB-SC signal reaching to the receiver is $m(t)\cos(\omega_c t)$. Assume locally generated carrier has phase error $\Delta\varphi$ and frequency error $\Delta\omega$, respectively. The product of two signals in synchronous detector yields:

$$\begin{aligned} e_d(t) &= m(t)\cos\omega_c t * \cos\left[(\omega_c + \Delta\omega)t + \Delta\varphi\right] \\ &= \frac{1}{2}m(t)\left\{\cos\left[(\Delta\omega)t + \Delta\varphi\right] + \cos\left[(2\omega_c + \Delta\omega)t + \Delta\varphi\right]\right\} \end{aligned} \tag{3.79}$$

When this signal is passed through the LPF, the output will be

$$e_o(t) = \frac{1}{2}m(t)\cos\left[(\Delta\omega)t + \Delta\varphi\right] \tag{3.80}$$

The baseband signal is multiplied with a slow time-varying signal $\cos\left[(\Delta\omega)t + \Delta\varphi\right]$ that distorts the message signal. Let discuss some special cases:

(i) When frequency error $\Delta\omega$ and phase error $\Delta\varphi$ both are zero, then

$$e_o(t) = \frac{1}{2}m(t) \text{ (No distortion in the output.)} \tag{3.81}$$

(ii) When there is only phase error, i.e., $\Delta\omega = 0$ & $\Delta\varphi \neq 0$, then output

$$e_o(t) = \frac{1}{2} m(t) \cos \Delta\varphi \tag{3.82}$$

If $\Delta\varphi$ is time-independent, there is no error, only attenuation. The output is maximum when $\Delta\varphi = 0^o$ and minimum when $\Delta\varphi = 90^o$.

(iii) When there is only frequency error, i. e., $\Delta\omega \neq 0$ & $\Delta\varphi = 0$, then

$$e_o(t) = \frac{1}{2} m(t) \cos \Delta\omega_c t \tag{3.83}$$

Here, the multiplying factor is time-dependent that causes distortion in demodulated or detected output.

(iv) When both errors are not zero, i.e., $\Delta\omega \neq 0$ & $\Delta\varphi \neq 0$. The distorted and attenuated output is obtained at the receiver end.

3.7 Single Sideband Suppressed Carrier (SSB-SC)

Carrier suppression in the DSB-SC signal improved the transmission efficiency, but still, bandwidth is occupied by both USB and LSB. Therefore, there is an unnecessary requirement of bandwidth. Since both sidebands contain the message signal, so only one sideband is sufficient to transmit the signal. Such type of modulation is called single sideband suppressed carrier (SSB-SC) modulation, as shown in Fig. 3.20.

Therefore, SSB-SC modulation is superior to conventional AM and DSB-SC modulations in terms of both transmission efficiency and bandwidth. As far as demodulation is concerned, we can coherently demodulate the SSB-SC signal.

3.7.1 SSB-SC Signal for Single Tone Modulation: Time Domain Description

Let a single tone modulating signal is expressed as

$$m(t) = \cos(\omega_m t) \tag{3.84}$$

The spectrum of this modulating signal is a pair of impulses at $\omega = \pm\omega_m$ as shown in Fig. 3.21(a). Let this signal modulates a carrier $\cos\omega_c t$ and generates the DSB-SC signal with spectrum shown in Fig. 3.21(b). In order to get the SSB-SC from DSB-SC, one of the sidebands is eliminated (either USB or LSB), as shown in Figs. 3.21(c) and 3.21(d).

Let the USB is eliminated, then the spectrum of SSB-SC consists of impulse function at $\omega = \pm(\omega_c - \omega_m)$ which is the spectrum of $\cos(\omega_c - \omega_m)t$ (Fig. 3.21(c)). Therefore, the expression of SSB-SC with LSB is

$$\phi_{\mathrm{SSB_L}}(t) = \cos(\omega_c - \omega_m)t = \cos(\omega_c t)\cos(\omega_m t) + \sin(\omega_c t)\sin(\omega_m t) \tag{3.85}$$

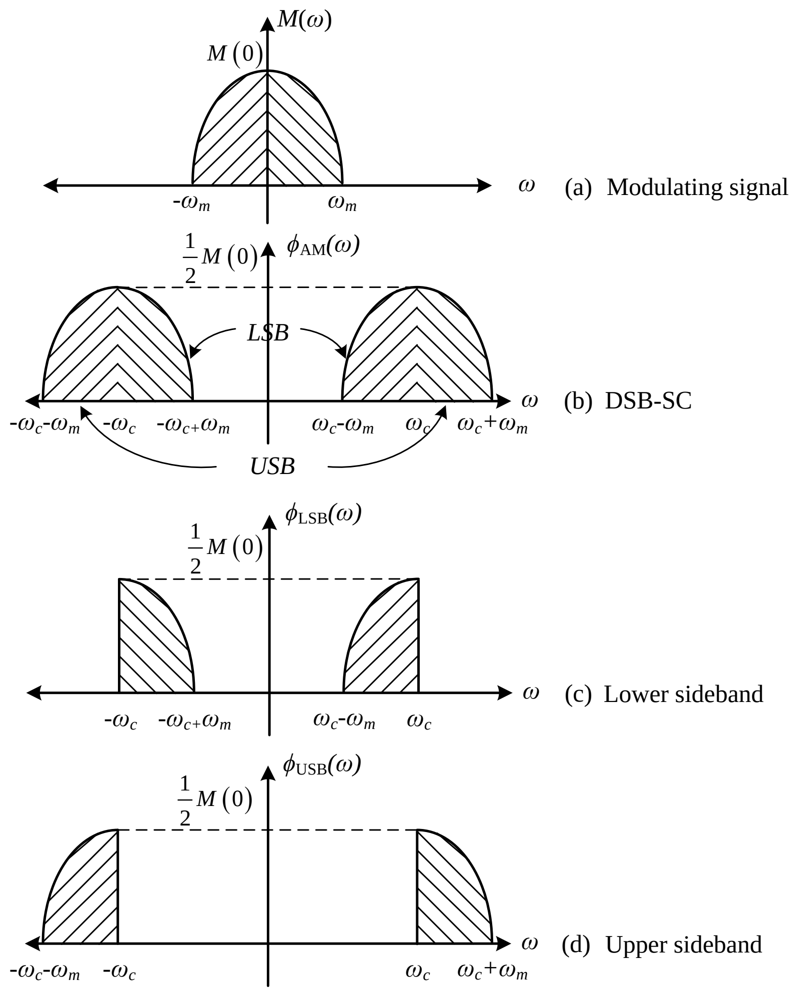


Fig. 3.20 Spectrum of (a) Modulating signal (b) DSB-SC (c) SSB-SC with lower sideband (d) SSB-SC with upper sideband

Similarly, the expression of SSB-SC with USB is given by

$$\phi_{\mathrm{SSB_U}}(t) = \cos(\omega_c + \omega_m)t = \cos(\omega_c t)\cos(\omega_m t) - \sin(\omega_c t)\sin(\omega_m t) \tag{3.86}$$

Hence, the SSB-SC signal is expressed as

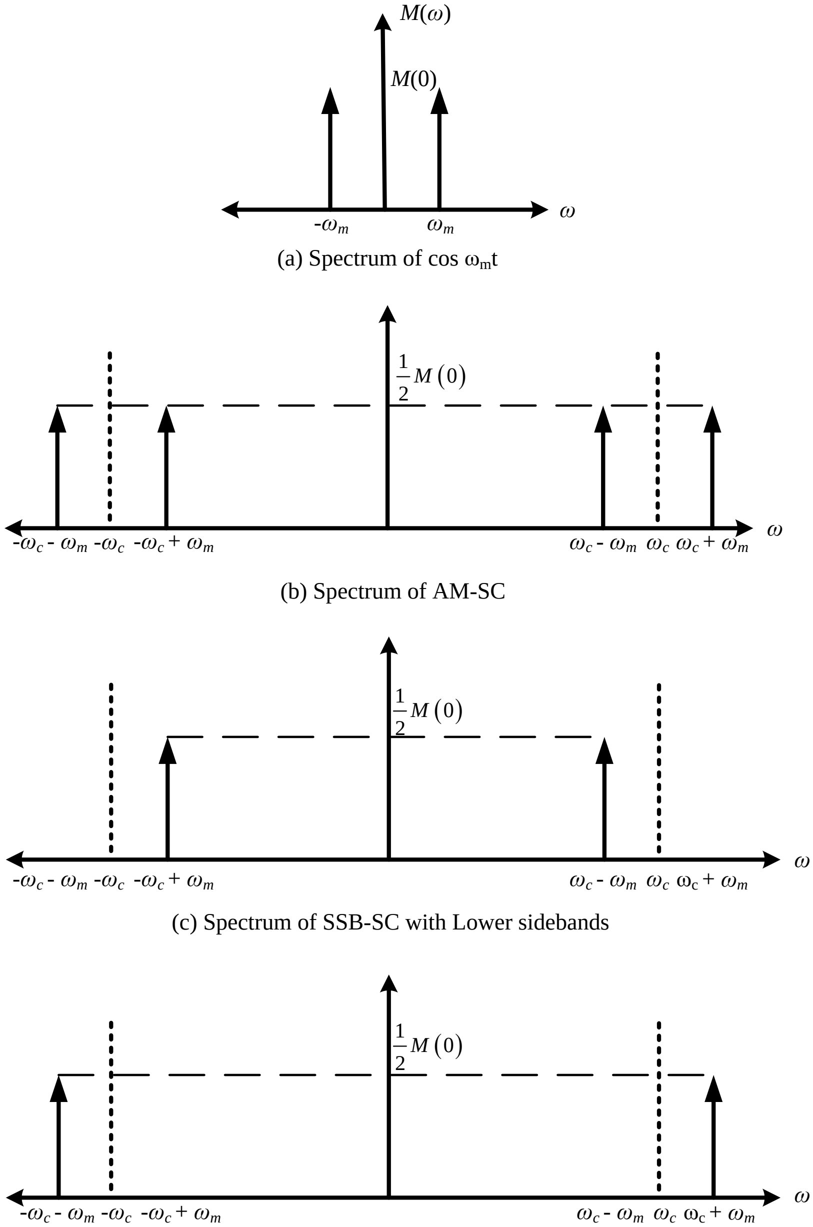
$$\begin{aligned}\phi_{\mathrm{SSB}}(t) &= \cos(\omega_m t)\cos(\omega_c t) \mp \sin(\omega_m t)\sin(\omega_c t)\\ &= m(t)\cos(\omega_c t) \mp m_h(t)\sin(\omega_c t)\end{aligned} \tag{3.87}$$

where, $m_h(t)$ is the Hilbert transform of the modulating signal $m(t)$.

3.7.2 Hilbert Transform

It can be seen that $m_h(t)$ is obtained by giving (-π/2) phase shift to each frequency component present in $m(t)$, therefore the Hilbert transform of $m(t)$ is defined as

$$m_h(t) = \frac{1}{\pi} m(t) * \frac{1}{t} = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{m(\tau)}{t-\tau} d\tau \tag{3.88}$$



(a) Spectrum of cos $\omega_m t$

(b) Spectrum of AM-SC

(c) Spectrum of SSB-SC with Lower sidebands

(d) Spectrum of SSB-SC with Upper sidebands

Fig. 3.21 Spectrum of (a) Modulating signal (b) DSB-SC (c) SSB-SC with lower sideband (d) SSB-SC with upper sideband

Properties of the Hilbert Transform

The properties of the signal $m(t)$ and its Hilbert transform $m_h(t)$ are as follows:

(i) Both have same autocorrelation function and same amplitude spectrum.

(ii) If Fourier transform exist then Hilbert transform also exists.

(iii) $m(t)$ and its Hilbert transform $m_h(t)$ are orthogonal.

(iv) Both have same energy spectral density.

3.7.3 Generation of SSB-SC

There are two methods for the generation of SSB-SC signals

1. Frequency discrimination or Selective filtering method
2. Phase discrimination or Phase-shift method

3.7.3.1 Selective Filtering Method (Frequency Discrimination)

This method is most common approach of SSB-SC signal generation. The basic idea behind this approach is as follows:

Step 1: Generate DSB-SC $\left(m(t)\cos\omega_c t\right)$ signal.

Step 2: Filter this signal through a BPF centered at $\pm\omega_c$.

The block diagram representation of the above approach is shown in Fig. 3.22.

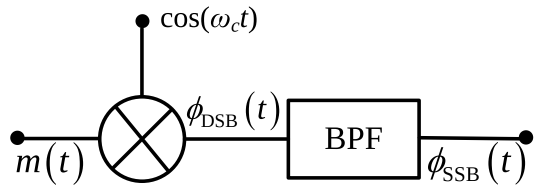


Fig. 3.22 Generation of SSB-SC using selective filtering approach

Limitations:

The frequency discrimination method has the following limitations:

1. **The baseband should be restricted at the lower edge to avoid overlapping between lower and upper sidebands.**

If it is not so, as shown in Fig. 3.23 (a), the filter should have a sharp cut-off boundary (highly selective).

In Fig. 3.23(b), the lower edge of the baseband is restricted at 300 Hz; therefore, the upper and lower sidebands are separated by 600 Hz. Now, either of the sidebands could be separated easily by a simple bandpass filter.

2. **The frequency discrimination method has another restriction that the baseband signal should be appropriately related to the carrier frequency**.

Let the voice signal (frequency range from 300 Hz to 3 kHz) modulates a carrier with a frequency of 10 MHz. Since, the upper and the lower sidebands are separated by 600 Hz, therefore, the percentage change in frequency to decide the selectivity of the bandpass filter is

$$\frac{600}{10\times10^6}\times100 = 0.006\,\%$$

Designing a bandpass filter with such a sharp selectivity is very complex. Therefore, frequency translation operation from baseband to carrier frequency is performed in several stages.

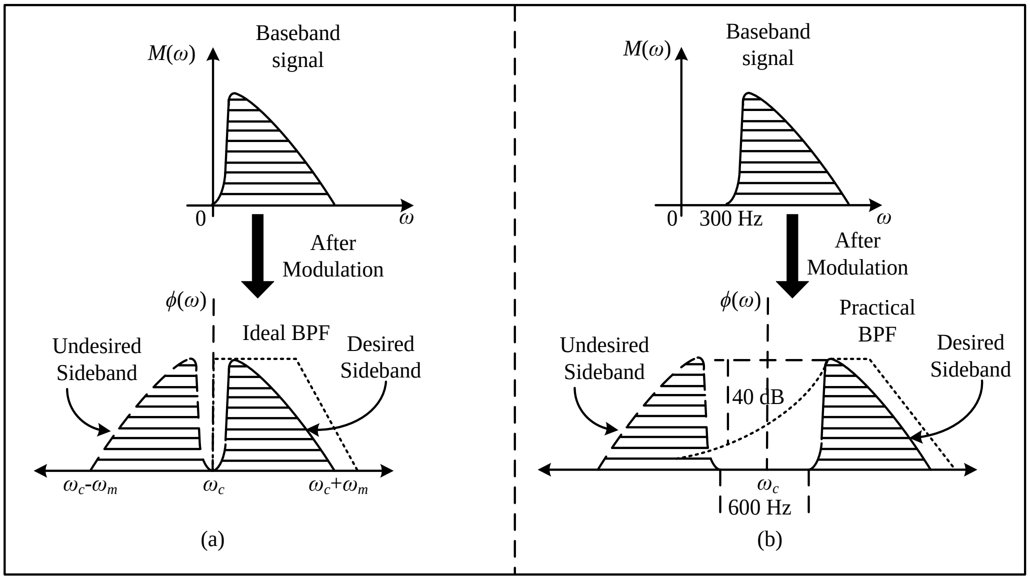


Fig. 3.23 Limitation of selective filtering approach

The above frequency translation operation is explained by an example given below:

SE3.1 Generate the SSB-SC signal for voice signal (from 300 Hz to 3 kHz) and the carrier frequency of 10 MHz by frequency discriminator approach in two stages.

Sol: Let the carrier frequency of the first stage is $f_{c1} = 100\ \text{kHz}$. The modulating signal is multiplied by this carrier signal in a balanced modulator.

Then lower sideband and upper sideband frequency ranges are

Lower sideband frequency range $(f_{c1} - f_m)$	Upper sideband frequency range $(f_{c1} + f_m)$
$\Rightarrow [(100-3) \quad (100-0.3)]\text{kHz}$ $\Rightarrow [97 \quad 99.7]\text{kHz}$	$\Rightarrow [(100+0.3) \quad (100+3)]\text{kHz}$ $\Rightarrow [100.3 \quad 103]\text{kHz}$

So, the selectivity of the filter $H_1(\omega)$ (with attenuation of 40 dB) for this stage is

$$\frac{600}{100\times10^3}\times100 = 0.6\ \%$$

Now, the upper sideband is selected by the filter, .i.e. $[100.3 \text{ to } 103]$ kHz is applied to the next modulator with the carrier frequency of 10 MHz. the upper and the lower sidebands frequency ranges at the output of the second balanced modulator is calculated in the table given below:

Lower sideband frequency range $(f_{c2} - f_m)$	Upper sideband frequency range $(f_{c2} + f_m)$
$\Rightarrow [(10000-103) \quad (10000-100.3)]\ \text{kHz}$ $\Rightarrow [9897 \quad 9899.7]\ \text{kHz}$	$\Rightarrow [(10000+100.3) \quad (10000+103)]\text{kHz}$ $\Rightarrow [10100.3 \quad 10103]\text{kHz}$

The separation between upper and lower sidebands is

$$10100.3 - 9899.7 = 200.6\ \text{kHz}$$

Then, the selectivity of the filter $H_2(\omega)$ (with attenuation of 40 dB) for the second stage is

$$\frac{200.6\times10^3}{10\times10^6}\times100 = 2\ \%$$

Thus, several stage operation for frequency translation makes the filter designing simple.

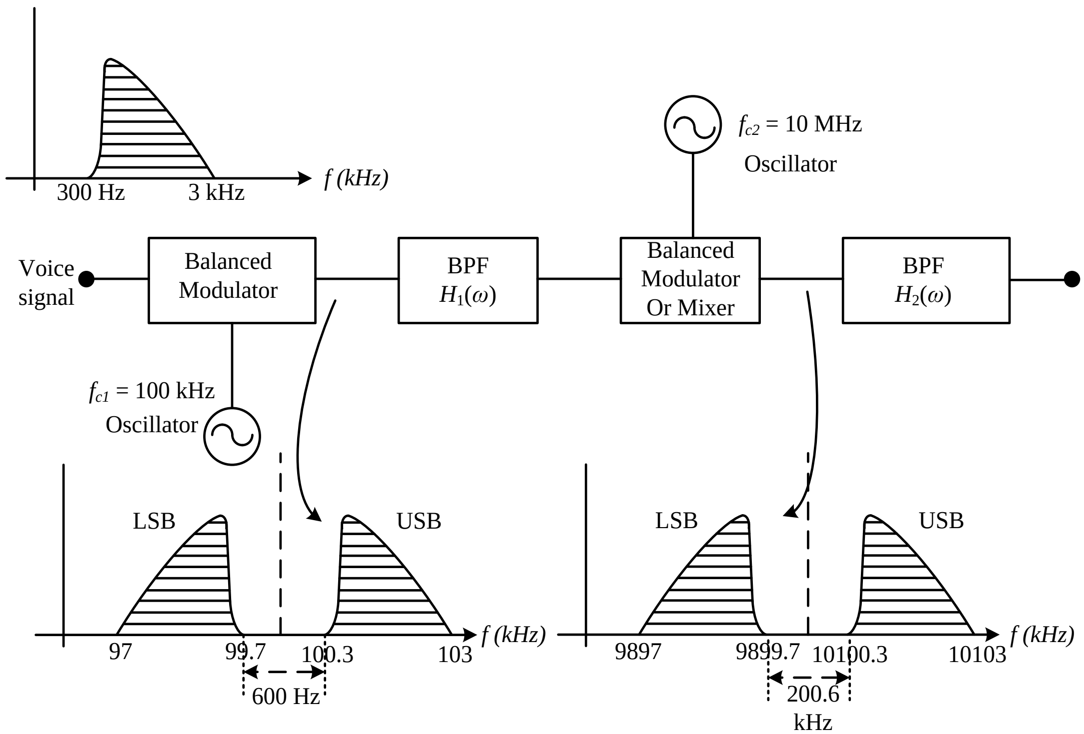


Fig. 3.24 SSB-SC with two transition stages

Instead of the balanced modulator, a mixer circuit can be used, but the drawback of the mixer circuit is that a mixer circuit generates not only the sum and difference of these frequencies but undesired input frequencies also. But here, this is not creating any problem as the input frequencies are removed by the bandpass filter.

3.7.3.2 Phase-Shift Method (Phase Discrimination)

The SSB-SC signal in the time domain is expressed by the Eq. (3.89) given below:

$$\phi_{\text{SSB}}(t) = m(t)\cos(\omega_c t) \mp m_h(t)\sin(\omega_c t) \tag{3.89}$$

The '+' sign is used for LSB, and the '-' sign is used for the USB spectrum.

DSB-SC or balanced modulators are used to generate the $m(t)\cos(\omega_c t)\big(\text{or } m_h(t)\sin(\omega_c t)\big)$.

Further, the - π/2 phase shift network is used to shift the entire frequency components by 90^o shift. The block diagram of the phase shift method for the SSB-SC generation is shown in Fig. 3.25.

The advantage of this method is that it does not require any sharp cut-off filter. But still, the phase shift method is not very popular due to the following reason:

1. Each phase-shifting network must provide an exact phase shift of π/2.

2. Each balanced modulator (DSB-SC modulator) must be balanced to suppress the carrier.
3. Each modulator must have equal sensitivity to the modulating signal.

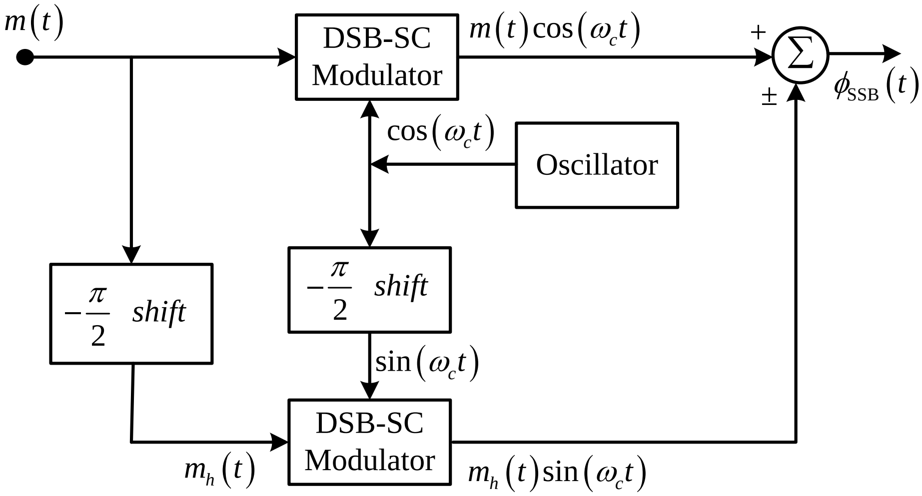


Fig. 3.25 SSB-SC signal generation using the phase-shift approach

3.7.4 Demodulation/Detection of the SSB-SC Signal

The message signal can be recovered from the SSB-SC signal by synchronous detection or coherent demodulation method. The block diagram of coherent detection is shown in Fig. 3.26.

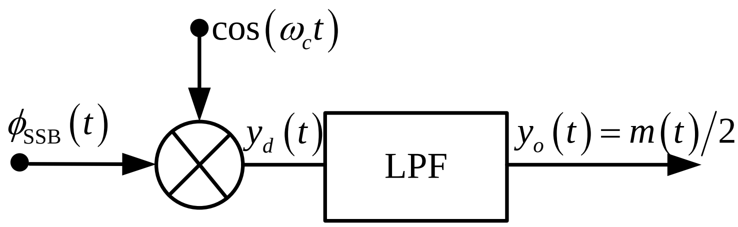


Fig. 3.26 Demodulation of SSB-SC by synchronous (coherent) detection

3.7.4.1 Coherent Demodulation

The locally generated carrier signal is multiplied with the modulated signal to retranslate the spectrum of the SSB-SC signal centred about $\omega = \pm\omega_c$ to centred about $\omega = 0$. Further, a low pass filter is used to recover the baseband (message) signal.

The output signal $y_d(t)$ of the multiplier network is given as

$$y_d(t) = \phi_{\text{SSB}}(t) * \cos(\omega_c t) \tag{3.90}$$

Substituting the value of $\phi_{\text{SSB}}(t)$

$$y_d(t) = \left[m(t)\cos(\omega_c t) \pm m_h(t)\sin(\omega_c t)\right]\cos(\omega_c t)$$

$$= m(t)\cos^2(\omega_c t) \pm m_h(t)\sin(\omega_c t)\cos(\omega_c t)$$

$$= \frac{1}{2}m(t)\left[1+\cos(2\omega_c t)\right] \pm \frac{1}{2}m_h(t)\sin(2\omega_c t)$$

$$= \frac{1}{2} m(t) + \frac{1}{2}\left[m(t)\cos(2\omega_c t) \pm m_h(t)\sin(2\omega_c t)\right] \tag{3.91}$$

Low pass filter allows only low-frequency component signal, i.e., $\frac{1}{2}m(t)$. Therefore, the output of the demodulator is given by

$$y_o(t) = \frac{1}{2}m(t) \tag{3.92}$$

The above expression shows the detected output is message signal with attenuation, but there is no distortion.

3.7.5 Effect of Phase and Frequency Errors in Synchronous Detection

Assume locally generated carrier has phase error and frequency error as $\Delta\varphi$ and $\Delta\omega$ respectively. The product of two signals in synchronous detector yields

$$\begin{aligned} e_d(t) &= \left[m(t)\cos\omega_c t + m_h(t)\sin\omega_c t\right] * \cos\left[(\omega_c + \Delta\omega)t + \Delta\varphi\right] \\ &= \frac{1}{2}m(t)\cos(\Delta\omega t + \Delta\varphi) + \frac{1}{2}m(t)\cos\left[(2\omega_c + \Delta\omega)t + \Delta\varphi\right] \\ &\quad - \frac{1}{2}m_h(t)\sin(\Delta\omega t + \Delta\varphi) + \frac{1}{2}m_h(t)\sin\left[(2\omega_c + \Delta\omega)t + \Delta\varphi\right] \end{aligned} \tag{3.93}$$

When this signal is passed through the LPF, the higher frequency terms of $\pm 2\omega_c$ are filtered out, and the output is obtained as

$$e_o(t) = \frac{1}{2}m(t)\cos(\Delta\omega t + \Delta\varphi) - \frac{1}{2}m_h(t)\sin(\Delta\omega t + \Delta\varphi) \tag{3.94}$$

Let discuss some special cases

(i) When frequency error $\Delta\omega$ and phase error $\Delta\varphi$ both are zero $(\Delta\omega = 0 \ \& \ \Delta\varphi = 0)$, then

$$e_o(t) = \frac{1}{2}m(t) \text{ (No distortion in the output.)} \tag{3.95}$$

(ii) When there is only phase error, i.e., $\Delta\omega = 0 \ \& \ \Delta\varphi \neq 0$, then output

$$e_o(t) = \frac{1}{2}m(t)\cos\Delta\varphi - \frac{1}{2}m_h(t)\sin\Delta\varphi \tag{3.96}$$

The undesirable term $m_h(t)\sin\Delta\varphi$ presents a phase distortion.

(iii) When there is only frequency error, i.e., $\Delta\omega \neq 0 \ \& \ \Delta\varphi = 0$, then

$$e_o(t) = \frac{1}{2}m(t)\cos(\Delta\omega)t - \frac{1}{2}m_h(t)\sin(\Delta\omega)t \tag{3.97}$$

Here, the multiplying factor is time-dependent that causes distortion in demodulated or detected output.

(iv) When both errors are not zero, i.e., $\Delta\omega \neq 0 \ \& \ \Delta\varphi \neq 0$. The distorted and attenuated output is obtained at the receiver end.

3.8 Vestigial Sideband SC (VSB-SC)

The advantage of the half bandwidth requirement in SSB-SC modulation as compared to DSB-SC is overwhelmed by the difficulty in filter designing for low-frequency signals such as television. This problem is overcome by another approach in which a small portion of the undesired sideband (vestige) is also allowed with the desired sideband. This scheme is called vestigial sideband (VSB) modulation. A VSB-SC signal is shown in Fig. 3.27.

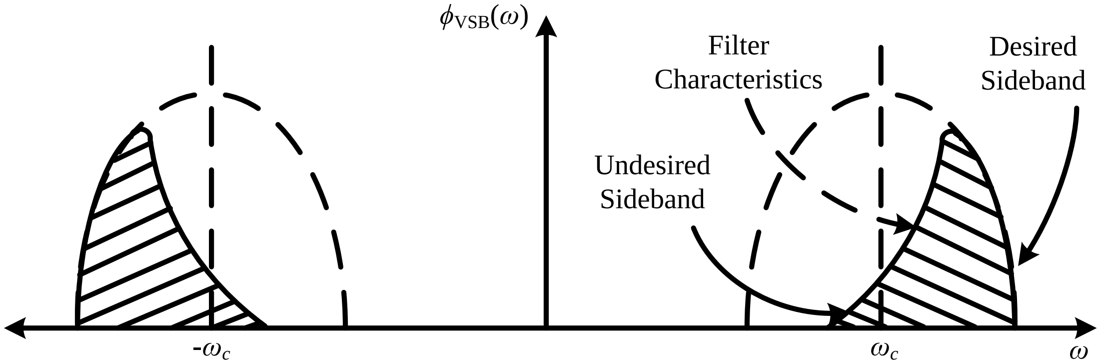


Fig. 3.27 Vestigial sideband SC

3.8.1 Generation and Detection of VSB-SC Signal

There are two methods to generate the VSB-SC signal

1. Filter method
2. Phase discrimination method

3.8.1.1 Filter Method

In this method, the VSB-SC signal is generated by passing the DSB-SC signal through an appropriate filter, as shown in Fig. 3.28(a). The only difference from DSB-SC generation is the characteristics of the filter.

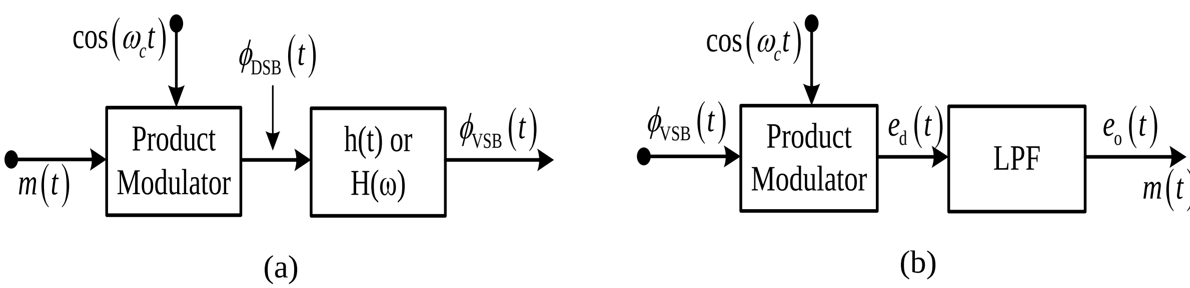


Fig. 3.28 (a) VSB-SC modulator (b) VSB-SC synchronous detector

Filter Characteristics

The spectrum of the DSB-SC signal is given by

$$\phi_{\text{DSB}}(\omega) = \frac{1}{2}\left[M(\omega + \omega_c) + M(\omega - \omega_c)\right] \tag{3.98}$$

The output of the filter $H(\omega)$ is given by

$$\phi_{\text{VSB}}(\omega)=\frac{1}{2}H(\omega)\left[M(\omega+\omega_c)+M(\omega-\omega_c)\right] \tag{3.99}$$

Now, this signal is passed through a product modulator for synchronous detection, as shown in Fig. 3.28(b).

$$e_d(t)=\phi_{\text{VSB}}(t)\cos(\omega_c t) \tag{3.100}$$

Taking Fourier transform of the Eq. (3.100)

$$E_d(\omega)=\frac{1}{2}\left[\phi_{\text{VSB}}(\omega+\omega_c)+\phi_{\text{VSB}}(\omega-\omega_c)\right] \tag{3.101}$$

Substituting the value of $\phi_{\text{VSB}}(\omega)$

$$E_d(\omega)=\frac{1}{4}\left[\left\{M(\omega+2\omega_c)+M(\omega)\right\}H(\omega+\omega_c)+\left\{M(\omega)+M(\omega-2\omega_c)\right\}H(\omega-\omega_c)\right] \tag{3.102}$$

The signal is passed through LPF, so high-frequency components are filtered out. Therefore,

$$E_o(\omega)=\frac{1}{4}M(\omega)\left[H(\omega+\omega_c)+H(\omega-\omega_c)\right] \tag{3.103}$$

Distortion less output is obtained when Eq. (3.103) follows the Eq. (3.104)

$$E_o(\omega)=K_1M(\omega) \tag{3.104}$$

Thus, the characteristics of the filter for VSB-SC modulation is

$$H(\omega+\omega_c)+H(\omega-\omega_c)=K_1(\text{Constant}) \tag{3.105}$$

3.8.1.2 Phase Discrimination Method

Like SSB-SC signals, the VSB-SC signal also can be generated by using the phase discrimination method.

Time-domain description of VSB-SC signal

The time-domain description of the VSB-SC signal is given as

$$\phi_{\text{VSB}}(t)=\frac{1}{2}m(t)\cos(\omega_c t)+\frac{1}{2}m_s(t)\sin(\omega_c t) \tag{3.106}$$

The VSB-SC signal generation is shown in Fig. 3.29.

Advantages:

The VSB-SC has the following advantages

(i) The required bandwidth for the VSB-SC is less than the DSB-SC.

(ii) As there is no need for high accuracy, therefore, filter design is easy for VSB-SC.

(iii) Highly efficient.

Disadvantages:

The VSB-SC has the following limitations

(i) The required bandwidth is less than that of the DSB-SC but greater than that of the SSB-SC.

(ii) Very complex demodulation process.

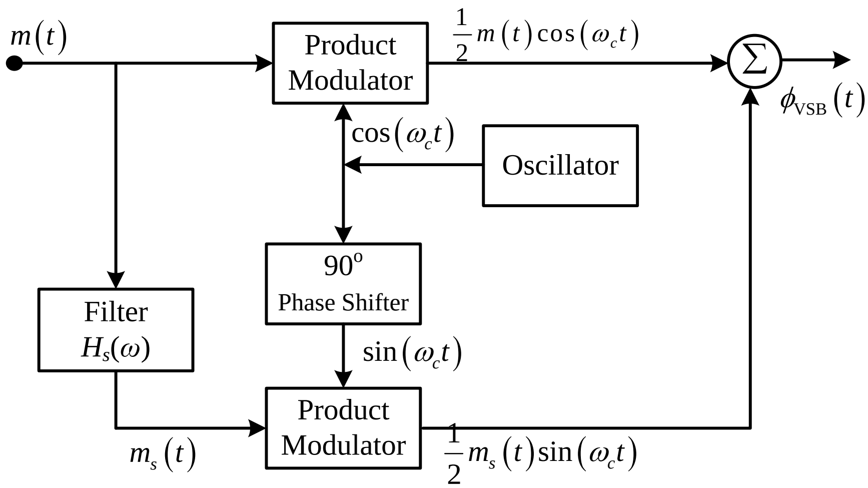


Fig. 3.29 Phase discrimination method of VSB-SC generation

3.9 Tuned Radio Frequency (TRF) Receiver

TRF receiver is a circuit configuration that performs the action of baseband detection from the received desired modulated signal. TRF receiver shows satisfactory performance for medium-range frequency. A TRF receiver performs all the functions of filtering, amplification, tuning and detecting at one radio frequency to which it is tuned. The block diagram of the TRF is shown in Fig. 3.30.

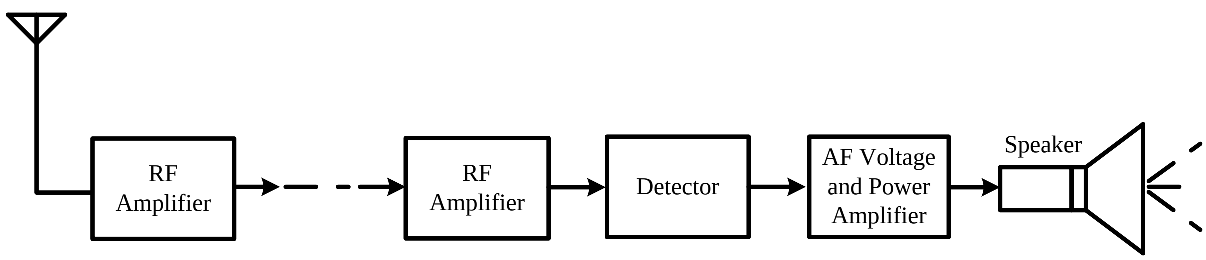


Fig. 3.30 TRF receiver

A TRF receiver consists of the following sections

(i) **Radio frequency amplifier:** This stage consists of one or more tuning and amplifying stages to provide the gain and the selectivity.

(ii) **Signal detector:** The detector stage enabled the TRF receiver to extract the audio signal. This stage normally uses envelop detector for audio signal extraction.

(iii) **Audio voltage and power amplifier:** These stages are utilized to enhance the power level of the extracted audio signal.

3.10 Superheterodyne Receiver

The heterodyning gives a far better performance than the TRF receiver (tuned radio frequency receiver). The block diagram of the superheterodyne receiver is shown in Fig. 3.31.

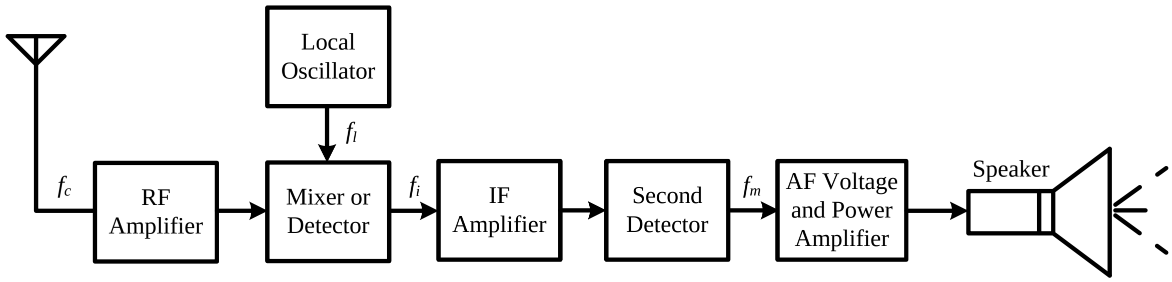


Fig. 3.31 Superheterodyne receiver

The significant features of the heterodyne receiver are as follows:

(i) The locally generated carrier is mixed with incoming carrier signal in mixer stage and further the sum and difference frequencies are obtained at the output.

(ii) The difference frequency is selected and amplified by the tuned IF amplifier stage.

(iii) All the radio frequency signals are converted into a single 455 kHz intermediate frequency (f_i) signal.

(iv) Since, the local oscillator frequency is higher than the carrier frequency, therefore, the receiver is called superheterodyne receiver.

The functions of the main elements of the superheterodyne receiver are as follows:

(i) RF Amplifier

The tuned RF stage with an optional RF amplifier is a class C tuned amplifier. The RF amplifier is used

1. To amplify the received signal to improve the *S/N* ratio and to provide better sensitivity.
2. To provide initial selectivity by rejecting the unwanted signals.

(ii) Image Signal

The image signal is the mirror image of the desired frequency signal (Fig. 3.32). It is defined as:

"A signal with the frequency above the local oscillator frequency (f_l) by the same amount as the desired signal frequency (f_c) below the local oscillator frequency"

So, the frequency of the image signal $(f_{c,image})$ is given as

$$f_l - f_c = f_i \tag{3.107}$$

$$f_l - f_i = f_c \tag{3.108}$$

$$\begin{aligned} f_{c,image} &= f_l + f_i = f_c + f_i + f_i \\ &= f_c + 2f_i \end{aligned} \tag{3.109}$$

Thus, the frequency difference between the desired carrier signal and the image signal is $2f_i$.

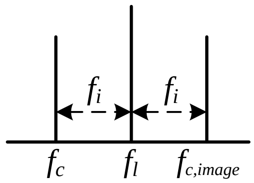


Fig. 3.32 Image signal

(iii) Local oscillator and mixer

The mixer stage is a non-linear device that is used to mix the incoming signal with a carrier signal from the local oscillator (LO).

(iv) IF amplifier

Intermediate frequency (IF) amplifier works as a tuned filter. The IF amplifier is tuned to a fixed frequency and reject all other incoming frequency by providing attenuation to all of them. This stage is responsible for most of the receiver gain; therefore, more than one IF amplifier are used to increase the sensitivity of the receiver.

Choice of Intermediate Frequency

The intermediate frequency has been fixed as 455 kHz. This is chosen for

(i) Adjacent channel selectivity and easy tracking for which the value of IF should be less.

(ii) Image signal rejection for which the value of IF should be high.

(v) Audio Amplifier

Audio amplifier is simply a combination of an *RC* voltage amplifier followed by a push-pull power amplifier. The more the bandwidth of this stage, the better the fidelity is.

3.11 Comparison between Amplitude Modulation Methods

A comparison between different types of amplitude modulations is shown in Table 3.1

Table 3.1 Comparison of Full AM, DSB-SC, SSB-SC and VSB-SC

S. N.	Parameters	Full AM	DSB-SC	SSB-SC	VSB-SC
1.	**Full-Form**	Amplitude Modulation	Double sideband suppressed carrier	Single sideband suppressed carrier	Vestigial sideband suppressed carrier
2.	**Transmitted signals**	Sidebands as well as carrier signals	Only sidebands (LSB and USB both)	Only one sideband (Either LSB or USB)	One sideband with some part of the vestigial band
3.	**Bandwidth**	$2\omega_m$	$2\omega_m$	ω_m	$\omega_m < \mathrm{BW} < 2\omega_m$
4.	**Transmitted Power**	More	Less than AM	Least	Less than DSB-SC but more than SSB-SC
5.	**Power Efficiency**	Poor (max. 33.3%)	Moderate	Maximum	Moderate
6.	**Demodulation**	Simple	Complex	Complex	Complex
7.	**Applications**	commercial AM radio broadcasting	point to point communication	long-distance voice transmission	TV and similar types of transmission

ADDITIONAL SOLVED EXAMPLES

SE3.6 A message signal $m(t) = \cos 2000\pi t + 4\cos 4000\pi t$ modulates the carriers $c(t) = \cos 2\pi f_c t$ where $f_c = 1$ MHz to produce an AM signal. For demodulating the generated AM signal using an envelope detector, the time constant RC of the detector circuit should satisfy **(GATE: 2011)**

(a) $0.5 \text{ ms} < RC < 1 \text{ ms}$ (c) $RC \ll 1 \text{ ms}$

(b) $1\ \mu\text{s} \ll RC < 0.5 \text{ ms}$ (d) $RC \gg 0.5 \text{ ms}$

Sol: According to Eq. (3.41)

$$\frac{1}{\omega_c} << RC < \frac{1}{\omega_m}$$

The maximum frequency component in $m(t)$ is

$$f_m = \frac{4000\pi}{2\pi} = 2000 \text{ Hz}$$

The carrier frequency is

$$f_c = 1 \text{ MHz}$$

Therefore

$$\frac{1}{1\times 10^6} << RC < \frac{1}{2000}$$

$$1\ \mu s << RC < 0.5 \text{ ms}$$

Hence, option (b) is correct.

SE3.7 In an amplitude modulation process, the carrier and modulating signal, respectively, are

$$e_c = E_c \sin \omega_c t$$

$$e_m = E_m \sin \omega_m t + \frac{E_m}{2}\sin 2\omega_m t + \frac{E_m}{3}\sin 3\omega_m t + \frac{E_m}{4}\sin 4\omega_m t$$

Derive an expression to show that for every modulating frequency component, the AM wave contains two sideband frequencies in addition to the carrier. Also, find the value of the composite modulation index. **(IES: 2009)**

Sol: Given data:

Carrier signal $e_c = E_c \sin \omega_c t$

Modulating signal $e_m = E_m \sin \omega_m t + \frac{E_m}{2}\sin 2\omega_m t + \frac{E_m}{3}\sin 3\omega_m t + \frac{E_m}{4}\sin 4\omega_m t$

The AM wave is expressed as a change in amplitude of the carrier wave in accordance with the instantaneous value of modulating signal. Hence,

$$\phi_{\text{AM}}(t) = (E_c + e_m)\sin\omega_c t$$

$$\phi_{\text{AM}}(t) = \left(E_c + E_m \sin\omega_m t + \frac{E_m}{2}\sin 2\omega_m t + \frac{E_m}{3}\sin 3\omega_m t + \frac{E_m}{4}\sin 4\omega_m t\right)\sin\omega_c t$$

$$= E_c\left(1 + \frac{E_m}{E_c}\sin\omega_m t + \frac{E_m}{2E_c}\sin 2\omega_m t + \frac{E_m}{3E_c}\sin 3\omega_m t + \frac{E_m}{4E_c}\sin 4\omega_m t\right)\sin\omega_c t$$

Since, $m_a = \dfrac{E_m}{E_c}$

Hence, $\phi_{\text{AM}}(t) = E_c\left(1 + m_a \sin\omega_m t + \dfrac{m_a}{2}\sin 2\omega_m t + \dfrac{m_a}{3}\sin 3\omega_m t + \dfrac{m_a}{4}\sin 4\omega_m t\right)\sin\omega_c t$

$$\phi_{\text{AM}}(t) = E_c \sin\omega_c t + m_a E_c \sin\omega_m t \sin\omega_c t + \frac{m_a E_c}{2}\sin 2\omega_m t \sin\omega_c t + \frac{m_a E_c}{3}\sin 3\omega_m t \sin\omega_c t$$
$$+ \frac{m_a E_c}{4}\sin 4\omega_m t \sin\omega_c t$$

$(\because 2\sin A \sin B = \cos(A-B) - \cos(A+B))$

$\therefore \quad \phi_{\text{AM}}(t) =$

$$E_c \sin\omega_c t + \frac{m_a E_c}{2}\left(\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t\right) + \frac{m_a E_c}{4}\left(\cos(\omega_c - 2\omega_m)t - \cos(\omega_c + 2\omega_m)t\right)$$
$$+ \frac{m_a E_c}{6}\left(\cos(\omega_c - 3\omega_m)t - \cos(\omega_c + 3\omega_m)t\right) + \frac{m_a E_c}{8}\left(\cos(\omega_c - 4\omega_m)t - \cos(\omega_c + 4\omega_m)t\right)$$

From above, it is clear that for every modulating frequency $\omega = \omega_m,\ 2\omega_m,\ 3\omega_m,\ 4\omega_m$, there are two sidebands around $(\omega_c \pm \omega)$.

Since composite modulation index is given by

$$m = \sqrt{m_{a1}^2 + m_{a2}^2 + m_{a3}^2 + m_{a4}^2}$$

Here, $m_{a1} = m_a, m_{a2} = \dfrac{m_a}{2}, m_{a3} = \dfrac{m_a}{3}, m_{a4} = \dfrac{m_a}{4}$

Therefore, $m = \sqrt{m_a^2 + \left(\dfrac{m_a}{2}\right)^2 + \left(\dfrac{m_a}{3}\right)^2 + \left(\dfrac{m_a}{4}\right)^2} = 1.19 m_a$ where $\left(m_a = \dfrac{E_m}{E_c}\right)$

SE3.8 A 10 kW carrier is sinusoidally modulated by two modulating signals corresponding to a modulation index of 30% and 40%, respectively. Find the total radiated power. **(IES: 1999)**

Sol: Given data: Carrier power $P_c = 10 \text{ kW}$

Modulation indices $m_{a1}(\%) = 30\% \Rightarrow m_{a1} = 0.3$

$m_{a2}(\%) = 40\% \Rightarrow m_{a2} = 0.4$

To calculate: Total power content $P_T = ?$

The net modulation index is given by

$$m_a = \sqrt{m_{a1}^2 + m_{a2}^2} = \sqrt{0.3^2 + 0.4^2} = 0.5$$

The total radiated power is given by

$$P_T = P_c\left(1 + \frac{m_a^2}{2}\right)$$

$$= 10\left(1 + \frac{0.5^2}{2}\right)$$

$$= 10 \times \frac{2.25}{2} = 11.25 \text{ kW}$$

SE3.9 In a superheterodyne receiver, the IF is 455 kHz, it is tuned to 1200 kHz, the image frequency will be **(IES: 2009)**

(a) 1655 kHz
(b) 754 kHz
(c) 2110 kHz
(d) 910 kHz

Sol: The image frequency is given by

$$f_c' = f_c + 2f_i$$

Putting the values of $f_c = 1200$ kHz and $f_i = 455$ kHz

$$f_c' = 1200 + 2 \times 455 = 2110 \text{ kHz}$$

Hence, option (c) is correct.

SE3.10 In a broadcast transmitter, the RF output is represented as

$$\phi_{\text{AM}}(t) = 50(1 + 0.89\cos 5000t + 0.30\sin 9000t)\cos\left(6\times10^6 t\right) \text{ volts.}$$

What are the sidebands of the signals in radians/sec?

Sol: The AM signal is expressed as

$$\phi_{\text{AM}}(t) = A_c\left(1 + m_{a1}\cos\omega_{m1}t + m_{a2}\sin\omega_{m2}t\right)\cos\omega_c t$$

Comparing the given expression with the standard equation

$$\omega_{m1} = 5000 \text{ rad/sec}; \quad \omega_{m2} = 9000 \text{ rad/sec}; \quad \omega_c = 6\times10^6 \text{ rad/sec}$$

The sidebands are obtained at $(\omega_c \pm \omega_{m1})$ and $(\omega_c \pm \omega_{m2})$.

So, the sidebands frequencies are at

$$(\omega_c \pm \omega_{m1}) = 6\times10^6 \pm 5\times10^3 \Rightarrow 6.005\times10^6 \text{ and } 5.995\times10^6$$

$$(\omega_c \pm \omega_{m2}) = 6\times10^6 \pm 9\times10^3 \Rightarrow 6.009\times10^6 \text{ and } 5.991\times10^6$$

SE3.11 A 1 MHz sinusoidal carrier is amplitude modulated by a symmetrical square wave of period 100 µsec. Which of the following frequencies will not be present in the modulated signal? **(GATE: 2002)**

(a) 990 kHz
(b) 1010 kHz
(c) 1020 kHz
(d) 1030 kHz

Sol: Given data: Carrier frequency $f_c = 1$ MHz

The time period of the square wave $T_m = 100\ \mu$sec

Therefore, the frequency of the square wave is

$$f_m = \frac{1}{T_m} = \frac{1}{100 \times 10^{-6}} = 10 \text{ kHz}$$

The Fourier transform of the symmetrical square wave is expressed as

$$M(\omega) = \sum_{n=1,3,5,..}^{\infty} \frac{4V_s}{n\pi} \sin(n\omega_m t)$$

where, V_s is amplitude and ω_m is the frequency of the square wave, respectively.

Therefore, it is concluded that the modulating signal consists of only odd harmonics like $\omega_m, 3\omega_m, 5\omega_m, ...$

Thus, the modulated signal has the following sidebands

$$f_c \pm f_m = 1000 \times 10^3 \pm 10 \times 10^3 = 1010 \text{ kHz and } 990 \text{ kHz}$$

$$f_c \pm 3 f_m = 1000 \times 10^3 \pm 30 \times 10^3 = 1030 \text{ kHz and } 970 \text{ kHz}$$

So, 1020 kHz signal will not be present in the modulated signal.

Hence, option (c) is correct.

SE3.12 A 4 GHz carrier is DSB-SC modulated by a low-pass message signal with a maximum frequency of 2 MHz. The resultant signal is to be ideally sampled. The minimum frequency of the sampling impulse train should be **(GATE: 1990)**

(a) 4 MHz
(b) 8 MHz
(c) 8 GHz
(d) 8.004 GHz

Sol: Given data: Carrier frequency $f_c = 4$ GHz

Modulating frequency $f_m = 2$ MHz

Since the bandwidth of the DSB-SC modulated signal is $\text{BW} = 2 f_m$.

So, the minimum frequency must be equal to Nyquist frequency, which is given as

$$f_{nyquist} = 2 \times \text{BW} = 4 f_m = 4 \times 2 = 8 \text{ MHz}$$

Hence, option (b) is correct.

SE3.13 A given AM broadcast station transmits a total power of 50 kW when the carrier is modulated by a sinusoidal signal with a modulation index of 0.707. Calculate:

(i) The carrier power, and (ii) The transmission efficiency

Sol: Given data: Total power $P_T = 50$ kW

Modulation index $m_a = 0.707$

(i) Since, $$P_T = P_c\left(1+\frac{m_a^2}{2}\right)$$

So, $$50 = P_c\left(1+\frac{0.707^2}{2}\right)$$

$$P_c = \frac{50}{1.25} = 40 \text{ kW}$$

(ii) The transmission efficiency $\eta = \dfrac{m_a^2}{2+m_a^2} = \dfrac{0.707^2}{2+0.707^2} = 0.2$

$$\eta(\%) = 20\%$$

SE3.14 Which of the following demodulator (s) can be used for demodulating the $x(t) = 5(1+2\cos 200\pi t)\cos(20000\pi t)$ signal. **(GATE: 1993)**

(a) Envelope demodulator

(b) Square-law demodulator

(c) Synchronous demodulator

(d) None of the above

Sol: Compare the given expression with standard AM expression

$$\phi_{\text{AM}}(t) = A_c(1+m_a\cos\omega_m t)\cos\omega_c t$$

The modulation index is $m_a = 2$

Since $m_a > 1$, then the signal has been detected by the synchronous detector.

Hence, option (c) is correct.

SE3.15 Prove that in AM, the maximum average power transmitted by an antenna is 1.5 times the carrier power.

Sol: Total power radiated by the antenna is given as

$$P_T = P_c\left(1+\frac{m_a^2}{2}\right)$$

As we know, the maximum value of the modulation index is1, i.e. $(m_a)_{\max} = 1$. So, the maximum transmitted power is

$$(P_T)_{\max} = P_c\left(1+\frac{1^2}{2}\right) = 1.5P_c$$

SE3.16 A message signal given by $m(t)=\left(\frac{1}{2}\right)\cos\omega_1 t-\left(\frac{1}{2}\right)\sin\omega_2 t$ is amplitude modulated with a carrier of frequency ω_c to generate $s(t)=[1+m(t)]\cos\omega_c t$. What is the power efficiency achieved by this modulation scheme?

(a) 8.33% (c) 20%

(b) 11.11% (d) 25% **(GATE: 2009)**

Sol: The given modulation is an example of multitone AM modulation

The modulated signal is

$$\phi_{\text{AM}}(t)=[1+m(t)]\cos\omega_c t \text{ with}$$

$$m(t)=\left(\frac{1}{2}\right)\cos\omega_1 t-\left(\frac{1}{2}\right)\sin\omega_2 t$$

Therefore, the modulation indices are

$$m_{aT}=\sqrt{m_{a1}^2+m_{a2}^2}=\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2}=\frac{1}{\sqrt{2}}$$

The power efficiency is

$$\eta=\frac{m_{aT}^2}{2+m_{aT}^2}=\frac{\left(1/\sqrt{2}\right)^2}{2+\left(1/\sqrt{2}\right)^2}=\frac{1}{5}=0.2=20\%$$

Hence, option (c) is correct.

SE3.17 A multitone AM signal is expressed as

$$\phi_{\text{AM}}(t)=10\cos\left(2\pi\times10^6 t\right)+5\cos\left(2\pi\times10^6 t\right)\cos\left(2\pi\times10^3 t\right)+2\cos\left(2\pi\times10^6 t\right)\cos\left(4\pi\times10^3 t\right)$$

Calculate total modulated power and net modulation index.

Sol: The expression of AM wave can be rewritten as

$$\phi_{\text{AM}}(t)=10\left[1+0.5\cos\left(2\pi\times10^3 t\right)+0.2\cos\left(4\pi\times10^3 t\right)\right]\cos\left(2\pi\times10^6 t\right)$$

Compare it with the standard equation of multitone AM wave

$$\phi_{\text{AM}}(t)=A_c\left[1+m_{a1}\cos\left(2\pi\times f_{m1}t\right)+m_{a2}\cos\left(4\pi\times f_{m2}t\right)\right]\cos\left(2\pi\times f_c t\right)$$

$$A_c=10,\ \ m_{a1}=0.5,\ \ m_{a2}=0.2$$

$$f_c=10^6\text{ Hz},\ \ f_{m1}=10^3\text{ Hz},\ \ f_{m2}=2\times10^3\text{ Hz}$$

Therefore, the unmodulated carrier power is $P_c=\frac{A_c^2}{2}=\frac{100}{2}=50\text{ W}$

The net modulation index $m_{aT}=\sqrt{m_{a1}^2+m_{a2}^2}=\sqrt{0.5^2+0.2^2}=0.539$

Total modulated power $P_T = P_c\left(1+\frac{m_{aT}^2}{2}\right)$

$$P_T = 50\left(1+\frac{0.539^2}{2}\right) = 50\times 1.145 = 57.25 \text{ W}$$

SE3.18 In a conventional AM, the modulating frequency is f_m and the carrier frequency is f_c. If 75% modulation is used, find the ratio of sidebands power and total modulated power.

Sol: The ratio of sidebands power is given as

Ratio = $\frac{P_s}{P_T} = \frac{m_a^2}{2+m_a^2}$ where, $m_a = 75\% = 0.75$

Ratio = $\frac{0.75^2}{2+0.75^2} = \frac{0.5625}{2.5625} = 0.2195$

SE3.19 Consider the amplitude modulated (AM) wave as,

$\phi_{\text{AM}}(t) = A_c \cos\omega_c t + 2\cos\omega_m t \cos\omega_c t$. For demodulating the signal using an envelope detector, the minimum value of A_c should be

(a) 2
(b) 1
(c) 0.5
(d) 0 **(GATE: 2008)**

Sol: The given AM signal is

$$\phi_{\text{AM}}(t) = A_c \cos\omega_c t + 2\cos\omega_m t \cos\omega_c t$$
$$= (A_c + 2\cos\omega_m t)\cos\omega_c t$$

For demodulation by an envelope detector, the modulation index must be

$$m_a \le 1$$

So, $m_a = \frac{A_m}{A_c} \le 1 \Rightarrow \frac{2}{A_c} \le 1$

$$2 \le A_c$$

Therefore, the minimum value of A_c is

$$(A_c)_{\min} = 2$$

Hence, option (a) is correct.

SE3.20 For a superheterodyne receiver, the intermediate frequency is 15 MHz and the local oscillator frequency is 3.5 GHz. If the frequency of the received signal is greater than the local oscillator frequency, then the image frequency (in MHz) is ___________

Sol: Given data: $f_i = 15$ MHz, $f_{LO} = 3.5$ GHz $= 3500$ MHz

$$f_c' = f_l + f_i = 3500 - 15$$
$$= 3485 \text{ MHz}$$

PROBLEMS

P3.1 Explain the need for modulation.

P3.2 Describe the operation of the square law modulator for the generation of AM signal with the help of proper circuit representation.

P3.3 Show that an AM signal can be recovered, irrespective of the value of percentage modulation, by using synchronous detection.

P3.4 Explain the operation of envelope detection and also compare its performance with a synchronous detector.

P3.5 Derive an expression for *RC* time constant for faithful recovery of modulating signal in AM wave through envelope detector.

P3.6 Describe switching modulator for DSB-SC signal generation?

P3.7 Explain the effect of phase and frequency errors in the synchronous detection method of DSB-SC.

P3.8 Prove that the balanced modulator removes the carrier and gives two sidebands only.

P3.9 Explain the generation of SSB-SC signal with the help of a suitable block diagram and expressions

P3.10 What is the limitation of the selective filter approach for SSB-SC signal generation and how can this limitation be improved?

P3.11 Derive an expression of filter characteristics for generation and detection of VSB-SC signal.

P3.12 Describe the following terms:

(i) Modulation index

(ii) Diagonal clipping

P3.13 Describe the following performance measures of radio receivers:

(i) Selectivity

(ii) Sensitivity

(iii) Fidelity

(iv) Image frequency rejection

P3.14 Draw the block diagram of the superheterodyne radio receiver and explain its operation. Also, mention the advantages of the superheterodyne receiver over TRF.

P3.15 Compare AM, DSB-SC, SSB-SC and VSB-SC signals.

NUMERICAL PROBLEMS

P3.16 Calculate the modulation index and percentage modulation if modulating signal and carrier are given as $m(t) = 40\sin(\omega_m t)$ and $c(t) = \sin(\omega_c t)$ respectively.

P3.17 A modulating signal is given by $m(t) = 2\cos(1000t) + \cos(2000t)$. Draw the spectrum of $m(t)$.

P3.18 An amplitude modulated signal is given by

$$\phi_{\text{AM}}(t) = 50(1 + 0.2\cos 100t + 0.01\cos 3500t)\cos(10^6 t)$$

Find the various frequency components present and also find the total modulated power, sidebands power, and net modulation index for AM signal.

P3.19 Calculate the percentage power saving when a carrier and one of the sidebands are suppressed in AM wave modulated to the depth of (i) 100% and (ii) 50%.

P3.20 An AM signal is generated by modulating the carrier of frequency $f_c = 800$ kHz by the message signal $m(t) = \sin(2000\pi t) + 5\cos(4000\pi t)$. The AM signal is expressed as

$$\phi_{\text{AM}}(t) = 100[1 + m(t)]\cos(2\pi f_c t).$$

(i) Draw the spectrum of the AM signal.

(ii) Determine the average power in the carrier and in the sidebands.

P3.21 A carrier signal $c(t) = A_c\cos(\omega_c t)$ is modulated by a single-tone modulating signal $m(t) = A_m\cos(\omega_m t)$. Calculate

(i) Total modulated power

(ii) Root mean square value of the modulated signal and

(iii)Transmission efficiency for a 100% modulation.

P3.22 A multiple-tone modulating signal $m(t)$ consisting of three frequency components is given by

$$m(t) = A_{m1}\cos\omega_1 t + A_{m2}\cos\omega_2 t + A_{m3}\cos\omega_3 t$$

where, $\omega_1 < \omega_2 < \omega_3$ & $A_{m1} > A_{m2} > A_{m3}$

The signal $m(t)$ modulates a carrier $c(t) = A_c\cos(\omega_c t)$.

(a) Derive an expression for AM wave.

(b) Draw a single-side spectrum, and find the bandwidth of the AM wave.

P3.23 The antenna current of an AM transmitter is 10 A when only the carrier is sent, but it increases to 10.63 A when the carrier is modulated by a single sine wave. Find the percentage of modulation. Determine antenna current when the percentage of modulation changes to 0.8.

P3.24 An amplitude modulated signal is given by:

$$\phi_{\text{AM}}(t) = 10\cos\left(2\pi\times10^6 t\right) + 5\cos\left(2\pi\times10^6 t\right)\cos\left(2\pi\times10^3 t\right) + 2\cos\left(2\pi\times10^6 t\right)\cos\left(4\pi\times10^3 t\right)\ \text{V}$$

Find the various frequency components present and the corresponding modulation indices. Also, find the total modulated power, sideband power, and net modulation index for AM signal.

P3.25 For a DSB-SC system with V_{max} = 20 V and V_{min} = 4 V. Determine the following:

(i) The amplitude of the carrier and message signal

(ii) Modulation index

(iii) Upper and Lower sidebands power

P3.26 A 400 W carrier is amplitude modulated to a depth of 100%. Calculate the total power in the case of AM and DSB-SC techniques. How much power saving is achieved for DSB-SC? If the depth of modulation is changed to 75%, then how much power is required for transmitting DSB-SC wave?

P3.27 An SSB-SC transmitter radiates 0.5 kW when modulation percentage is 60%. How much of carrier power is required if we want to transmit the same message by an AM transmitter.

P3.28 Show that the coherent demodulation scheme shown in Fig. 3.33 given below can demodulate the AM signal $\left[A_c + m(t)\right]\cos\left(2\pi f_c t\right)$ regardless of the value of A_c.

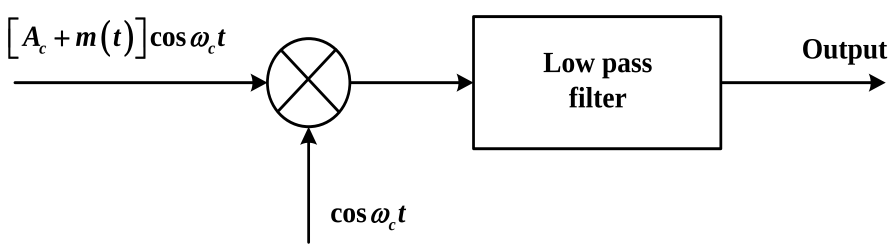


Fig. 3.33 Figure of problem P3.28

P3.29 A modulating signal is given as $m(t) = 2\cos(300\pi t) + \cos(100\pi t)$.

(i) Sketch the spectrum of $m(t)$.

(ii) Find and sketch the spectrum of the DSB-SC signal $\phi_{\text{DSB}} = 2m(t)\cos(1000\pi t)$.

(iii) From the spectrum obtained in (ii), suppress the LSB spectrum to get the USB spectrum.

(iv) Knowing the USB spectrum in (iii), write the expression for ϕ_{USB}.

MULTIPLE-CHOICE QUESTIONS

MCQ3.1. Amplitude modulation is

(a) Change in the amplitude of the carrier according to modulating signal
(b) Change in frequency of the carrier according to modulating signal
(c) Change in the amplitude of the modulating signal according to the carrier signal
(d) Change in the amplitude of the carrier according to modulating signal frequency

MCQ3.2. The standard intermediate frequency used for AM receiver is

(a) 455 MHz (c) 455 kHz
(b) 455 Hz (d) 455 GHz

MCQ3.3. A 3 GHz carrier is DSB-SC modulated by a signal with a maximum frequency of 2 MHz. The minimum sampling frequency required for the signal so that the signal is ideally sampled is

(a) 4 MHz (c) 6 MHz
(b) 6.004 GHz (d) 6 GHz

MCQ3.4. The total power in an amplitude modulated signal if the carrier of an AM transmitter is 800 W and it is modulated 50 percent.

(a) 850 W (c) 1000.8 kW
(b) 750 W (d) 900 W

MCQ3.5. The frequency range of amplitude modulation is between __________

(a) 540-1600 Hz (c) 140-1600 Hz
(b) 940-1600 Hz (d) 240-1600 Hz

MCQ3.6. The circuit which is used to produce the amplitude modulation is___

(a) Modulator (c) Both a and b
(b) Duplexer (d) None of the above

MCQ3.7. The ……….. modulation consists of both lower and upper sidebands, whereas the ………. modulation consists of only lower or only upper sideband.

(a) SSB-SC, DSB-SC
(b) VSB-SC, DSB-SC
(c) DSB-SC, SSB-SC
(d) Both (a) and (c)

MCQ3.8. The number of sidebands in amplitude modulation is

(a) 4 (b) 2 (c) 3 (d) 1

MCQ3.9. The Hilbert transform is

(a) Linear system (c) Both (a) and (b)
(b) Non-linear system (d) None of the above

MCQ3.10. The maximum transmission efficiency of an AM signal is

(a) 33.3% (c) 66.7%
(b) 50% (d) 100%

MCQ3.11. What is the percentage of modulation if the modulating signal is 6V and the carrier is 10V?

(a) 20% (c) 60%
(b) 40% (d) 80%

MCQ3.12. When does over-modulation occur?

(a) Modulating signal voltage < Carrier voltage
(b) Modulating signal voltage > Carrier voltage
(c) Modulating signal voltage = Carrier voltage
(d) Modulating signal voltage = 0

MCQ3.13. What is the modulation index value if V_{max}= 8 V and V_{min} = 4 V.
(a) 30% (c) 66.7%
(b) 33.3% (d) 90%

MCQ3.14. The modulating signal voltage in problem MCQ3.13 is
(a) 8 V (c) 4 V
(b) 6 V (d) 2 V

MCQ3.15. The ratio between modulating signal voltage and the carrier voltage is called?
(a) Modulation index
(b) Amplitude modulation
(c) Noise factor
(d) Ratio of modulation

MCQ3.16. The resultant modulation index for a multi-tone AM modulated signal with modulation indices of 0.3 and 0.4 is
(a) 0.3 (c) 0.5
(b) 0.4 (d) 0.6

MCQ3.17. A 2 kW carrier is sinusoidally modulated by two carriers corresponding to a modulation index of 40% and 30%, respectively. The total radiated power is
(a) 1.25 kW (c) 3.25 kW
(b) 2.25 kW (d) 4.25 kW

MCQ3.18. In a DSB-SC system with 100% modulation, the power saving is
(a) 30% (c) 66.7%
(b) 33.3% (d) 90%

MCQ3.19. An AM signal has total radiated power of 1 kW. The carrier power and each sidebands power for 100% modulation are
(a) 0.5 kW, 0.25 kW
(b) 666.67 kW, 166.67 kW
(c) 166.67 W, 666.67 W
(d) 666.67 W, 166.67 W

MCQ3.20. A carrier signal $c(t) = 50\sin(2\pi \times 10^5 t)$ is amplitude modulated by $m(t) = 10\sin(2\pi \times 500t)$. The upper and lower sidebands frequencies and bandwidth of modulated signal are
(a) 100.5 kHz, 99.5 kHz, 1 kHz
(b) 99.5 kHz, 100.5 kHz, 1 kHz
(c) 100.5 kHz, 99.5 kHz, 2 kHz
(d) 99.5 kHz, 100.5 kHz, 2 kHz

MCQ ANSWERS

MCQ3.1	(a)	MCQ3.11	(c)
MCQ3.2	(c)	MCQ3.12	(b)
MCQ3.3	(b)	MCQ3.13	(b)
MCQ3.4	(d)	MCQ3.14	(d)
MCQ3.5	(a)	MCQ3.15	(a)
MCQ3.6	(a)	MCQ3.16	(c)
MCQ3.7	(c)	MCQ3.17	(b)
MCQ3.8	(b)	MCQ3.18	(c)
MCQ3.9	(b)	MCQ3.19	(d)
MCQ3.10	(a)	MCQ3.20	(a)

CHAPTER 4

ANGLE MODULATION

Definition

Angle modulation is the process in which the carrier signal's total phase angle is varied in accordance with the instantaneous value of information or baseband or modulating signal.

Highlights

Expression

$$\phi_{\mathrm{FM}}(t) = A_c \cos\left(\omega_c t + k_f \int m(t)dt\right)$$

$$\phi_{\mathrm{PM}}(t) = A_c \cos\left(\omega_c t + k_P m(t)\right)$$

4.1 Introduction

Instead of amplitude, the other parameters of the carrier wave are phase and frequency. Any change in these properties in accordance with the instantaneous value of modulating signal is called phase modulation (PM) and frequency modulation (FM) respectively. Both modulations have similar properties; therefore, both are studied as angle modulation or exponential modulation.

4.2 Angle Modulation

Angle modulation (or exponential modulation) is the process in which the carrier signal's total phase angle is varied in accordance with the instantaneous value of information or baseband or modulating signal. In angle modulation, the amplitude of the carrier signal remains constant. Unlike AM, angle modulation is nonlinear modulation.

Let the carrier signal is expressed as

$$c(t) = A_c \cos(\omega_c t) \tag{4.1}$$

Here, the angle of the carrier signal is given as $\theta = \omega_c t$. Since angle θ itself is also a function of the carrier frequency ω_c, the angle modulation is categorized into following two categories:

1. Frequency modulation (FM)
2. Phase modulation (PM)

Waveforms of different types of modulation techniques are shown in Fig. 4.1.

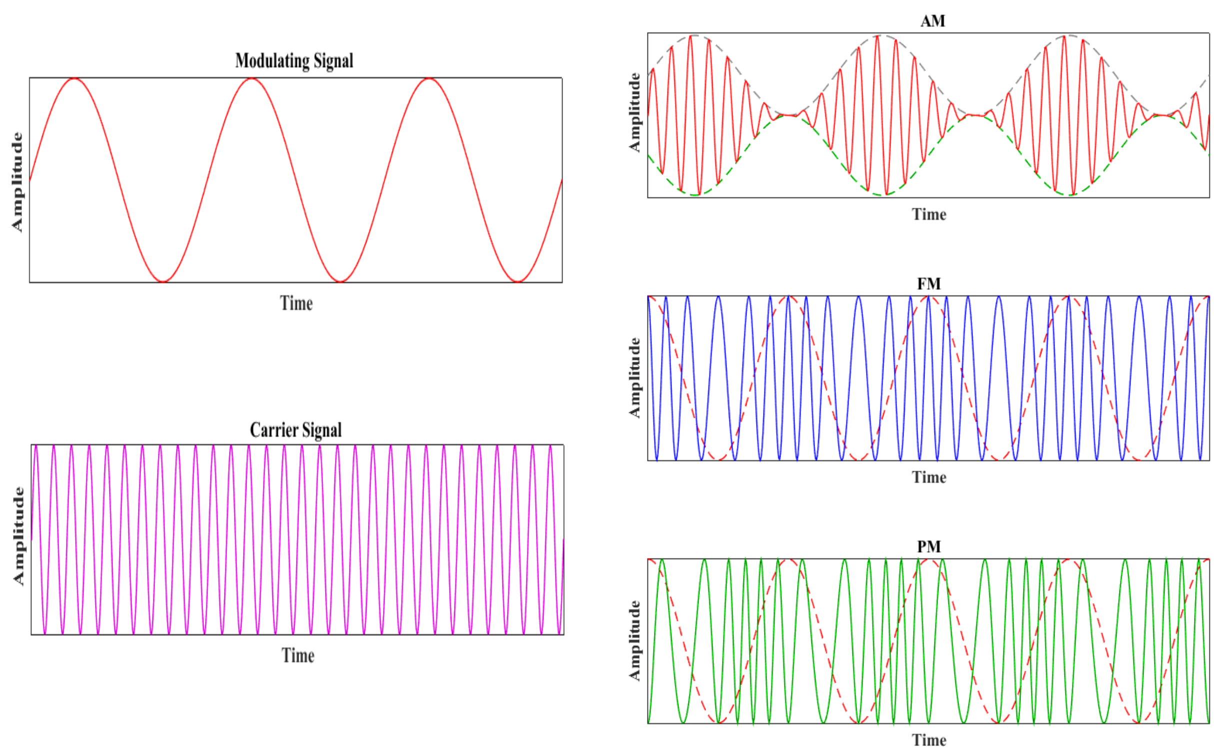


Fig. 4.1 Waveforms of different types of modulation techniques

4.2.1 Frequency Modulation (FM)

The procedure of varying the carrier frequency in accordance with the instantaneous value of the modulating signal is called frequency modulation (FM).

Let the expression of the carrier wave is

$$c(t) = A_c \cos(\omega_c t + \varphi) \tag{4.2}$$

The total phase angle of the carrier signal is given as

$$\psi_c = (\omega_c t + \varphi) \tag{4.3}$$

Differentiating Eq. (4.3)

$$\frac{d\psi_c}{dt} = \omega_c \tag{4.4}$$

In Eq. (4.4), the derivative term $\frac{d\psi_c}{dt}$ is constant with time for the unmodulated carrier. In general, the derivative term may not be constant with time; instead, it may vary with time. This time-dependent term is called instantaneous frequency and represented as ω_i. Therefore,

$$\frac{d\psi_c}{dt} = \omega_i \Rightarrow \psi_c = \int \omega_i dt \tag{4.5}$$

In FM, the instantaneous value of carrier frequency varied in accordance with modulating signal, therefore, ω_i is given by

$$\omega_i = \omega_c + k_f m(t) \tag{4.6}$$

where, k_f is called frequency sensitivity and expressed in Hz/volt.

Hence, the total phase angle of the carrier wave is

$$\begin{aligned} \psi_c &= \int \omega_i \, dt = \int \left(\omega_c + k_f m(t)\right) dt \\ &= \omega_c t + k_f \int m(t)\, dt \end{aligned} \tag{4.7}$$

Therefore, the expression of the FM wave is represented as

$$\phi_{\text{FM}}(t) = A_c \cos\left(\omega_c t + k_f \int m(t) dt\right) \tag{4.8}$$

4.2.2 Phase Modulation (PM)

The procedure of varying the phase angle of the carrier signal in accordance with the instantaneous value of the modulating signal is called phase modulation (PM).

The instantaneous phase angle of the carrier wave is given by

$$\psi_i = \omega_c t + k_P m(t) \tag{4.9}$$

Therefore, the PM wave equation is expressed as

$$\phi_{\text{PM}}(t) = A_c \cos\left(\omega_c t + k_P m(t)\right) \tag{4.10}$$

where, k_p is called phase sensitivity and expressed in radians/volt.

4.2.3 Relationship between PM and FM

From Eqs. (4.8) and (4.10), it is clear that both FM and PM are closely related to each other. In other words, both modulation techniques are a form of phase angle variation, so both are closely related to each other. FM is obtained by the phase modulator if modulating signal is first integrated and further, this integrated signal is applied at the input of the phase modulator, as shown in Fig. 4.2(a).

Similarly, if the modulating signal is firstly differentiated and then applied to the frequency modulator, the output would be phase modulated wave, as shown in Fig. 4.2(b)

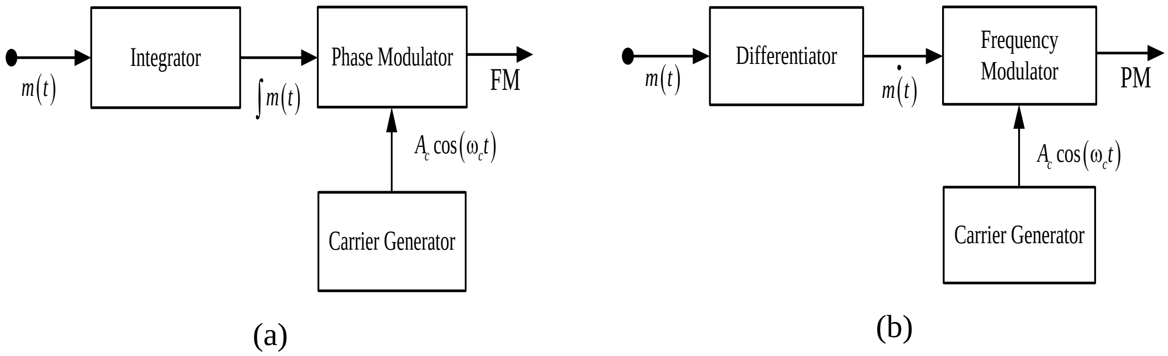


Fig. 4.2 (a) FM generation using phase modulator (b) PM generation using frequency modulator

4.3 Frequency Deviation

A single tone modulating signal is given as

$$m(t) = A_m \cos(\omega_m t) \tag{4.11}$$

From Eq. (4.4) and Eq. (4.8), The instantaneous frequency of the modulated signal for FM is expressed as

$$\omega_i = \left(\omega_c + k_f m(t)\right) = \omega_c + k_f A_m \cos(\omega_m t) \tag{4.12}$$

The deviation around the centre frequency ω_c is defined as frequency deviation and given as

$$\Delta\omega = \left|k_f A_m \cos(\omega_m t)\right|_{\max} = k_f A_m \tag{4.13}$$

Hence, instantaneous frequency is

$$\omega_i = \omega_c + \Delta\omega \cos(\omega_m t) \tag{4.14}$$

The phase angle of the modulated wave is obtained by integrating the instantaneous frequency and given as

$$\psi_i = \int \omega_i \, dt = \int \left[\omega_c + \Delta\omega \cos(\omega_m t)\right] dt$$

$$= \omega_c t + \frac{\Delta\omega}{\omega_m} \sin(\omega_m t)$$

$$= \omega_c t + \beta \sin(\omega_m t) \tag{4.15}$$

Here, the term β is defined as the deviation ratio. For single tone modulation, β is also called frequency modulation index $\left(m_f\right)$. Therefore, frequency modulation index m_f is given as

$$m_f = \frac{\Delta\omega}{\omega_m} = \frac{k_f A_m}{\omega_m} \tag{4.16}$$

For multiple tone signals, the frequency deviation is expressed as

$$\Delta\omega = \left|k_f m(t)\right|_{\max} \tag{4.17}$$

According to the sign of $k_f m(t)$, the frequency deviation will be either positive or negative, as shown in Fig. 4.3.

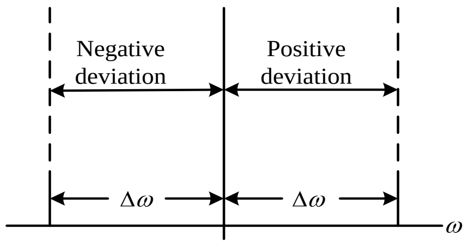


Fig. 4.3 Frequency deviation

4.4 The Bandwidth of Angle Modulation

The angle modulated waveform is represented in the exponential waveform as

$$\phi_{\text{EM}}(t) = \text{Re}\left[e^{j\left(\omega_c t + k_f \int m(t)dt\right)}\right] \tag{4.18}$$

Let $x = \int m(t)dt$

So, $$\phi_{\text{EM}}(t) = \text{Re}\left[e^{j\left(\omega_c t + k_f x\right)}\right] \tag{4.19}$$

$$e^{j\left(\omega_c t + k_f x\right)} = e^{jk_f x} e^{j\omega_c t} \tag{4.20}$$

Further expansion of the exponential term gives

$$e^{j\left(\omega_c t + k_f x\right)} = \left[1 + jk_f x(t) - \frac{k_f^2}{|\underline{2}} x^2(t) + \ldots\ldots + j^n \frac{k_f^n}{|\underline{n}} x^n(t) + \ldots\right] e^{j\omega_c t} \tag{4.21}$$

$$\begin{aligned}\phi_{EM}(t) &= \text{Re}\left\{\left[1 + jk_f x(t) - \frac{k_f^2}{|\underline{2}} x^2(t) + \ldots\ldots + j^n \frac{k_f^n}{|\underline{n}} x^n(t) + \ldots\right] e^{j\omega_c t}\right\} \\ &= A\left[\cos\omega_c t - k_f x(t)\sin\omega_c t - \frac{k_f^2}{|\underline{2}} x^2(t)\cos\omega_c t + \frac{k_f^3}{|\underline{3}} x^3(t)\sin\omega_c t + \ldots\ldots\right]\end{aligned} \tag{4.22}$$

From Eq. (4.22), it is clear that the angle modulated waveform consists of an unmodulated carrier and a number of amplitude-modulated waveforms ($x(t)\sin\omega_c t,\ x^2(t)\cos\omega_c t,\ x^3(t)\sin\omega_c t$). If $M(\omega)$ is bandlimited to ω_m, then $x(t)$ is bandlimted to ω_m, $x^2(t)$ is bandlimted to $2\omega_m$ and so on. Therefore, the theoretical bandwidth of angle modulated waveform is infinity.

4.4.1 Narrowband Frequency Modulation (NBFM)

If k_f is very small, i.e. $\left|k_f m(t)\right| << 1$, then the value of higher-order terms would be very small and may be ignored. Then, the expression of NBFM is

$$\phi_{\text{NBFM}}(t) = A_c\left[\cos\omega_c t - k_f x(t)\sin\omega_c t\right] \text{ where, } \left(x(t) = \int m(t)\,dt\right) \tag{4.23}$$

Like amplitude modulation, NBFM is a linear modulation with the bandwidth of $2\omega_m$. The only difference is that the sidebands spectrum in NBFM has $\pi/2$ phase shift with carrier wave instead of the same phase like AM. Further, FM and AM have different waveforms also. In AM, the frequency remains constant and amplitude varies with time, whereas amplitude remains constant and frequency varies with time in FM.

Similarly, narrowband phase modulation (NBPM) is represented by the Eq. (4.24)

$$\phi_{\text{NBPM}}(t) = A_c\left[\cos\omega_c t - k_p m(t)\sin\omega_c t\right] \tag{4.24}$$

NBPM also has approximate bandwidth of $2\omega_m$.

4.4.2 Wideband Frequency Modulation (WBFM)

If k_f is large enough such that $\left|k_f m(t)\right| >> 1$, then the higher-order terms could not be ignored. Such a case of FM modulation is called wideband frequency modulation (WBFM). Now, we can better approximate the bandwidth of the FM wave.

4.5 Sidebands in Frequency Modulation

Any modulation techniques of the carrier wave always generates sidebands. In AM modulation, sidebands generation is straightforward and responsible for the bandwidth of the modulated signal. Unlike AM, the sidebands generation is somewhat different in angle modulation.

Unlike AM, the frequency modulated signal consists of a carrier signal and the infinite number of sidebands at integer multiples of baseband (or modulating) frequency, i.e. $\omega_c \pm \omega_m$, $\omega_c \pm 2\omega_m$, …, $\omega_c \pm n\omega_m$. The FM sidebands are dependent on both the level of deviation and the modulation index.

Single-tone FM signal is represented by the Bessel’s function, i.e.

$$\cos\left(2\pi f_c t + m_f \sin(2\pi f_m t)\right) = \sum_{n=-\infty}^{\infty} J_n(m_f)\cos\left(2\pi(f_c + nf_m)t\right) \tag{4 25}$$

where, $J_n(m_f)$ represents the strength of n^{th} sideband at $\omega = \omega_c \pm n\omega_m$ for given value of modulation index m_f.

From Fig. 4.4, it is clear that for small values of modulation index m_f, Bessel's coefficient $J_n(m_f)$ decays quickly; therefore, only the first component will be dominated.

For large values of m_f, Bessel's coefficient $J_n(m_f)$ increases to a maximum and then decays like one over square root of the index.

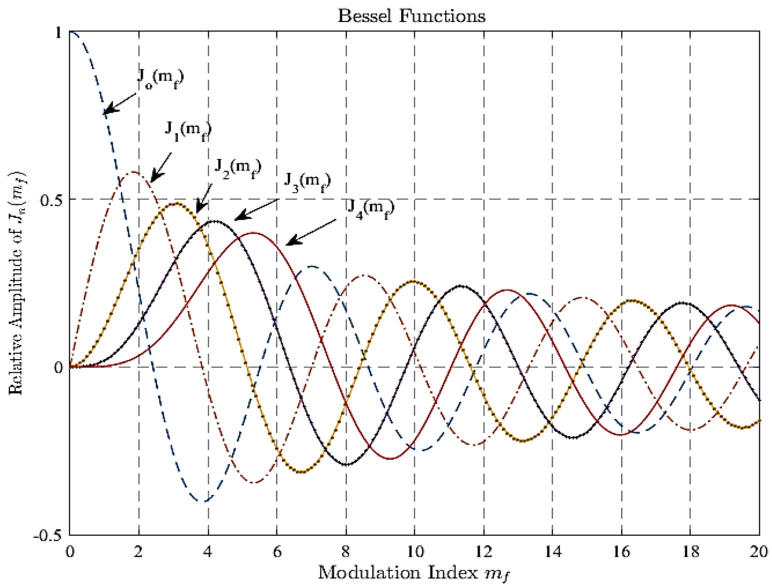


Fig. 4.4 Variations of $J_n(m_f)$ as a function of *n* for various values of m_f

For a fixed value of m_f and large *n*, the approximated value of $J_n(m_f)$ is given as

$$J_n(m_f) = \frac{m_f^n}{2^n n!} \tag{4.26}$$

For $m_f = 1$ and $n = 3$, the approximated value of $J_n(m_f)$ is

$$J_n(m_f) = \frac{1^3}{2^3 3!} = \frac{1}{48} = 0.021$$

The numerical strength of $J_n(m_f)$ for different values of m_f is shown in Table 4.1.

The value of $J_n(m_f)$ is almost negligible for $n > m_f + 1$. Therefore, a significant number of sidebands would be effective for bandwidth calculation.

Hence, the bandwidth of the FM wave is given by

$$\begin{aligned}(\mathrm{BW})_{\mathrm{FM}} &= 2nf_m \\ &= 2(m_f + 1) f_m\end{aligned} \tag{4.27}$$

Since, FM wave consists of an infinite number of sidebands, i.e. $n \to \infty$. Therefore, the bandwidth of FM is theoretically infinite.

Table 4.1 Relative sideband's amplitude for different values of m_f

Sidebands / m_f	Relative Sidebands Amplitude					
	0	**1**	**2**	**3**	**4**	**5**
0.0	1.00					
0.25	0.98	0.12				
0.5	0.94	0.24	0.03			
1.0	0.77	0.44	0.11	0.02		
2.0	0.22	0.58	0.35	0.13	0.03	
2.41	0.0	0.52	0.43	0.20	0.06	0.02

4.6 Carson's Rule for FM Bandwidth

According to Carson's rule, the approximated bandwidth of the FM signal is expressed as

BW = *2* (*Highest modulated frequency* + *Maximum frequency deviation*)

$$(\mathrm{BW})_{\mathrm{FM}} = 2(\omega_m + \Delta\omega) \tag{4.28}$$

Since, $m_f = \dfrac{\Delta\omega}{\omega_m} \Rightarrow \omega_m = \dfrac{\Delta\omega}{m_f}$

Substituting the value of ω_m in Eq. (4.28)

$$(\mathrm{BW})_{\mathrm{FM}} = 2\left(\frac{\Delta\omega}{m_f} + \Delta\omega\right)$$

$$= 2\Delta\omega\left(\frac{1}{m_f} + 1\right) \tag{4.29}$$

Or, $$(\mathrm{BW})_{\mathrm{FM}} = 2\Delta f\left(\frac{1}{m_f} + 1\right) \text{ Hz} \tag{4.30}$$

In another way, $m_f = \dfrac{\Delta\omega}{\omega_m}$

So, $$(\mathrm{BW})_{\mathrm{FM}} = 2\omega_m\left(\frac{\Delta\omega}{\omega_m} + 1\right) = 2\omega_m\left(m_f + 1\right) \tag{4.31}$$

Or, $$(\mathrm{BW})_{\mathrm{FM}} = 2 f_m\left(m_f + 1\right) \text{ Hz}$$

Special Cases:

(i) When $\Delta\omega << \omega_m$, Narrowband FM (NBFM), $m_f << 1$

$$(\mathrm{BW})_{\mathrm{FM}} = 2\omega_m \tag{4.32}$$

(ii) When $\Delta\omega >> \omega_m$, Wideband FM (WBFM), $m_f >> 1$

$$\left(\mathrm{BW}\right)_{\mathrm{FM}} = 2\Delta\omega \qquad (4.33)$$

4.7 Bandwidth of PM

According to Carson's rule, the bandwidth of the PM signal is given by

$$\left(\mathrm{BW}\right)_{\mathrm{PM}} = 2\Delta\omega = 2k_p A_m \omega_m \qquad (4.34)$$

The phase modulation index m_p (or phase deviation θ_d) is given by

$$m_p = k_p A_m \qquad (4.35)$$

4.8 Power of an Angle Modulated Wave

The amplitude of the angle modulated signal remains constant (given as A_c); therefore, the power of the angle modulated waveform is given by $A_c^2/2$ regardless the value of k_p (for PM) or k_f (for FM).

4.9 International Regulation for Frequency Modulation

To avoid interference, the values for the commercial FM broadcast stations are prescribed by CCIR (Consultative Committee for International Radio), which are as follows:

(a) Allowable bandwidth/channel = 200 kHz

(b) Maximum frequency deviation = $\pm 75\,\mathrm{kHz}$

(c) Frequency stability of the carrier = $\pm 2\,\mathrm{kHz}$

Example: FM Radio

The frequency band of FM radio = 88 MHz to 108 MHz.

The frequency deviation = 75 kHz

The upper and lower guard band = 25 kHz.

Therefore,

Frequency separation /channel = Total deviation + Upper guard band + Lower guard band

So, $\text{Frequency separation /channel} = 2\times 75 + 25 + 25 = 200 \text{ kHz}$

Let, the range of is 88.1 MHz to 108.1 MHz

Hence, $\text{Total number of channels} = \dfrac{(108.1-88.1)\times 10^6}{200\times 10^3} = \dfrac{20000}{200} = 100 \text{ channels}$

AM Radio

The frequency band of FM radio = 535 kHz to 1605 kHz. (with guard band 540 to 1600 kHz)

Channels are assigned = 10 kHz

Therefore, $\text{Total number of channels} = \dfrac{(1600-540)\times 10^3}{10\times 10^3} = \dfrac{1060}{10} = 106 \text{ channels}$

Points to Remember

(1) *FM has better noise immunity and higher efficiency in comparison to AM. But these are achieved at the cost of larger bandwidth.*

(2) *In FM radio, the bandwidth and the efficiency both depend on the modulation index and the maximum modulating frequency.*

4.10 Generation of Narrowband FM (NBFM)

The expression for NBFM is given by:

$$\phi_{FM}(t) = A_c \cos(\omega_c t) - A_c k_f g(t) \sin(\omega_c t) \tag{4.36}$$

Similarly, the expression for NBPM is given by:

$$\phi_{PM}(t) = A_c \cos(\omega_c t) - A_c k_p m(t) \sin(\omega_c t) \tag{4.37}$$

The block diagrams for generation of NBFM and NBPM are shown in Figs 4.5(a)-(b).

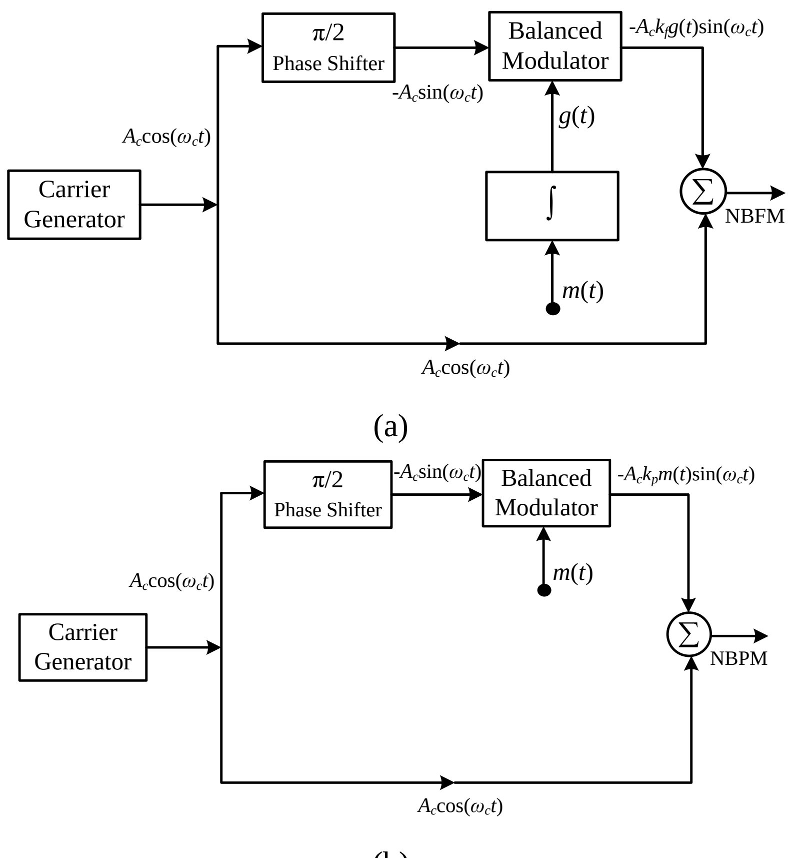


Fig. 4.5 (a) Generation of NBFM (b) Generation of NBPM

4.11 Generation of FM Waves: FM Modulators and Transmitter

There are two ways to generate FM waves which are

1. Direct generation or Parameter variation method
2. Indirect generation or Armstrong FM transmitter

Further, the classification of FM generation is shown in Fig. 4.6.

4.11.1 Direct Generation

The modulating (baseband) signal directly modulates the carrier in a direct generation approach. The carrier signal is generated by an electronic oscillator circuit that uses a parallel resonant tuned *LC* circuit. The oscillating frequency is given by:

$$\omega_c = \frac{1}{\sqrt{LC}} \tag{4.38}$$

The carrier frequency ω_c can be made varied according to the modulating signal $m(t)$ if *L* or *C* is changed according to $m(t)$.

In a voltage-controlled oscillator (VCO), the frequency is controlled by an external voltage, i.e. modulating signal. The oscillating frequency varies linearly with the control voltage by using a voltage variable capacitor in the tuned circuit. This voltage variable capacitor is known as varicap or varactor. Thus, the instantaneous frequency of the FM signal is given as

$$\omega_i = \omega_c + k_f m(t) \tag{4.39}$$

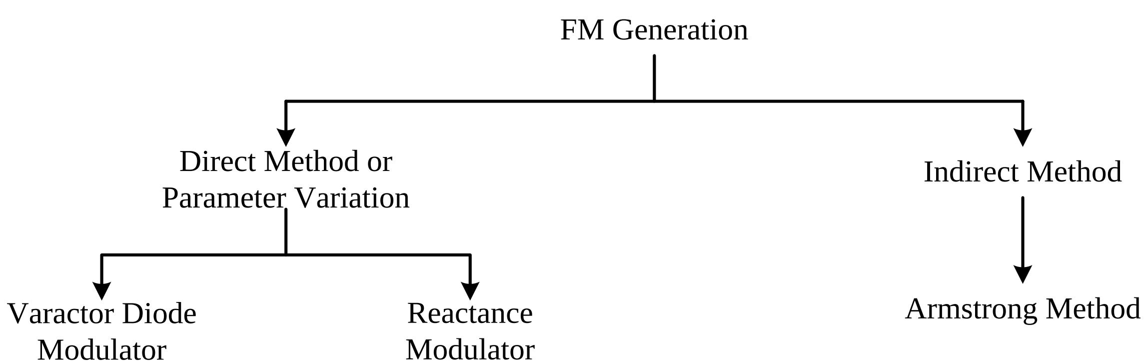


Fig. 4.6 FM generation methods

4.11.1.1 FM Modulation Circuit using Varactor Diode

The varactor diode is a semiconductor diode whose junction capacitance changes with dc bias voltage. The capacitor C_d of the diode is given by the relation:

$$C_d = K(v_D)^{-1/2} \tag{4.40}$$

where,

$$v_D = V_o + m(t) \tag{4.41}$$

If $C \ll C_d$, the total capacitance of the tank circuit is $C_o + C_d$ and hence the instantaneous frequency of oscillation ω_i is given by

$$\omega_i = \frac{1}{\sqrt{L_o(C_o + C_d)}} \tag{4.42}$$

Substituting C_d

$$\omega_i = \left[L_o \left(C_o + K v_D^{-1/2} \right) \right]^{-1/2} \tag{4.43}$$

Therefore, the oscillator frequency ω_i is dependent on the modulating signal $m(t)$ and thus, frequency modulated waveform is generated by varactor diode method (Fig. 4.7).

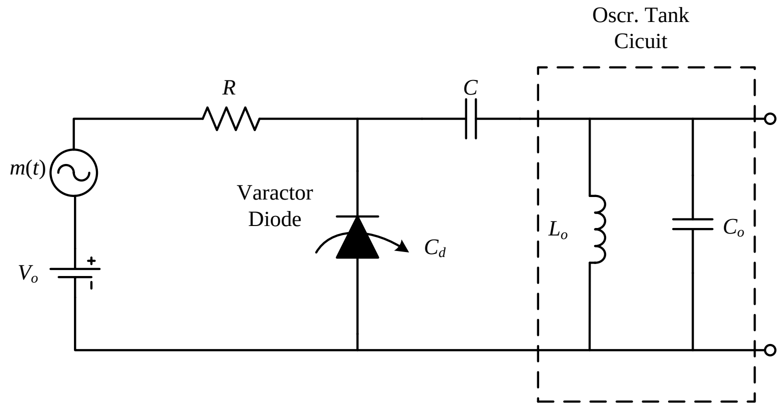


Fig. 4.7 Varactor diode FM modulator

4.11.1.2 Demerits of Direct Methods

1. In this method, the carrier generation is directly dependent on the modulating signal; therefore, the use of a crystal oscillator type stable oscillator is complicated. Hence, higher-order stability in carrier frequency cannot be obtained easily.
2. The FM signal is distorted due to harmonics of the modulating signal which are generated because of the non-linearity of the circuit configuration.

4.11.1.3 FM Transmitter using Direct Method of Frequency Modulation

The direct modulated FM transmitter is shown in Fig. 4.8. In this transmitter, a pre-emphasis circuit is used to eliminate the effect of noise at higher audio frequencies for threshold improvement. The operation of the pre-emphasis circuit is further explained in this chapter.

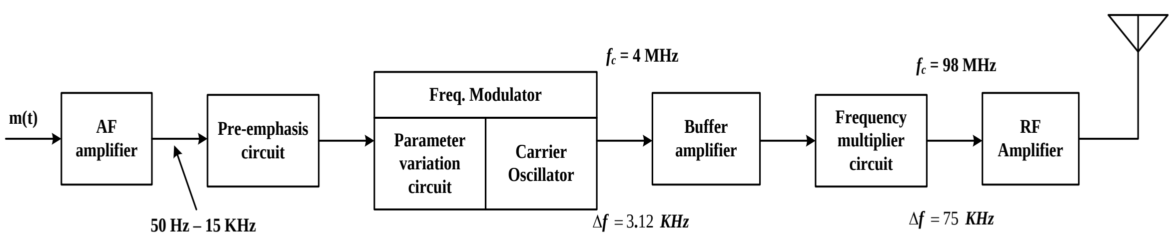


Fig. 4.8 Direct modulated FM transmitter

4.11.2 Indirect Generation or Armstrong Method

In this approach, the NBFM signal is generated indirectly by the use of a phase modulator. This NBFM signal is used to generate the WBFM signal, as shown in Fig. 4.9. Moreover, multiple stage frequency multipliers are used for the NBFM to the WBFM conversion. A 12th fold increase in frequency deviation for the WBFM generation is achieved either by one 12th order nonlinear device or two second-order and one third-order nonlinear devices; thereafter, a bandpass filter centered on is used to select only appropriate term for WBFM.

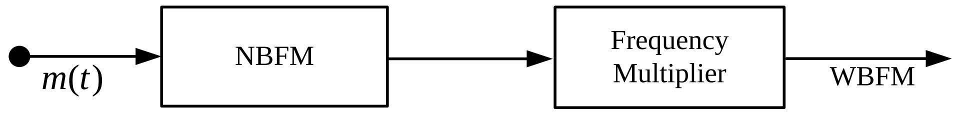


Fig. 4.9 Simplified block diagram of Armstrong indirect FM wave generator

The block diagram of a commercial FM transmitter using the indirect approach or Armstrong's method is shown in Fig. 4.10.

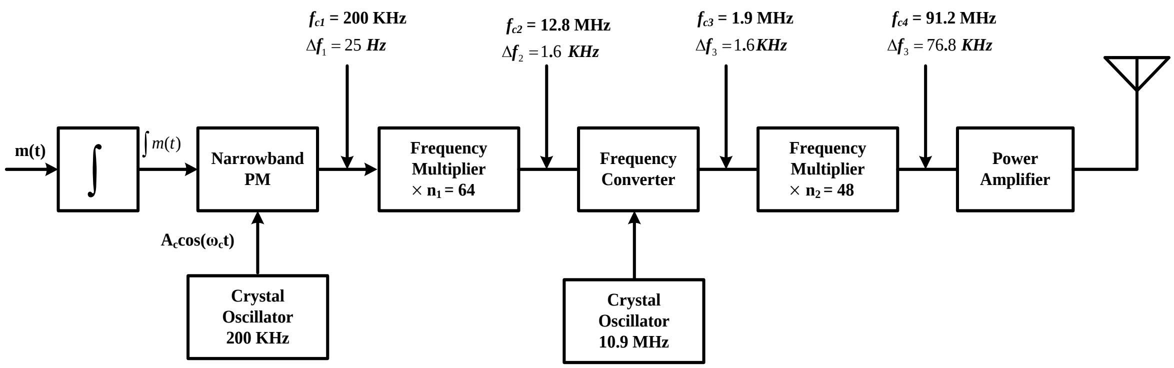


Fig. 4.10 Armstrong indirect FM transmitter

The working principle of WBFM generation by Armstrong's method can be explained by the example given below:

SE4.1 Design an Armstrong FM modulator to generate an FM carrier with f_c = 90 MHz and Δf = 75 kHz. There is the availability of a narrow band FM generator with $f_{c,NB} = 200$ kHz and $\Delta f_{NB} = 25$ Hz. The crystal oscillator generates a signal of 10.9 MHz.

Sol: NBFM generator generates a signal with the following frequency and frequency deviation value

$$f_{c,NB} = 200 \text{ kHz} \quad \& \quad \Delta f_{NB} = 25 \text{ Hz}$$

The desired values of the carrier frequency and the deviation at the transmitter output are as follows:

$$f_c = 90\,\text{MHz} \quad \& \quad \Delta f = 75 \text{ kHz}$$

Therefore, multiplication factors needed

(i) For the desired deviation, is $\dfrac{75 \text{ kHz}}{25 \text{ Hz}} = 3000$; whereas

(ii) For the desired carrier, is $\dfrac{90 \text{ MHz}}{200 \text{ kHz}} = 450$

Now, if the multiplication factor is selected as 3000 in a single stage to achieve the desired frequency deviation, the carrier frequency at the output of the transmitter would be 600 MHz which is very far from the desired value of 90 MHz.

Therefore, the whole multiplication operation is performed into two stages (with multiplication factors as n_1 and n_2) which are as follows:

1. Total multiplication factors should be chosen in such a way so that total multiplication could achieve the desired derivation. Therefore,

$$n_1 \times n_2 = 3000 \tag{4.44}$$

2. Before, second stage multiplication, the carrier frequency of the first stage multiplication output is shifted downward to $\left(n_1 f_{c1,NB} - f_{co}\right)$ by frequency converter (mixer).

3. Now, the output of the frequency converter or mixer is increased n_2 times to obtain the desired carrier frequency of f_c = 90 MHz. So,

$$n_2\left(n_1 f_{c1,NB} - f_{co}\right) = f_c \tag{4.45}$$

Put $f_{c1,NB} = 200$ kHz, f_{co} = 10.9 MHz & f_c = 90 MHz

$$n_2\left(0.2n_1 - 10.9\right) = 90$$

$$n_2\left(\frac{0.2 \times 3000}{n_2} - 10.9\right) = 90$$

$$\Rightarrow \left(600 - 10.9n_2\right) = 90 \Rightarrow 10.9n_2 = 510$$

$$\Rightarrow n_2 = \frac{510}{10.9} = 46.78$$

$$n_1 \times n_2 = 3000$$

$$\Rightarrow n_1 = \frac{3000}{n_2} = \frac{3000}{46.78}$$

$$\Rightarrow n_1 = 64.12$$

The values are rounded off to make the multiplication factors as the factors of 2 and 3 to make the design of the multiplication circuit simplified. Therefore, multiplication factors are chosen as n_1 = 64 = 2^6 and n_2 = 48 = 3 x 2^4.

Although indirect FM generation has the benefit of frequency stability, still it suffers from two types of distortion, which are frequency distortion and amplitude distortion. The NBFM generated by Armstrong's method has some distortion. This method has some amplitude modulation in its output. Amplitude limiting in frequency multipliers removes most of the distortion.

4.12 FM Demodulators

The modulating signal from the FM signal is extracted by the FM demodulator or detector in the following two steps:

1. The first step is to convert the FM signal into a corresponding AM signal by a frequency-dependent circuit (i.e. a circuit whose output voltage depend on an input frequency). Such circuit is named as frequency discriminators.
2. The second step is to recover the original baseband signal $m(t)$ from above AM signal by using a linear diode envelope detector.

Note: A simple *RL* circuit can be used as a discriminator, but this circuit has poor sensitivity as compared to a tuned *LC* circuit.

4.12.1 Types of FM Discriminators

The FM discriminators are categorized into the following two types:

1. Slope detectors
2. Phase difference discriminators

4.12.1.1 Slope detector

The operating principle of a slope detector depends on the slope of the frequency response characteristics of a frequency selective network. Two main slope detectors come under this category:

1. Simple slope detector or Single tuned discriminator circuit
2. Balanced slope detector or Stagger tuned discriminator

Simple slope detector or Single tuned discriminator circuit

One of the simplest forms of the FM demodulator is a single tuned discriminator circuit. The block diagram of the simple slope detector is shown in Fig. 4.11. This circuit depends on the selectivity of the receiver itself to provide the demodulation. Such type of detector is not much effective as the selectivity curve is not linear in the whole response curve. Therefore, nonlinearity in the curve comes in the form of distortion in output which makes the receiver sensitive to amplitude variations. So, a simple slope detector has very limited applications.

The characteristic is slightly off-tuned at ω_c. A small variation in the frequency $(\Delta\omega)$ of the input signal will produce a change in the amplitude E_{AM} by an amount $\Delta E_{AM} = \alpha(\Delta\omega)$, as shown in Fig. 4.12. This process produces the corresponding AM wave. Further, the envelope detector detects the modulating signal.

Although the circuit is simple and inexpensive, it suffers from the following demerits

1. The circuit's nonlinear characteristics cause harmonic distortion.
2. It does not eliminate amplitude variations and the output is sensitive to any amplitude variations in the input FM signal, which is not a desirable feature.

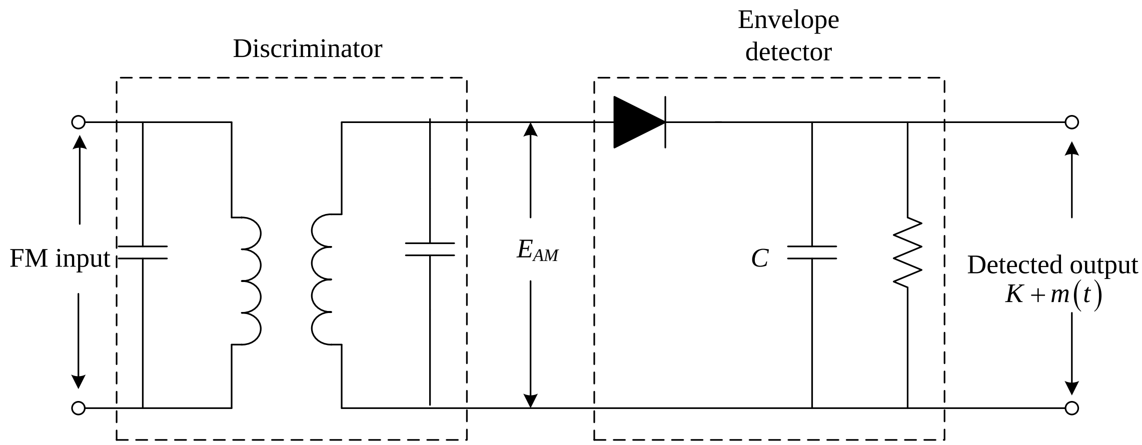


Fig. 4.11 Simple slope detector

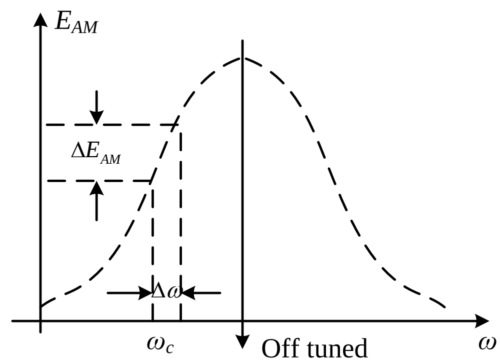


Fig. 4.12 Simple slope detector characteristics

Balanced slope demodulator or Stagger tuned discriminator

The balanced slope modulator circuit consists of two *LC* circuits, as shown in Fig. 4.13. The characteristics of balanced slope modulator is shown in Fig. 4.14. The two tuned circuits are in the staggered tuned mode, one is tuned above the carrier frequency ω_c (curve e_1) and the other is tuned below ω_c (curve e_2). The resultant is linear, as shown by the dotted line.

The disadvantage of this circuit is that the linear characteristic is limited to a slight frequency deviation $(\Delta\omega)$.

4.12.1.2 Phase difference Discriminators

There are two types of phase difference discriminators circuit for FM demodulator

1. Foster-Seeley discriminator
2. Ratio detector

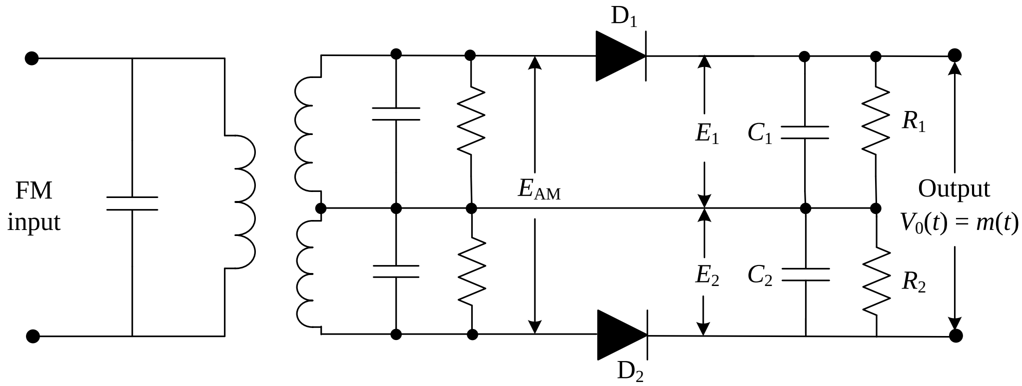


Fig. 4.13 Balanced slope demodulator

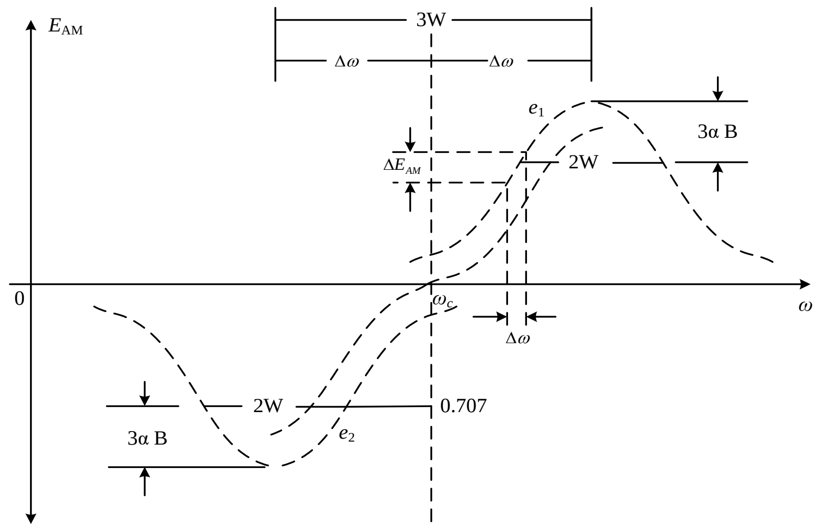


Fig. 4.14 Characteristics of balanced slope demodulator

Foster-Seeley discriminator

This circuit is also called a center-tuned discriminator as the center of the secondary is connected to the top of the primary through a capacitor *C*. The circuit arrangement is shown in Fig. 4.15(a). The functions of this capacitor *C* are as follows:

1. It blocks the dc from primary to secondary.

It couples the signal from primary to centre tapping of the secondary.

The working operation of the Foster-Seeley discriminator is as follows

The secondary voltages across half winding will be equal in magnitude but with opposite polarities. Therefore, the voltages applied at the diodes input terminals are

$$V_{D1} = V_L + V_{s1}$$
$$V_{D2} = V_L - V_{s2}$$

The diodes are so arranged that the output voltage V_0 is equal to the arithmetic difference

$$V_0 = |V_{02}| - |V_{01}| \tag{4.46}$$

The voltage V_0 will vary with the instantaneous frequency in accordance with the difference $|V_{02}|-|V_{01}|$ shown by the dotted line in Fig. 4.15(b). This is called the discriminator characteristics. It is zero at resonance, negative below resonance and positive above resonance. The linear region (between peaks V_{D1} to V_{D2}) of the discriminator characteristics is called the peak separation region.

The disadvantage is that any variation in the amplitude of the input FM signal due to noise modifies the discriminator characteristic.

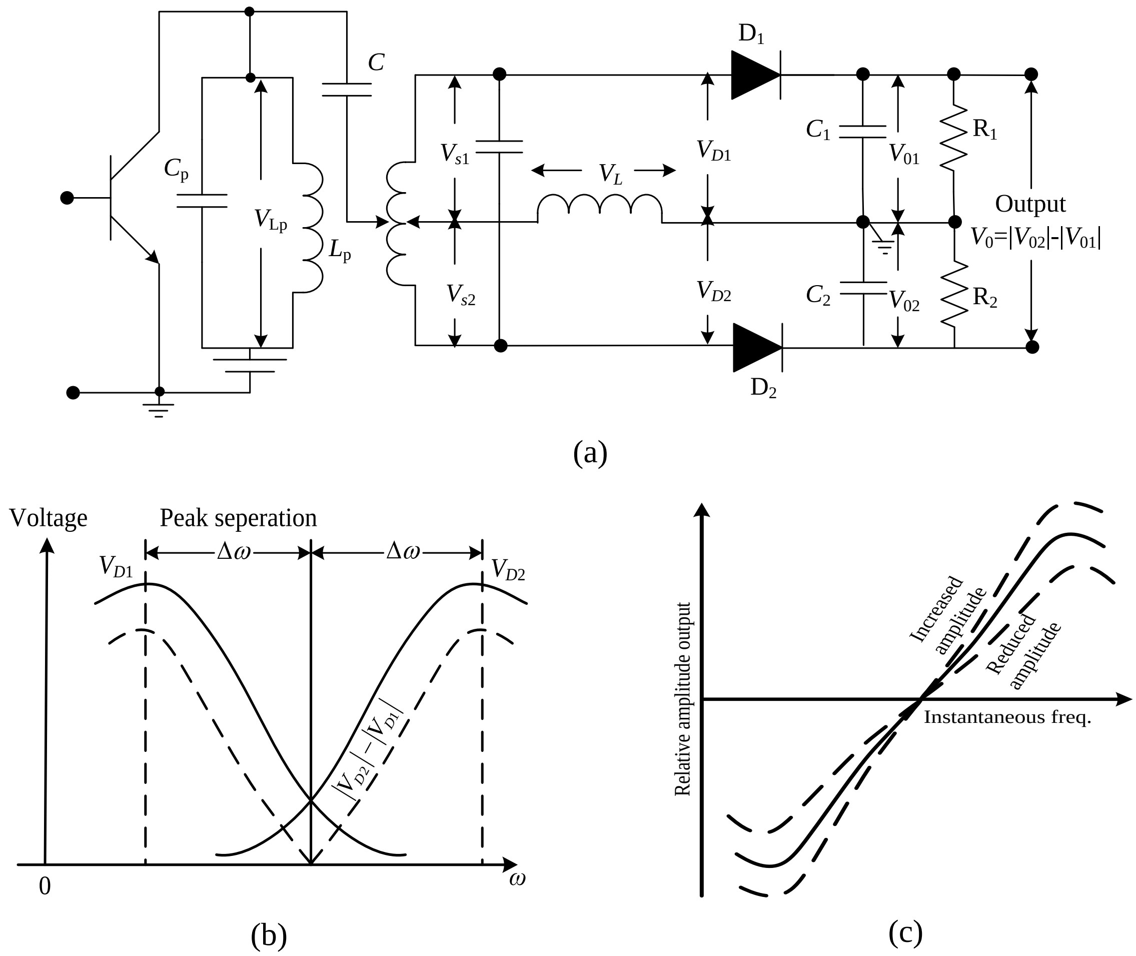


Fig. 4.15 (a) Foster Seeley circuit (b) and (c) Discriminator characteristics

Ratio detector

This is an improvement of Foster Seeley discriminator. The difference from Foster Seeley configuration is

1. The polarity of the diode D_2 is reversed and
2. The output is taken from the center of the resistor *R*.

The ratio detector uses two diodes along with a few resistors and capacitors, as shown in Fig. 4.16. This circuit does not respond to amplitude variations since *R* and *C* suppressed the amplitude fluctuation; therefore, there is no need for a limiter circuit. The working operation of the ratio detector is similar to Foster Seeley except for the output voltage V_R across the resistor *R*, which is now the sum of V_{01} and V_{02}.

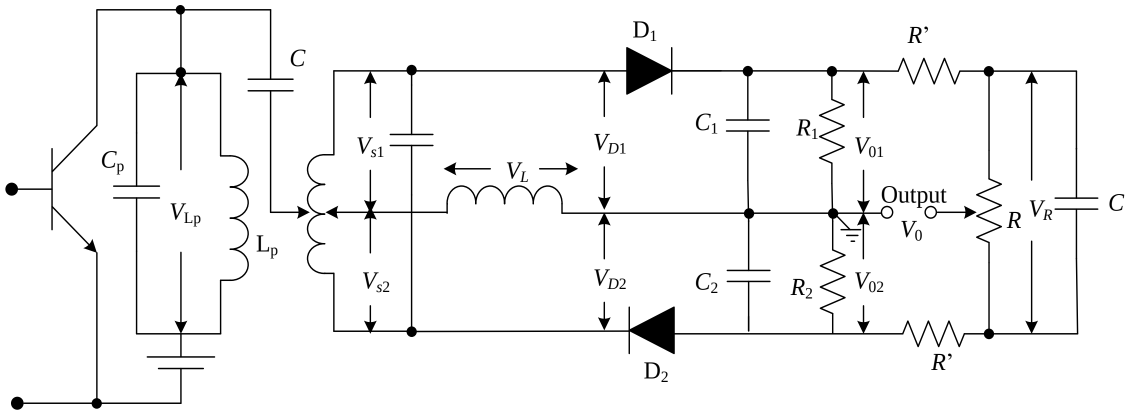


Fig. 4.16 Ratio detector

$$V_R = |V_{01}| + |V_{02}| \tag{4.47}$$

The output voltage $V_0 = |V_{02}| - \left|\frac{V_R}{2}\right|$ (4.48)

$$V_0 = V_{02} - \frac{V_{02} + V_{01}}{2} = \frac{V_{02} - V_{01}}{2} \tag{4.49}$$

Although the ratio detector performed well, it uses the transformer as well as other wound components; therefore, this is an expensive form of the FM detector.

4.12.1.3 Zero-Crossing Detector

The zero-crossing detector works on the principle of the instantaneous frequency of FM signal, which is given as

$$f = \frac{1}{2\Delta t} \tag{4.50}$$

where, Δt is the time difference between two adjacent zero-crossing points as shown in Fig. 4.17 Consider a time duration *T* such that

(i) $T < \frac{1}{f_m}$ and

(ii) $T > \frac{1}{f_c}$

where, f_m and f_c are the modulating frequency and carrier frequency, respectively.

If the total number of zero-crossing points in time duration T is N, then Δt is given by

$$\Delta t = \frac{T}{N} \tag{4.51}$$

Therefore, the instantaneous frequency is obtained as

$$f_i = \frac{1}{2\Delta t} = \frac{N}{2T} \tag{4.52}$$

Thus, the baseband signal is recovered if N is known.

The zero crossing detector along with pulse generator and low pass filter are used as FM demodulator. The block diagram of FM demodulator using zero crossing detector and the waveforms at different points of demodulator are shown in Fig. 4.18 and Fig. 4.19, respectively.

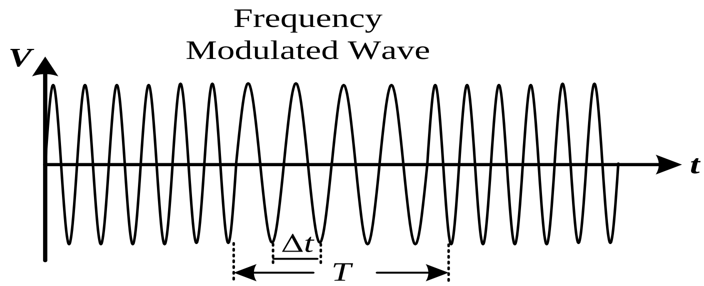


Fig. 4.17 Time difference between two adjacent zero-crossing points

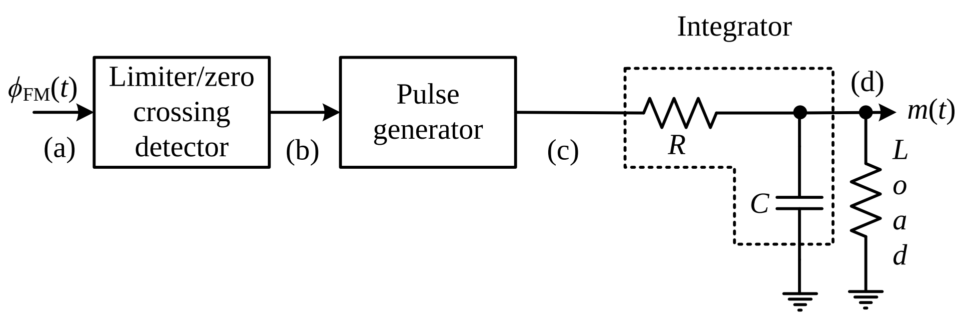


Fig. 4.18 The block diagram of zero crossing detector application as FM demodulator

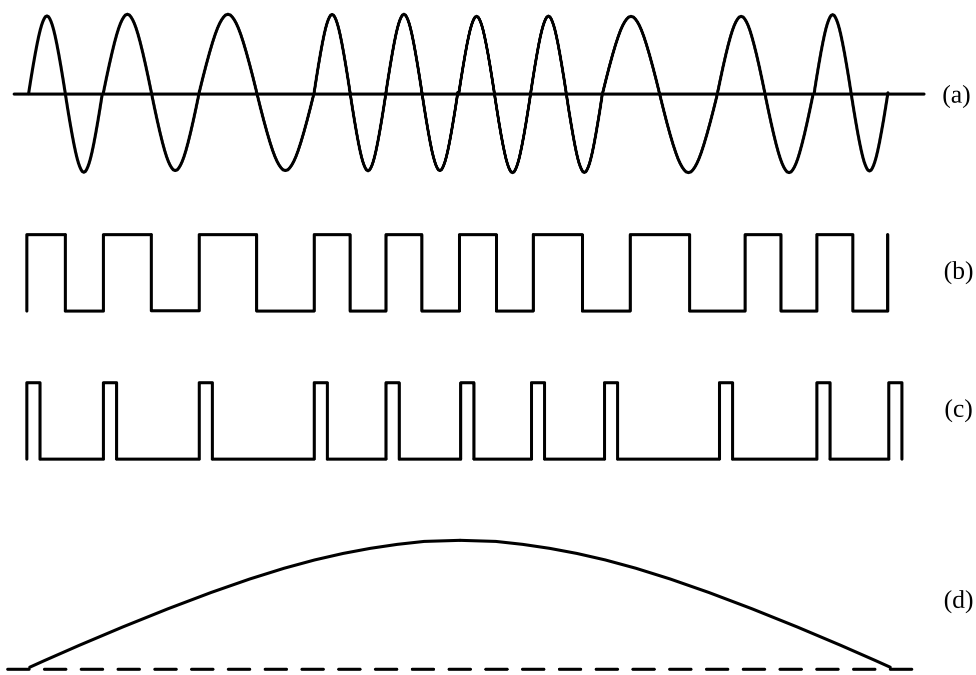


Fig. 4.19 The waveforms at different points of Fig. 4.18

A comparison between different types of FM detectors is given in Table 4.2.

Table 4.2 A comparison of different detector methods of FM detectors

S. N.	Parameters	Ratio detector	Balanced slope detector	Phase discriminator
1.	**Alignment/tuning**	Not critical	Critical as three circuits are to be tuned at different frequencies	Not critical
2.	**Output characteristics depends on**	The primary and secondary phase relation	The Primary and secondary frequency relation	The primary and secondary phase relation
3.	**Linearity of the output characteristics**	Good	Poor	Very good
4.	**Amplitude Limiting**	provided by the ratio detector	Not provided inherently	Not provided inherently
5.	**Applications**	TV receiver sound section, narrowband FM receivers	Not used in practice	FM radio, satellite station receiver etc

4.13 Pre-emphasis and De-emphasis

(a) Pre-emphasis

The high frequencies components in FM modulation are most affected by the noise. Therefore, a signal loss occurs during the transmission and the received signal at the receiver end is less than the required sensitivity.

So, a pre-emphasis circuit is used to increase the signal strength to improve the noise immunity of the FM signals at higher frequencies. This is possible by increasing the amplitude of the higher frequencies FM signals. Amplification in amplitude is achieved by the pre-emphasis circuit shown in Fig. 4.20(a). The pre-emphasis circuit is a high pass filter with the characteristics shown in Fig. 4.20(b).

The working principle of the pre-emphasis circuit is as follows:

The modulating signal is applied to a high pass RC filter which allows only high frequencies components to go forward to the input of the FM modulator. The reactance of capacitor C is given as

$$X_C = \frac{1}{j2\pi fC}$$

Therefore, the reactance of capacitance decreases with increment in frequency and therefore, modulating voltage applied to the FM modulator also increases. The frequency response characteristics of HPF is shown in Fig. 4.20(b). The amplification or boosting is done according to this curve. Thus we can say

"The pre-emphasis circuit is a high pass filter which is used for artificial boosting of higher frequencies components of FM signal."

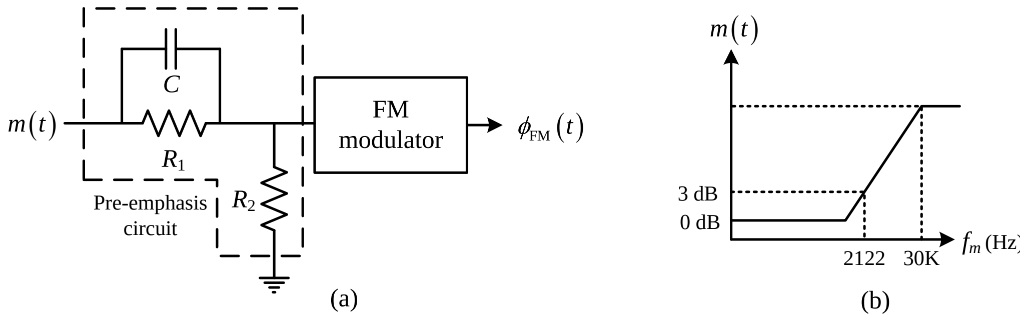


Fig. 4.20 (a) Pre-emphasis circuit (b) Characteristics of pre-emphasis circuit

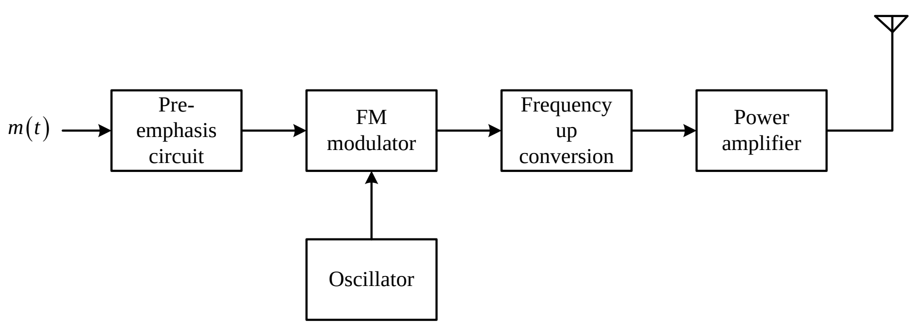


Fig. 4.21 A pre-emphasis circuit at the transmitter side

The use of a pre-emphasis circuit at the transmitter side is shown by the block diagram in Fig. 4.21. Standardized 75 μsec of pre-emphasis is used in the US for FM transmission.

(b) De-emphasis

The effect of pre-emphasis is nullified at the receiver end by the use of the de-emphasis circuit. Therefore, de-emphasis is defined as a process at the receiver end to compensate the artificial boosting of higher frequency components done by the pre-emphasis, i.e. the process to bring the artificially boosted higher frequencies components to their original amplitude is called de-emphasis.

The use of a de-emphasis circuit at the receiver end and its characteristics are shown by the block diagram shown in Fig. 4.22(a)-(b).

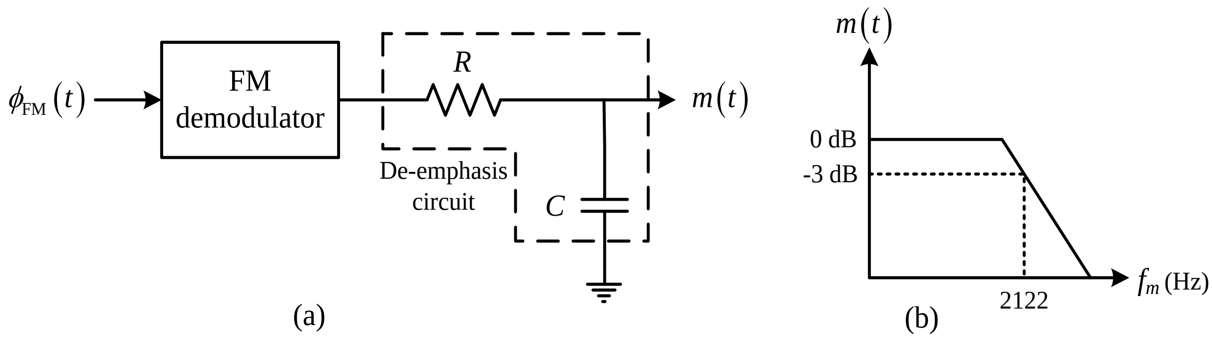


Fig. 4.22 (a) De-emphasis circuit and (b) its characteristics

The de-emphasis circuit is a low pass RC filter. The reactance of capacitor C decreases with increment in f_m; therefore, the output of the de-emphasis circuit reduces. If $75\ \mu\text{sec}$ of de-emphasis is desired, then it is required to design an LPF with the time constant $RC = 75\ \mu\text{sec}$. Hence, the higher cut-off frequency of the de-emphasis circuit is given by

$$f = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 75 \times 10^{-6}} = 2122 \text{ Hz}$$

4.14 FM Receiver

The function of the FM receiver is to receive the incoming signal from the FM transmitter, to intercept this signal and to recover the original baseband signal. The functions of each elements of the FM receiver (as shown in Fig. 4.23) are as follows:

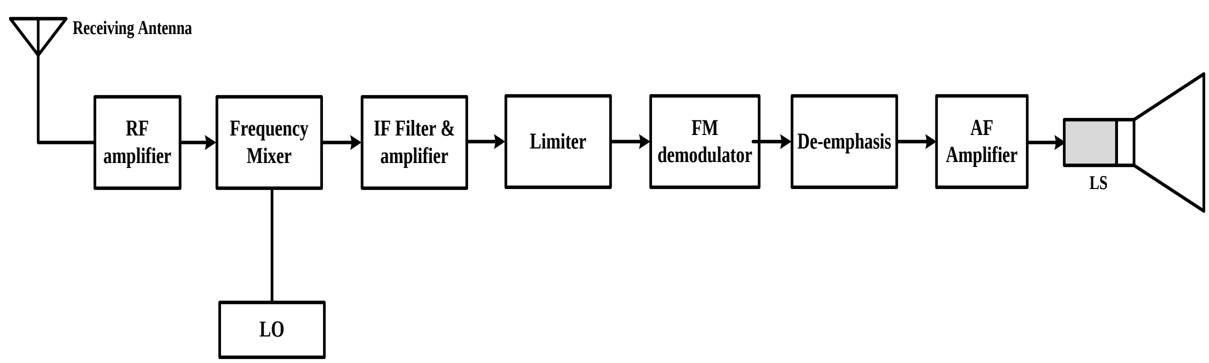


Fig. 4.23 Block diagram of the FM receiver

(i) RF amplifier

Like the RF stage of AM receiver, this stage of FM receiver rejects the image signal. Another function of the RF amplifier is to amplify the received radio signal. The required bandwidth for the RF amplifier may be as large as 150 kHz.

(ii) Frequency Mixer and Local Oscillator

The local oscillator is used to generate a constant frequency signal. The frequency mixer has two inputs: one is the received signal and the other one is from the local oscillator. The frequency mixer mixes these two signals and generates two different frequency signals $f_r + f_{LO}$ and $f_r - f_{LO}$, which is called intermediate frequency.

(iii) IF Filter & Amplifier

This stage is normally a bandpass filter that passes and amplifies the desired frequency signal. The IF amplifier improves the signal to noise ratio by suppressing unwanted higher frequency signals such as noise. Therefore, the IF amplifier is responsible for the selectivity and sensitivity of the FM receiver.

(iv) Limiter

The limiter may be ignored if the ratio detector has been used. The limiter suppresses all the fluctuation in amplitude due to noise and keeps the output at a pre-determined value. A combination of the diode and amplifying device is used to make the limiter circuit.

(v) FM Demodulator & De-emphasis circuit

FM detector recovers the original baseband signal from the IF amplifier and further, the de-emphasis circuit does the opposite function of the pre-emphasis circuit.

(vi) AF Amplifier and Speaker

The signal obtained by the FM detector and de-emphasis circuit is amplified by the audio frequency (AF) amplifier. Further, this electrical signal is converted back into an audio signal by the loudspeaker.

The comparative analysis of NBFM vs. WBFM, FM vs. PM and FM vs. AM are given in Table 4.3 to Table 4.5, respectively.

Table 4.3 A comparison between FM and AM

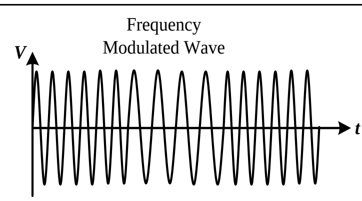
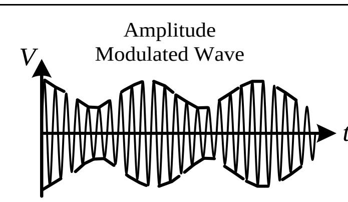
S. N.	Parameters	FM	AM
1.	**Full form**	Frequency Modulation	Amplitude Modulation
2.	**Expression**	$A_c \cos\left(\omega_c t + k_f \int m(t)\,dt\right)$	$\left(A_c + m(t)\right)\cos\omega_c t$
3.	**Modulating property of carrier signal**	Frequency	Amplitude
4.	**Constant parameters**	Phase and amplitude	Phase and frequency
5.	**Number of sidebands**	Depends upon the modulation index	Two
6.	**Efficiency**	More efficient than AM	Poor
7.	**Frequency range**	88.1 to 108.1 MHz. or up to 1200 to 2400 bits per second.	from 535 to 1700 kHz or up to 1200 bits per second
8.	**Bandwidth**	$2(\Delta f + f_m)$	$2 f_m$
9.	**Bandwidth requirement**	Greater and depends on the modulation index.	Less than FM and does not depend upon modulation index.
10.	**Noise Immunity**	Better than AM	Poor
11.	**Sound quality**	Better	Poor
12.	**Complexity**	More complex than AM	Less complex than FM and PM, but synchronization is needed in the case of SSBSC carriers.
13.	**Applications**	MW (Medium wave), SW (short wave) band broadcasting, video transmission in T.V.	Broadcasting FM, audio transmission in T.V
14.	**Waveform**		

Table 4.4 A comparison between NBFM and WBFM

S. N.	Parameters	NBFM	WBFM
1.	**Modulation index**	Slightly less than 1	>1
2.	**Maximum modulation Index**	Slightly greater than 1	5 to 2500
3.	**Range of modulating frequency**	30 Hz to 3 kHz	30 Hz to 15 kHz
4.	**Maximum deviation**	5 kHz	75 kHz
5.	**Bandwidth**	Same as AM	About 15 times higher than AM
6.	**Pre-emphasis and De-emphasis**	Required	Required
7.	**Applications**	FM mobile and speech communication	High-quality music transmission

Table 4.5 A comparison between FM and PM

S. N.	Parameters	FM	PM
1.	**Full form**	Frequency Modulation	Amplitude Modulation
2.	**Expression**	$A_c \cos\left(\omega_c t + k_f \int m(t)\,dt\right)$	$A_c \cos\left(\omega_c t + k_P m(t)\right)$
3.	**Modulating property**	Frequency	Phase
4.	**Constant parameters**	Phase and amplitude	Frequency and amplitude
5.	**Noise immunity**	Better than AM and FM both	Better than AM but worse than FM
6.	**SNR**	Better than that of PM	Inferior to that of FM
7.	**Signal quality**	High	Low
8.	**Frequency deviation**	proportional to the modulating voltage only	proportional to the modulating voltage as well as modulating frequency
9.	**Interchangeable receiver**	Possible to receive FM on PM receiver	Possible to receive PM on FM receiver
10.	**Applications**	In audio system	In the mobile communication system

ADDITIONAL SOLVED EXAMPLES

SE4.2 A carrier signal $A_c \cos(\omega_c t)$ is modulated by a modulating signal

$$m(t) = 2\cos(10^4.2\pi t) + 5\cos(10^3.2\pi t) + 3\cos(10^4.4\pi t)$$

Find the bandwidth of the FM signal by using Carson's rule. Assume k_f = 15 x 10^3 Hz/V.

Also, find modulation index m_f .

Sol: The maximum frequency component in $m(t)$ is 20 kHz. The second tone in $m(t)$ has the maximum amplitude of A_m = 5V.

Therefore, the frequency deviation Δf is given by

$$\Delta f = k_f A_m = 15\times10^3\times5 = 75 \text{ kHz}$$

So, the bandwidth of the FM signal is given by

$$\text{BW} = 2(\Delta f + f_m)$$
$$= 2(75+20)$$
$$= 2\times95$$
$$\text{BW} = 190 \text{ kHz}$$

The modulation index is given by

$$m_f = \frac{\Delta f}{f_m} = \frac{75}{20} = \frac{7.5}{2}$$

$$\text{BW} = 2\times75\left(1+\frac{2}{7.5}\right) = 190\,\text{kHz}$$

SE4.3 An angle modulated signal with carrier frequency $\omega_c = 2\pi\times10^5$ is described by

$$\phi_{\text{EM}}(t) = 10\cos(\omega_c t + 0.1\sin 2000\pi t)$$

(a) Find the power of the modulated signal

(b) Find the frequency deviation Δf

(c) Find the deviation ration β

(d) Find the phase deviation $\Delta\theta$

(e) Estimated bandwidth of $\varphi_{EM}(t)$

Sol: Given data:

Angle modulated signal $\phi_{EM}(t) = 10\cos(\omega_c t + 0.1\sin 2000\pi t)$

Carrier frequency $f_c = \frac{\omega_c}{2\pi} = \frac{2\pi\times10^5}{2\pi} = 10^5$ Hz

Amplitude A_c = 10 V

Modulating signal frequency $f_m = \frac{\omega_m}{2\pi} = \frac{2000\pi}{2\pi} = 1000\ \text{Hz}$

(a) Power $P = \frac{A_c^2}{2} = \frac{10^2}{2} = 50$

(b) Frequency deviation is a deviation in frequency around the carrier frequency. So instantaneous frequency is

$$\omega_i = \frac{d\theta(t)}{dt} = \frac{d}{dt}\left(\omega_c t + 0.1\sin 2000\pi t\right) = \omega_c + \left(0.1*2000\pi\right)\cos 2000\pi t$$

The frequency deviation is

$$\Delta\omega = \left[\left(0.1*2000\pi\right)\cos 2000\pi t\right]_{\max} = 0.1*2000\pi = 200\pi$$

$$\Delta f = \frac{\Delta\omega}{2\pi} = \frac{200\pi}{2\pi} = 100\ \text{Hz}$$

(c) Deviation ratio $\beta = \frac{\Delta f}{f_m} = \frac{100}{1000} = 0.1$

(d) Since $\theta(t) = \left(\omega_c t + 0.1\sin 2000\pi t\right)$

So, phase deviation $\Delta\theta = \left(0.1\sin 2000\pi t\right)_{\max} = 0.1\ \text{rad}$

(e) Bandwidth $\text{BW} = 2\left(\Delta f + f_m\right) = 2(100 + 1000) = 2200\ \text{Hz}$

SE4.4 Consider an FM wave

$$f(t) = \cos\left[2\pi f_c t + \beta_1 \sin 2\pi f_1 t + \beta_2 \sin 2\pi f_2 t\right]$$ **(GATE: 2014)**

Find the maximum deviation of the instantaneous frequency from the carrier frequency f_c.

Sol: The FM wave is given by:

$$f((t)) = \cos\left[\underbrace{2\pi f_c t}_{\substack{\text{carrier}\\ \text{frequency}}} + \underbrace{\beta_1 \sin 2\pi f_1 t + \beta_2 \sin 2\pi f_2 t}_{\text{Deviation terms } \Delta\theta}\right]$$

Therefore, maximum deviation is

$$\Delta\omega_{\max} = \left.\frac{d(\Delta\theta)}{dt}\right|_{\max} = \left.\frac{d\left[\beta_1 \sin 2\pi f_1 t + \beta_2 \sin 2\pi f_2 t\right]}{dt}\right|_{\max}$$

$$= \left[2\pi\beta_1 f_1 \cos 2\pi f_1 t + 2\pi\beta_2 f_2 \cos 2\pi f_2 t\right]\Big|_{\max} = 2\pi\left(\beta_1 f_1 + \beta_2 f_2\right)$$

Hence, $\Delta f_{\max} = \frac{\Delta\omega}{2\pi} = \frac{2\pi\left(\beta_1 f_1 + \beta_2 f_2\right)}{2\pi} = \left(\beta_1 f_1 + \beta_2 f_2\right)$

SE4.5 Design an Armstrong indirect FM modulator to generate an FM carrier with a carrier frequency of 96 MHz and Δf = 20 kHz. A narrow band FM generator with f_c = 200 kHz and adjustable Δf in the range of 9 to 10 Hz is available. An oscillator with adjustable frequency in the range of 9 to 10 MHz is also available. There is a bandpass filter with any center frequency and only frequency doublers are available.

Sol: Given data:

Carrier frequency $f_c = 96$ MHz

Frequency deviation $\Delta f_c = 20$ kHz

Let the multiplication factor of two stages are n_1 and n_2, respectively. The narrowband generator has adjustable Δf in the range of 9 to 10 Hz. Therefore, the range of multiplication factor is

$$\left[\frac{\Delta f}{\Delta f|_{\max}}, \frac{\Delta f}{\Delta f|_{\min}}\right] = \left[\frac{20\times 10^3}{10}, \frac{20\times 10^3}{9}\right] = [2000,\ 2222]$$

Since only frequency doublers are available, so we can select a multiplication factor that could be made only with frequency doublers that is

$$2000 < 2^{11} = 2048 < 2222$$

Therefore, $n_1 \times n_2 = 2048$

Before the second multiplication, the carrier frequency of the output of the first multiplication is shifted downward to $(n_1 f_{c1} - f_{co})$. The shifted frequency is increased n_2 times by the second multiplier to get desired carrier frequency of f_c = 90 MHz. So,

$$n_2 (n_1 f_{c1} - f_{co}) = f_c$$

Put f_{c1} = 200 kHz & f_c = 96 MHz

$$n_2 (0.2 n_1 - f_{co}) = 96$$

$$n_2 \left(\frac{0.2 \times 2048}{n_2} - f_{co}\right) = 96$$

$$\Rightarrow \left(\frac{409.6}{n_2} - f_{co}\right) = \frac{96}{n_2}$$

$$\Rightarrow f_{co} = \left(\frac{409.6}{n_2} - \frac{96}{n_2}\right)$$

$$\Rightarrow f_{co} = \left(\frac{313.6}{n_2}\right)$$

Since f_{c2} is variable between 9 MHz to 10 MHz, therefore,

$$9 < \frac{313.6}{n_2} < 10$$

$$\Rightarrow 31.36 < n_2 < 34.88$$

Since only frequency doublers are available, n_2 is selected as 32 $\left(31.36 < 2^5 = 32 < 34.88\right)$

Hence $n_1 = \frac{2048}{n_2} = \frac{2048}{32} = 64$

Therefore, the frequency deviation $\Delta f = \frac{20 \times 10^3}{64 \times 32} = 9.7656$

and $\quad f_{co} = \frac{313.6}{32} = 9.8$

The block diagram of Armstrong indirect FM modulator for SE4.5 is shown in Fig. 4.24.

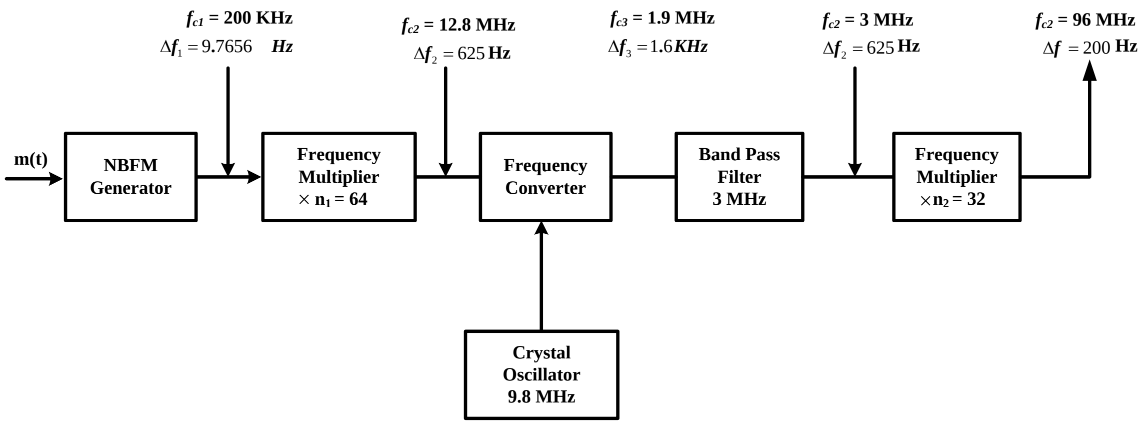


Fig. 4.24 Armstrong indirect FM modulator

SE4.6 FM broadcast standards specify a maximum deviation of frequency to be equal to 75 kHz and a maximum modulating frequency of 15 kHz. What is the modulation index of FM wave? **(IES: 2007)**

Sol: Given data:

Maximum frequency deviation $\quad \Delta f = 75$ kHz

Modulating frequency $\quad f_m = 15$ kHz

The modulation index is given as

$$m_f = \frac{\text{Maximum frequency deviation}}{\text{Maximum modulating frequency}} = \frac{75}{15} = 5.$$

SE4.7 A signal $m(t) = 5\cos(2\pi 100t)$ frequency modulates a carrier. The resulting FM signal is $10\cos\left\{\left(2\pi 10^5 t\right) + 15\sin\left(2\pi 100t\right)\right\}$. What would be the approximate bandwidth? **(IES: 2000)**

Sol: Given data: Modulation index $\quad m_f = 15$

Modulating frequency $\quad f_m = 100\text{Hz}$

According to Carson's rule, the approximated bandwidth is given as

$$\text{BW} = 2\left(m_f + 1\right) f_m .$$

So, the bandwidth is

$$\text{BW} = 2\left(15+1\right)\times 100 = 3200 = 3.2 \text{ kHz}$$

SE4.8 An angle modulated signal is expressed by

$$f_a\left(t\right) = \cos\left\{\left(2\pi 10^8 t\right) + 75\sin\left(2\pi 10^3 t\right)\right\}$$

What is the peak frequency deviation of the carrier? **(IES: 2001)**

Sol: Given data:

Modulation index $m_f = 75$

Modulating frequency $f_m = 10^3$ Hz=1 kHz

Peak frequency deviation is given as

$$\Delta f = m_f f_m = 75\times 1 = 75 \text{ kHz} .$$

SE4.9 Consider the frequency modulated signal

$$10\left\{\cos\left(2\pi\times 10^5\right)t + 5\sin\left(2\pi\times 1500\right)t + 7.5\sin\left(2\pi\times 1000\right)t\right\}$$

with the carrier frequency of 10^5 Hz. What is the modulation index? **(GATE: 2008)**

Sol: The instantaneous angular frequency is

$$\omega_i = \frac{d\theta}{dt} = \omega_c + \frac{d}{dt}\left\{5\sin\left(2\pi\times 1500\right)t + 7.5\sin\left(2\pi\times 1000\right)t\right\}$$

So, the frequency deviation is given by

$$\Delta f = 5\times 2\pi\times 1500\cos\left(2\pi\times 1500\right)t + 7.5\times 2\pi\times 1000\cos\left(2\pi\times 1000\right)$$

Therefore. The maximum frequency deviation is

$$\left(\Delta\omega\right)_{\max} = 5\times 2\pi\times 1500 + 7.5\times 2\pi\times 1000 = 2\pi\times 15000$$

So, $$\left(\Delta f\right)_{\max} = \frac{\left(\Delta\omega_m\right)_{\max}}{2\pi} = 15000$$

Hence, the modulation index is

$$m_f = \frac{\Delta f}{f_m} = \frac{15000}{1500} = 10$$

SE4.10 An FM modulated signal is represented as $\phi_{\text{FM}}\left(t\right) = 20\cos\left(5\times 10^6 t + 10\sin 1000t\right)$.

Determine

(a) Modulating frequency

(b) Carrier frequency

(c) The modulation index

(d) Maximum deviation

(e) The power dissipated in 10Ω resistor

Sol: The FM wave is expressed as

$$\phi_{\text{FM}}(t) = A_c \cos\left(\omega_c t + k_f \int m(t)\,dt\right)$$

The given FM wave is

$$\phi_{\text{FM}}(t) = 20\cos\left(5\times10^6 t + 10\sin 1000t\right)$$

Compare the given equation with the above standard equation

$$A_c = 20, \quad \omega_c = 5\times10^6$$

(a)Modulating frequency

$$\omega_m = 2\pi f_m = 1000$$

$$\Rightarrow f_m = \frac{1000}{2\pi} = 159.15 \text{ Hz}$$

(b) Carrier frequency

$$\omega_c = 2\pi f_c = 5\times10^6$$

$$\Rightarrow f_m = \frac{5\times10^6}{2\pi} = 795.77 \text{ kHz}$$

(c)The modulation index

$$m_f = 10$$

(d) The maximum deviation is obtained by

$$m_f = \frac{\Delta\omega}{\omega_m} = \frac{\Delta f}{f_m}$$

$$\Rightarrow \Delta f = m_f f_m = 10\times159.19 = 1.5919 \text{ kHz}$$

(e)Power dissipated

$$P = \frac{\left(20/\sqrt{2}\right)^2}{10} = \frac{200}{10} = 20 \text{ W}$$

SE4.11 If the carrier swing of the FM signal is 200 kHz and the modulating frequency is 16 kHz, determine the modulation index.

Sol: The modulation index is given as

$$m_f = \frac{\Delta f}{f_m} = \frac{100}{16} = 6.25$$

SE4.12 A carrier wave of frequency 100 MHz is frequency modulated by a sinusoidal wave of amplitude 20V and frequency 100 kHz. The frequency sensitivity of the modulator is 25 kHz per volt. Determine the appropriate bandwidth of the FM signal. **(UPTU: 2002-03)**

Sol: Given data:

Carrier frequency	$f_c = 100$ MHz
Modulating frequency	$f_m = 100$ kHz
Amplitude	$A_m = 20$
Frequency sensitivity	$k_f = 25$ kHz/volt

According to Carson's rule, the approximated bandwidth is given as

$$\text{BW} = 2(\Delta f + f_m)$$

The frequency deviation is

$$\Delta f = k_f A_m = 25 \times 20 = 500 \text{ kHz}$$

So, $\text{BW} = 2(500 + 100) = 1200$ kHz

SE4.13A carrier that attains a peak voltage of 5V has a frequency of 100 MHz. This carrier is modulated by a sinusoidal waveform of frequency 2 kHz to such an extent that frequency deviation from carrier frequency is 75 kHz. The modulated waveform passes through zero and increasing at time $t = 0$. Write the expression of modulated carrier waveform. **(UPTU:2003-04)**

Sol: Given data:

Carrier amplitude	$A_c = 5$V
Carrier frequency	$f_c = 100$ MHz
Modulating frequency	$f_m = 2$ kHz
Frequency deviation	$\Delta f = 75$ kHz
So, modulation index	$m_f = \dfrac{\Delta f}{f_m} = \dfrac{75}{2} = 37.5$

The FM wave is expressed as

$$\phi_{\text{FM}}(t) = A_c \cos\left[\omega_c t + m_f \sin \omega_m t\right]$$
$$= 5\cos\left[2\pi 10^8 t + 37.5 \sin 4\pi 10^3 t\right]$$

SE4.14 Given an angle modulated signal

$$\phi_{EM}(t) = 10\cos\left(2\pi 10^8 t + 200\cos 2\pi 10^3 t\right)$$

What will be the bandwidth of this angle modulated signal?

Sol: The instantaneous angular frequency is

$$\omega_i = \frac{d\theta}{dt} = \frac{d}{dt}\left\{2\pi 10^8 t + 200\cos 2\pi 10^3 t\right\} = 2\pi 10^8 - 4\pi 10^5 \sin 2\pi 10^3 t$$

Therefore, the frequency deviation is

$$\Delta\omega = 2\pi\Delta f = 4\pi 10^5$$

$$\Delta f = 2\times 10^5 \text{ Hz}$$

The modulating frequency is

$$\omega_m = 2\pi f_m = 2\pi\times 10^3$$

$$\Rightarrow f_m = 10^3 \text{Hz}$$

Since, $\Delta f \gg f_m$

So, bandwidth is given as

$$\text{BW} = 2\Delta f_m = 2\times 2\times 10^5 = 400 \text{ kHz}$$

SE4.15 Determine the relative power of the carrier wave and side frequencies when modulation index $m_f = 0.20$ for 10 kW FM transmitter.

Sol: From Bessel's table,

Only one significant sideband at $f_c \pm f_m$ with relative amplitude of 0.099.

Hence, the carrier power is

$$P_c = J_o^2\left(m_f\right)\times P_{FM}$$

$$= 0.99^2\times 10 = 9.9 \text{ kW}$$

The power of each significant sideband is

$$P_{s1} = P_{s2} = J_1^2\left(m_f\right)\times P_{FM}$$

$$= 0.099^2\times 10 = 98 \text{ W}$$

Another approach

In another way, the value of $J_n\left(m_f\right)$ is almost negligible for $n > m_f + 1$ i.e.

For $m_f = 0.20$, $n > 0.2+1 \Rightarrow n > 1$, the amplitude of the sidebands other than first is almost negligible.

The approximated amplitude is obtained by

$$J_n\left(m_f\right) = \frac{m_f^n}{2^n n!}$$

The amplitude of the carrier is obtained by $\left(m_f = 0.2,\ \ n = 0\right)$

$$J_0(m_f) = \frac{0.2^0}{2^0 0!} = 1$$

The amplitude of the first sideband is obtained by $(m_f = 0.2, \; n = 1)$

$$J_1(m_f) = \frac{0.2^1}{2^1 1!} = \frac{0.2}{2} = 0.1$$

Hence, the approximated carrier power is

$$P_c = J_o^2(m_f) \times P_{FM}$$
$$= 1^2 \times 10 = 10 \text{ kW}$$

The power of each significant sideband is

$$P_{s1} = P_{s2} = J_1^2(m_f) \times P_{FM}$$
$$= 0.1^2 \times 10 = 0.1 \text{ kW} = 100 \text{ W}$$

The approximated result is almost equal to the exact result.

SE4.16 Consider an FM wave $f(t) = \cos\left[2\pi f_c t + \beta_1 \sin 2\pi f_1 t + \beta_2 \sin 2\pi f_2 t\right]$. The maximum deviation of the instantaneous frequency from the carrier frequency f_c is

(a) $\beta_1 f_1 + \beta_2 f_2$ (c) $\beta_1 + \beta_2$

(b) $\beta_1 f_2 + \beta_2 f_1$ (d) $f_1 + f_2$ **(GATE: 2014)**

Sol: The instantaneous angular frequency is

$$\omega_i = \frac{d\theta}{dt} = \frac{d}{dt}\left\{2\pi f_c t + \beta_1 \sin 2\pi f_1 t + \beta_2 \sin 2\pi f_2 t\right\}$$
$$= \underbrace{2\pi f_c}_{\text{Carrier frequency}} + \underbrace{\beta_1 2\pi f_1 \cos 2\pi f_1 t + \beta_2 2\pi f_2 \cos 2\pi f_2 t}_{\text{Deviation terms}}$$

So, the frequency deviation is given by

$$\Delta\omega = \beta_1 2\pi f_1 \cos 2\pi f_1 t + \beta_2 2\pi f_2 \cos 2\pi f_2 t$$

Therefore. The maximum frequency deviation is

$$(\Delta\omega)_{\max} = (2\pi\Delta f)_{\max} = 2\pi(\beta_1 f_1 + \beta_2 f_2)$$
$$\Rightarrow (\Delta f)_{\max} = (\beta_1 f_1 + \beta_2 f_2)$$

Hence, option (a) is correct.

SE4.17 Consider an angle modulation signal

$x(t) = 6\cos\left[2\pi \times 10^3 t + 2\cos(8000\pi t) + 4\cos(8000\pi t)\right]$ V. The average power of $x(t)$ is

(a)10 W (c)20 W

(b) 18 W (d) 28 W **(GATE: 2010)**

Sol: The average power of $x(t)$ is

$$P = \frac{A_c^2}{2} = \frac{6^2}{2} = 18 \text{ W} \qquad (\because A_c = 6)$$

Hence, option (b) is correct.

SE4.18 The signal $m(t)$ as shown is applied both to a phase modulator (with k_p as the phase constant) and a frequency modulator with (k_f as the frequency constant) having the same carrier frequency. The ratio k_p / k_f (in rad/Hz) for the same maximum phase deviation is

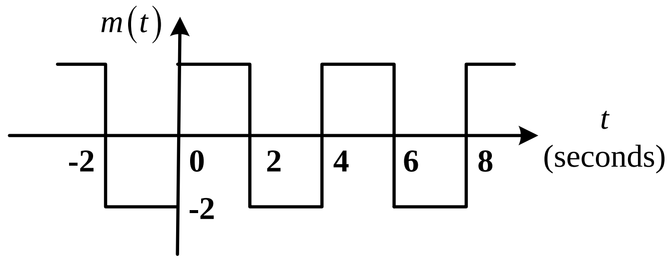


Fig. 4.25 Figure of problem SE4.19.

(a) 8π (c) 2π

(b) 4π (d) π **(GATE: 2012)**

Sol: The phase-modulated wave is expressed as

$$\phi_{\text{PM}}(t) = A_c \cos\left[\omega_c t + k_p m(t)\right]$$

The instantaneous value of phase angle is

$$\theta_i = \omega_c t + k_p m(t)$$

Therefore, the maximum phase deviation is

$$(\Delta\theta)_{\max} = k_p \left(m(t)\right)_{\max}$$

From Fig. 4.25, $\left(m(t)\right)_{\max} = 2$

So, $(\Delta\theta)_{PM,\max} = 2k_p$

The frequency modulated wave is expressed as

$$\phi_{FM}(t) = A_c \cos\left[\omega_c t + k_f \int m(t)\,dt\right]$$

The instantaneous value of angular frequency is

$$\omega_i = \omega_c + 2\pi k_f m(t)$$

Therefore, the total phase is

$$\theta = \int \omega_i dt = \int \left(\omega_c + 2\pi k_f m(t)\right) dt$$
$$= \omega_c t + 2\pi k_f \int m(t)\,dt$$

So, $\left(\Delta\theta\right)_{\text{FM,max}} = 2\pi k_f \int_0^2 2dt = 8\pi k_f$

For the same maximum phase deviation

$$\left(\Delta\theta\right)_{\text{PM,max}} = \left(\Delta\theta\right)_{\text{FM,max}}$$

$$2k_p = 8\pi k_f$$

$$\Rightarrow \frac{k_p}{k_f} = \frac{8\pi}{2} = 4\pi$$

Hence, option (b) is correct.

SE4.19 A device with input $x(t)$ and output $y(t)$ is characterized by: $y(t) = x^2(t)$. An FM signal with frequency deviation of 90 kHz and modulating signal bandwidth of 5 kHz is applied to this device. The bandwidth of the output signal is

(a) 370 kHz

(b) 190 kHz

(c) 380 kHz

(d) 95 kHz

(GATE: 2005)

Sol: Given data

Frequency deviation $\Delta f = 90$ kHz

Modulating signal bandwidth $f_m = 5$ kHz

When the FM signal is applied to doubler, the frequency deviation becomes double.

Therefore, the bandwidth of the output signal

$$\text{BW} = 2\left(\Delta f + f_m\right)$$

$$= 2\left(180 + 5\right) = 370 \text{ kHz}$$

Hence, option (a) is correct.

PROBLEMS

P4.1 Explain the FM modulation and demodulation with suitable waveforms.

P4.2 Define the following terms with respect to FM.

(i) Frequency deviation

(ii) Modulation Index

(iii)Deviation Ratio

P4.3 Distinguish between NBFM and WBFM.

P4.4 Derive an expression for single tone FM wave and Narrowband FM wave.

P4.5 Why is FM more immune to the effects of noise?

P4.6 Derive Carson's rule for Narrowband FM.

P4.7 How is FM generated with the help of a varactor diode? Explain with the help of a neat diagram.

P4.8 Draw the block diagram of the Armstrong method and explain its operation for FM generation. Why this method is called the indirect method?

Or

Draw and explain the block diagram for the indirect method of FM generation.

P4.9 In an FM system, if m_f is doubled by having the same modulating frequency, what will be the effect on the maximum deviation?

P4.10 Draw the schematic diagram for a Foster Seeley discrimination and describe its operation.

P4.11 Draw the schematic diagram for a ratio detector and describe its operation

P4.12 Write the advantages and disadvantages of Foster Seeley discrimination method?

P4.13 Describe the role of Pre-emphasis and De-emphasis in case FM.

P4.14 Compare AM and FM modulations.

P4.15 Draw the block diagram of the FM receiver and explain the working of each element.

NUMERICAL PROBLEMS

P4.16 The maximum deviation allowed in an FM broadcast system is 75 kHz. If the modulating signal is a single tone sinusoid of 10 kHz, find the bandwidth of the FM signal.

P4.17 What will be the change in the bandwidth of the system given in P4.16 if the modulating frequency is doubled? Determine the bandwidth when modulating signal amplitude is also doubled?

P4.18 Consider an angle modulated signal $\phi_{\text{EM}}(t) = 20\cos\left[10^6 \pi t + 10\sin\left(10^3 \times 2\pi t\right)\right]$. Find the maximum phase deviation and maximum frequency deviation.

P4.19 The carrier $c(t) = 100\cos(2\pi f_c t)$ is FM modulated by the signal $m(t) = 5\cos(2000\pi t)$, where, $f_c = 10^8$ Hz. Determine

(i) The amplitude and frequency of all signal components that have a power level of at least 10% of the power of the unmodulated carrier.

(ii) Find the bandwidth of the FM signal using Carson's rule.

P4.20 An FM radio link has a frequency deviation of 30 kHz. The modulating frequency is 3 kHz. Calculate the bandwidth needed for the link. What will be the bandwidth if the deviation is reduced to 15 kHz?

P4.21 Find the bandwidth of a commercial FM transmission if frequency deviation $\Delta f = 75$ kHz and modulating frequency $f_m = 15$ kHz.

P4.22 A frequency modulated signal with carrier frequency $\omega_c = 2\pi \times 10^5$ is described by the equation

$$\phi_{\text{FM}}(t) = 10\cos\left(\omega_c t + 5\sin 3000t + 10\sin 2000\pi t\right)$$

(i) Find the power of the modulated signal

(ii) Find the frequency deviation Δf

(iii) Find the deviation ratio β

P4.23 An angle modulated signal with carrier frequency $\omega_c = 2\pi \times 10^5$ is described by the equation

$$\phi_{\text{EM}}(t) = 10\cos\left(\omega_c t + 0.1\sin 2000\pi t\right)$$

(i) Determine the power of the modulated signal

(ii) Calculate the frequency deviation Δf.

(iii) Calculate the phase deviation.

(iv) Estimate the bandwidth.

P4.24 A sinusoidal modulating waveform of amplitude 5V and a frequency of 2 kHz is applied to the FM generator, which has a frequency sensitivity of 40 Hz/volt. Calculate the frequency deviation, modulation index, and bandwidth.

P4.25 A single-tone FM is represented by the voltage equation

$$\phi_{\text{FM}}(t) = 12\cos\left(6\times10^8 t + 5\sin 1250t\right)$$

Determine the following:

(i) Carrier frequency

(ii) Modulation index

(iii) Modulating frequency

(iv) Maximum deviation

(v) What power this FM wave will dissipate in a 10 Ω resistor

P4.26 Find the instantaneous frequency in Hz of each of the following signals

(i) $10\cos\left(20\pi t + \pi/2\right)$

(ii) $10\cos\left(200\pi t + \pi/3\right)$

(iii) $\cos\left(200\pi t\right)$

P4.27 A 107.6 MHz carrier signal is frequency modulated by a 7 kHz sine wave. The resultant FM signal has a frequency deviation of 50 kHz. Determine the following:

(i) Carrier swing of FM signal

(ii) Highest & lowest frequencies attained by the modulated signal

(iii)The modulation index of FM wave

P4.28 Design an Armstrong indirect FM modulator to generate an FM carrier with a carrier frequency of 96 MHz and Δf = 20 kHz. A narrow band FM generator with f_c = 200 kHz and adjustable Δf in the range of 9 to 10 Hz is available. An oscillator with adjustable frequency in the range of 9 to 10 MHz is also available. There is a bandpass filter with any center frequency and only frequency doublers are available.

P4.29 An FM transmission has a frequency deviation of 20 kHz. Determine:

(i) Percentage modulation of this signal if it is broadcasted in the 88 – 108 MHz band.

(ii) Calculate the % modulation if this signal is broadcasted as the audio portion of a TV broadcast

P4.30 In an FM system, the modulating frequency, f_m = 1 kHz, the modulating voltage A_m = 2 volt and the deviation is 6 kHz. If the modulating voltage is raised to 4 volts, then what will be the new deviation? If the modulating voltage is further raised to 8 volts and the modulating frequency is reduced to 500 Hz, what will be the new deviation?

MULTIPLE-CHOICE QUESTIONS

MCQ4.1 The two most important classes of angle modulations are

(a) SSB-SC and DSB-SC
(b) DSB-SC and Amplitude modulation
(c) Phase modulation and Amplitude modulation
(d) Phase modulation and Frequency modulation

MCQ4.2 Which of the following is not a technique for FM demodulation?

(a) Product demodulator
(b) Zero crossing detector
(c) Slope detector
(d) All of the above

MCQ4.3 In FM modulation, when the modulation index increases, the transmitted power is

(a) Unchanged
(b) Increased
(c) Decreased
(d) None of the above

MCQ4.4 Theoretically, an FM signal has …………. Bandwidth

(a) Zero (c) 1 MHz
(b) One kHz (d) Infinite

MCQ4.5 In an FM system, the modulating frequency is 2 kHz and the maximum frequency deviation allowed is 2000. The modulation index β is

(a) 2 (c) 2000
(b) 1 (d) 4000

MCQ4.6 The NBFM signal is $\phi_{\text{FM}}(t) = 25\cos\left(2000\times 2\pi\, \text{t} + 0.5\sin\left(200\times 2\pi\, \text{t}\right)\right)$. The instantaneous frequency (Hz) is

(a) $2000 + 0.5\sin\left(200\,\text{t}\right)$
(b) $2000 + 0.5\sin\left(\text{t}\right)$
(c) $2000 + 100\cos\left(200\times 2\pi\,\text{t}\right)$
(d) $2000 + 100\sin\left(200\times 2\pi\,\text{t}\right)$

MCQ4.7 Varactor diode and reactance modulator are used for ……….. generation.

(a) SSB-SC (c) DSB-SC
(b) FM (d) All of the above

MCQ4.8 In an FM system, the modulating frequency f_m = 4 kHz, the modulating voltage V_{m}= 5 V and the deviation is 8 kHz. If the modulating voltage is raised to 10 V, then what is the new deviation?

(a) 8 kHz (c) 4 kHz
(b) 16 kHz (d) 12 kHz

MCQ4.9 An FM signal is given as $\phi_{FM}(t) = A_m \sin\left(1500\times 2\pi\,\text{t}\right)$. The frequency modulation index is 2. The frequency deviation is

(a) 3 kHz (c) 6 kHz
(b) 1.5 kHz (d) 7.5 kHz

MCQ4.10 A carrier signal $\left(f_c = 100\text{ MHz}\right)$ is frequency modulated by a sinusoidal signal $\left(f_m = 600\text{ kHz}\right)$. If the maximum frequency deviation is $\Delta f = 10\text{ kHz}$, the approximate transmission bandwidth of the FM signal is

(a) 600 kHz (c) 1200 kHz

(b) 20 kHz (d) 100 MHz

MCQ4.11 An FM signal is represented as $\phi_{\text{FM}}(t) = 12\sin\left(6\times 10^8 t + 5\sin 1250t\right)$.

The carrier frequency and frequency deviation, respectively, are

(a) 191 MHz and 665 Hz

(b) 99.5 MHz and 995 Hz

(c) 191 MHz and 995 Hz

(d) 95.5 MHz and 665 Hz

MCQ4.12 Which one of the following statements is not correct?

(a) Bandwidth increases with increment in modulation depth

(b) Sideband power increases with increment in modulation depth

(c) The modulation index for FM is always greater than one

(d) FM has an infinite number of side-bands

MCQ4.13 In PM, phase deviation is proportional to

(a) Message signal

(b) Carrier amplitude

(c) Carrier phase

(d) Message signal frequencies

MCQ4.14 An FM wave uses a 2-5 V, 100 Hz modulating frequency and has a modulation index of 20. The deviation is

(a) 500 Hz (c) 1.2 kHz

(b) 1.0 kHz (d) 2.0 kHz

MCQ4.15 A way to derive FM from PM is:

(a) integrate the signal out of the PM oscillator

(b) differentiate the signal out of the PM oscillator

(c) integrate the modulating signal before applying to the PM oscillator

(d) differentiate the modulating signal before applying to the PM oscillator

MCQ4.16 FM bandwidth can be approximated by

(a) Bessel's Rule

(b) Carson's Rule

(c) Armstrong's Rule

(d) None of the above

MCQ4.17 Pre-emphasis is used to

(a) increase the signal to noise ratio for lower audio frequencies

(b) increase the signal to noise ratio for higher audio frequencies

(c) allow the stereo audio to be carried by FM stations

(d) increase the signal to noise ratio for all audio frequencies

MCQ4.18 As the FM modulation index increases, the number of significant sidebands

(a) increases

(b) decreases

(c) does not change

(d) None of the above

MCQ4.19 For specific values of m_f, such as 2.4, the amplitude of the carrier frequency

(a) goes to infinity

(b) becomes unity

(c) goes to zero

(d) becomes half

MCQ4.20 Using Carson's rule, what is the approximate bandwidth of an FM signal with a modulation index of 3 being modulated by a 10 kHz signal?

(a) 3 kHz

(b) 30 kHz

(c) 60 kHz

(d) 80 kHz

MCQ ANSWERS

MCQ4.1	(d)	MCQ4.11	(b)
MCQ4.2	(a)	MCQ4.12	(c)
MCQ4.3	(a)	MCQ4.13	(a)
MCQ4.4	(d)	MCQ4.14	(d)
MCQ4.5	(b)	MCQ4.15	(c)
MCQ4.6	(c)	MCQ4.16	(b)
MCQ4.7	(b)	MCQ4.17	(b)
MCQ4.8	(b)	MCQ4.18	(a)
MCQ4.9	(a)	MCQ4.19	(c)
MCQ4.10	(c)	MCQ4.20	(d)

CHAPTER 5

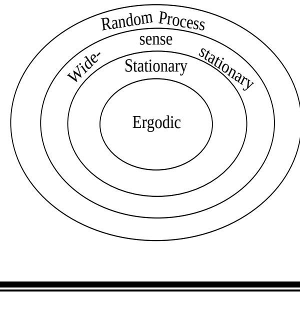
PERFORMANCE OF ANALOG MODULATION SYSTEMS

Definition

A random process is an assembly of an infinite number of RVs.

Highlights

Representation



5.1 Introduction

The temperature x of a city is measured at noon and complete statistics of the temperature data is recorded for many days at noon time. In this statistics, the recorded temperature x measured at noon time is called a random variable. Moreover, the temperature is a function of time also; therefore, if the temperature is measured at 2 pm daily, it will be an entirely different distribution from noon temperature. So, random variable x is represented as a time-dependent function $\mathrm{x}(t)$.

5.2 Random Process

A random process is an assembly of an infinite number of RVs. In other words, an RV as a function of time is called a random process or stochastic process. The noise waveform or unpredictable signals are an example of a random process. In this chapter, the roman time (x) is used to denote a random variable (RV) and italic type, i.e. x_1, x_2,….,x_n , to denote the value of x. The probability function $P_{\mathrm{x}}(x_i)$ represents the probability of RV taking the value x_i .

5.2.1 Autocorrelation Function of Random Variables

The autocorrelation function is the spectral information of a random process. It is a measure of similarity of amplitudes at two different time instants t_1 and t_2. Let the random variables RVs, $\mathrm{x}_1(t_1)$ and $\mathrm{x}_2(t_2)$ are x_1 and x_2, respectively, then the autocorrelation function of these RVs is given as

$$R_{\mathrm{x}}(t_1,t_2)=\overline{\mathrm{x}(t_1)\mathrm{x}(t_2)}=\overline{\mathrm{x}_1\mathrm{x}_2} \tag{5.1}$$

where, $t_2=t_1+\tau$.

Thus autocorrelation function is calculated by multiplication of amplitudes of the samples at t_1 and t_2 and averaging the product over the ensemble. A slowly varying and a rapidly varying signals with their respective autocorrelation functions are shown in Fig. 5.1(a) and Fig. 5.1(b), respectively. It is clear that the product of $\mathrm{x}_1\mathrm{x}_2$ for a small value of τ is positive, whereas $\mathrm{y}_1\mathrm{y}_2$ will be positive or negative. Therefore, the average value $\overline{\mathrm{x}_1\mathrm{x}_2}$ is greater than $\overline{\mathrm{y}_1\mathrm{y}_2}$. Fig. 5.1(b) shows that the x_1 and x_2 show correlation for larger value of τ whereas y_1 and y_2 lost correlation even for small value of τ .

5.2.2 Classification of Random Process

The classifications of random processes are as follows:

1. **Stationary and Non-stationary Random Process**

If the statistical characteristics of any random process do not alter with time, the random process is called a stationary (or strict sense) random process. Therefore, time origin shift cannot be detected for a stationary random process. Therefore,

$$p_x(x;\ t) = p_x(x) \qquad \text{(Independent of time)} \tag{5.2}$$

In other words, the autocorrelation function $R_x(t_1, t_2)$ of the stationary random process depends only on the time difference. So,

$$\begin{aligned} &R_x(t_1, t_2) = R_x(t_2 - t_1) = R_x(\tau) \\ &\Rightarrow R_x(\tau) = \overline{x(t_1)x(t_2)} = \overline{x(t)x(t+\tau)} \end{aligned} \tag{5.3}$$

Therefore, for a stationary process, the joint PDF must depend only on time differences.

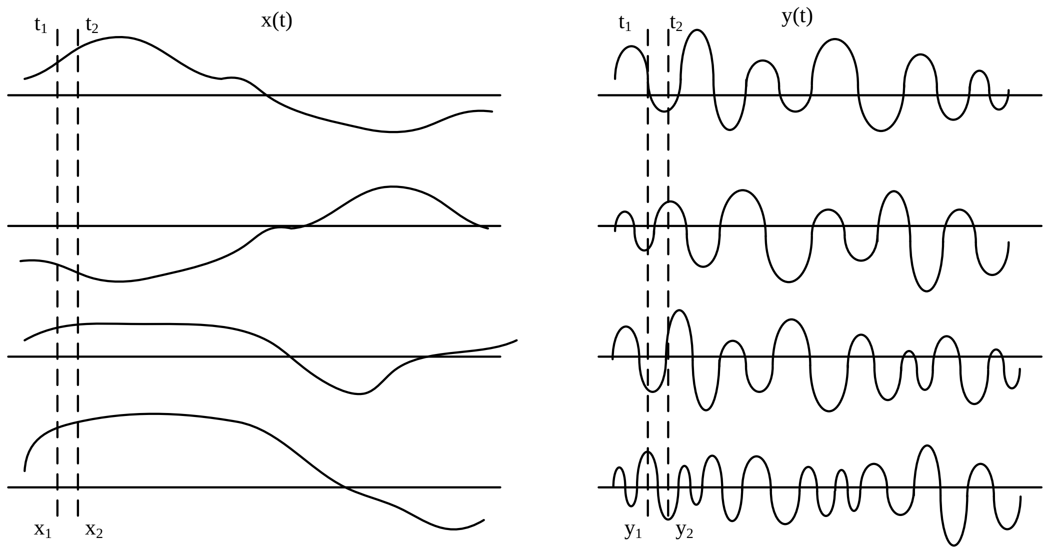


Fig. 5.1 (a) A slowly varying signal x(t) and a rapidly varying signal y(t)

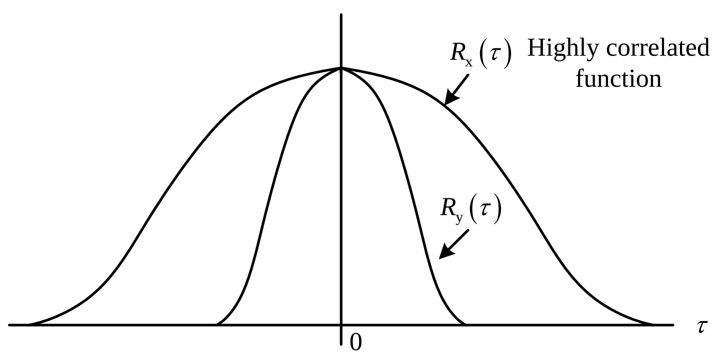


Fig. 5.1 (b) Autocorrelation function for rapidly varying and slowly varying functions

2. Wide-Sense (Weakly) Random Process

Although a stationary random process is independent of the time shift of origin, it may have an autocorrelation function and a mean value, i.e.

$$R_x(t_1, t_2) = R_x(\tau) \qquad \tau = t_1 - t_2$$

and (5.4)

$$\overline{x(t)} = \text{constant}$$

If any random process satisfies the above conditions, the random process is termed as a weakly stationary or wide-sense process.

3. Ergodic Process

If ensemble averages and time averages of any random process are equal, that random process is named as Ergodic process. That means

$$R_x(\tau) = \mathbb{R}_x(\tau)$$
and (5.5)
$$\overline{x(t)} = x(t)$$

The classification of the random process is shown in Fig. 5.2.

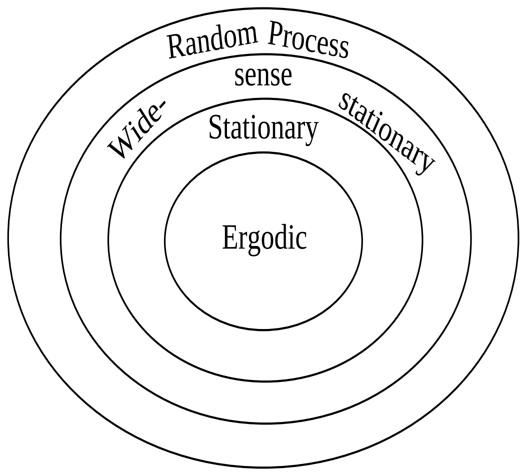


Fig. 5.2 Classification of the random process

Note:

(i) A truly stationary process starts at the time instant of $t = -\infty$.

(ii) All stationary processes are wide-sense stationary, but converse may or may not be true.

(iii) All possible ensemble averages and time averages are equal for an Ergodic process.

(iv) The time-mean $x(t)$ and time autocorrelation $\mathbb{R}_x(\tau)$ are expressed as

$$x(t) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\,dt \qquad \text{and}$$
$$\mathbb{R}_x(\tau) = x(t)x(t+\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x(t+\tau)\,dt \tag{5.6}$$

5.2.3 Power Spectral Density (PSD)

Whenever randomness is involved, it is meaningful to predict the information in terms of averages. Therefore, the random process' PSD is defined as the weighted average of all samples' PSDs and is expressed as:

$$S_x(\omega) = \lim_{T\to\infty} \overline{\left[\frac{|X_T(\omega)|^2}{T} \right]} \tag{5.7}$$

where, bar sign represents the ensemble average and $X_T(\omega)$ is FT of the truncated random process $\mathrm{x}(t)\mathrm{rect}(t/T)$.

Further, PSD is the Fourier transform (FT) of the autocorrelation function and is expressed as:

$$S_{\mathrm{x}}(\omega)=\int_{-\infty}^{\infty} R_{\mathrm{x}}(\tau)e^{-j\omega\tau}d\tau \tag{5.8}$$

The autocorrelation function is obtained by the inverse Fourier transform given as:

$$R_{\mathrm{x}}(\tau)=\frac{1}{2\pi}\int_{-\infty}^{\infty} S_{\mathrm{x}}(\omega)e^{j\omega\tau}d\omega \tag{5.9}$$

5.2.4 Properties of Autocorrelation Function

The properties of the autocorrelation function are enlisted as

(i) The mean square value of the random process $\mathrm{x}(t)$ is given by

$$\overline{x^2(t)}=\overline{\mathrm{x}(t)\mathrm{x}(t)}=R_{\mathrm{x}}(0) \tag{5.10}$$

Here, the mean $\overline{\mathrm{x}(t)}$ is the ensemble average of the sample.

(ii) The $R_{\mathrm{x}}(\tau)$ is an even function

$$R_{\mathrm{x}}(\tau)=\overline{\mathrm{x}(t)\mathrm{x}(t+\tau)}$$

So, $R_{\mathrm{x}}(-\tau)=\overline{\mathrm{x}(t)\mathrm{x}(t-\tau)}$

Let $(t-\tau)=\mu$

Then,

$$R_{\mathrm{x}}(-\tau)=\overline{\mathrm{x}(\mu+\tau)\mathrm{x}(\mu)}=R_{\mathrm{x}}(\tau) \tag{5.11}$$

5.2.5 The Power of a Random Process

The mean square value of wide sense stationary process x(t) is the average power P_{x} and is given by

$$P_{\mathrm{x}}=\overline{\mathrm{x}^2}$$

The autocorrelation is given by

$$R_{\mathrm{x}}(\tau)=\overline{\mathrm{x}(t)\mathrm{x}(t+\tau)}$$

Putting $\tau=0$

$$R_{\mathrm{x}}(0)=\overline{\mathrm{x}(t)\mathrm{x}(t)}=\overline{\mathrm{x}^2(t)}=\overline{\mathrm{x}^2}$$

Substituting $\tau=0$ in Eq. (5.9)

$$R_{\mathrm{x}}(0)=\frac{1}{2\pi}\int_{-\infty}^{\infty} S_{\mathrm{x}}(\omega)d\omega \tag{5.12}$$

Therefore, the mean square value of a random process $x(t)$ is $R_x(0)$. Hence,

$$P_x = \overline{x^2} = R_x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_x(\omega)d\omega \tag{5.13}$$

Let,

$$\begin{aligned} &\omega = 2\pi f \\ &\Rightarrow d\omega = 2\pi df \end{aligned} \qquad (f = \text{frequency in Hz}) \tag{5.14}$$

Therefore,

$$P_x = \overline{x^2} = 2\int_{0}^{\infty} S_x(\omega)df \tag{5.15}$$

It is concluded that the power of the random process is the area under PSD.

5.3 Bandpass Random Process

If the PSD of a random process is band limited to a specific band only, then the process is called the bandpass random process. Any random bandpass process is expressed as its quadrature components. The random bandpass process is expressed as

$$x(t) = x_c(t)\cos(\omega_c t + \theta) + x_s(t)\sin(\omega_c t + \theta) \tag{5.16}$$

If we consider $\theta = 0$, then

$$x(t) = x_c(t)\cos(\omega_c t) + x_s(t)\sin(\omega_c t) \tag{5.17}$$

where, $x_c(t)$ and $x_s(t)$ are band-limited random process to f_m Hz with PSDs as

$$S_{x_c}(\omega) = S_{x_s}(\omega) = \begin{cases} S_x(\omega+\omega_c) + S_x(\omega-\omega_c) & |\omega| \le 2\pi f_m \\ 0 & |\omega| > 2\pi f_m \end{cases} \tag{5.18}$$

It is observed that the area is the same under the PSDs of $S_x(\omega)$, $S_{x_c}(\omega)$ and $S_{x_s}(\omega)$. Hence,

$$\overline{x_c^2(t)} = \overline{x_s^2(t)} = \overline{x^2(t)} \tag{5.19}$$

The white Gaussian process $n(t)$ is shown in Fig. 5.3(a). The PSD and the bandwidth of $n(t)$ is $\mathcal{N}/2$ and of $2\omega_m$ respectively and it is represented in terms of quadrature component as

$$n(t) = n_c(t)\cos\omega_c t + n_s(t)\sin\omega_c t$$

with PSDs obtained by Eq. (5.18) as

$$S_{n_c}(\omega) = S_{n_s}(\omega) = \begin{cases} \mathcal{N} & |\omega| \le \omega_m \\ 0 & |\omega| > \omega_m \end{cases}$$

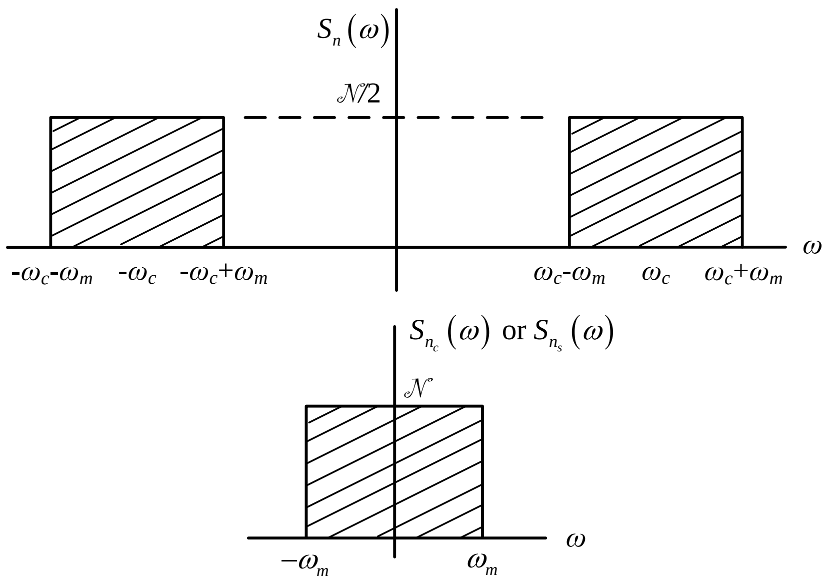
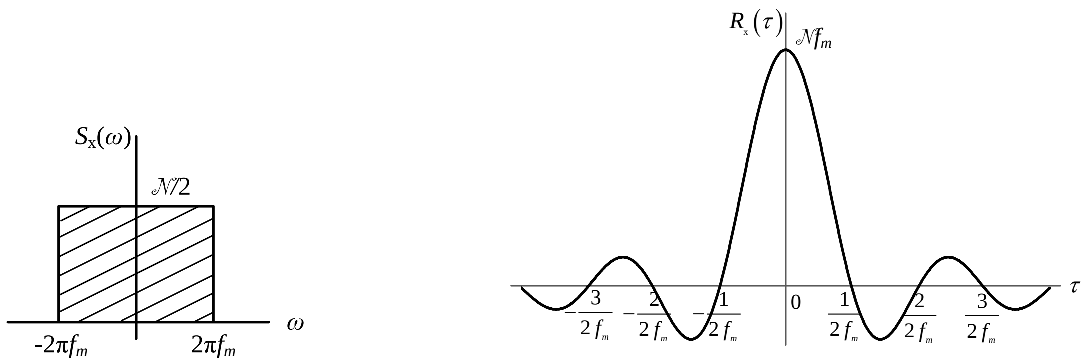


Fig. 5.3 (a) PSD of white Gaussian process (b) PSDs of quadrature components

SE5.1 Calculate the power and the autocorrelation function of the random process with PSD shown in Fig. 5.4.



(a) (b)

Fig. 5.4 (a) PSD of a random process (b) Autocorrelation function of SE5.1

Sol: The PSD of the random process is a rectangular function given by

$$S_x(\omega) = \frac{\mathcal{N}}{2}\text{rect}\left(\frac{\omega}{4\pi f_m}\right)$$

The autocorrelation function is the inverse Fourier transform of PSD as

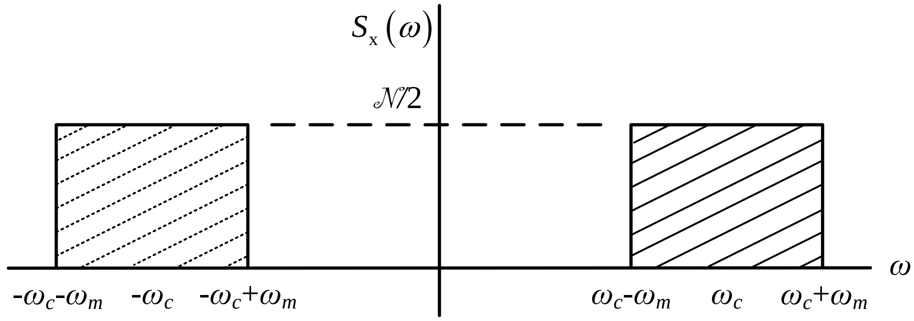
$$R_x(\tau) = \mathcal{N} f_m \text{sinc}(2\pi f_m \tau)$$

Power of random process is given as $P_x = \overline{x^2} = 2\int_0^\infty S_x(\omega)df$

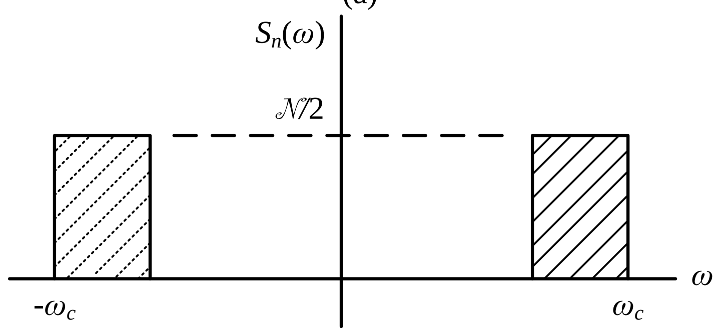
$$= 2\int_0^{f_m} \frac{\mathcal{N}}{2}df$$

$$= 2\times\frac{\mathcal{N}}{2}\times[f_m - 0] = \mathcal{N} f_m$$

SE5.2 The PSDs of a bandpass white noise for a DSB channel and an SSB channel with a lower sideband are shown in Figs. 5.5(a) and (b) respectively.



(a)



(b)

Fig. 5.5 PSD of a white noise (a) for a DSB channel (b) for a DSB channel

Determine powers for each channel.

Sol: (a) For a DSB-SC channel

The PSD $S_{n_c}(\omega)$ and $S_{n_s}(\omega)$ are obtained by shifting $S_n(\omega)$ of DSB-SC (shown in Fig. 5.5(a)), i.e.

$$S_{n_c}(\omega) = S_{n_s}(\omega) = \begin{cases} S_n(\omega+\omega_c) + S_n(\omega-\omega_c) & |\omega| \le 2\pi f_m \\ 0 & |\omega| > 2\pi f_m \end{cases}$$

$$= \begin{cases} N & |\omega| \le 2\pi f_m \\ 0 & |\omega| > 2\pi f_m \end{cases}$$

The PSD of quadrature components is shown in Fig. 5.6.

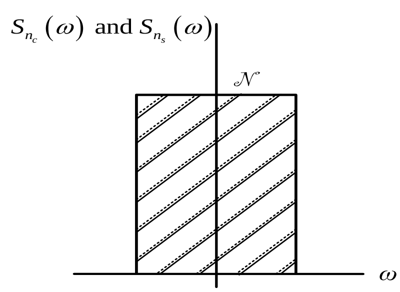


Fig. 5.6 PSD of quadrature components for DSB-SC channel noise

The power is given by

$$P_n = \overline{n^2} = 2\int_0^{\infty} S_{n_c}(\omega)df$$
$$= 2\int_0^{f_m} \mathcal{N} df$$
$$= 2\times\mathcal{N}\times[f_m - 0] = 2\mathcal{N} f_m$$

So, $\overline{n_c^2} = \overline{n_s^2} = 2\mathcal{N} f_m$

(b) The PSD $S_{n_c}(\omega)$ and $S_{n_s}(\omega)$ are obtained by shifting $S_n(\omega)$ of SSB-SC (shown in Fig. 5.5(b)), i.e.

$$S_{n_c}(\omega) = S_{n_s}(\omega) = \begin{cases} S_n(\omega+\omega_c) + S_n(\omega-\omega_c) & |\omega| \le 2\pi f_m \\ 0 & |\omega| > 2\pi f_m \end{cases}$$
$$= \begin{cases} \mathcal{N}/2 & |\omega| \le 2\pi f_m \\ 0 & |\omega| > 2\pi f_m \end{cases}$$

The resulting PSD of quadrature components is shown in Fig. 5.7.

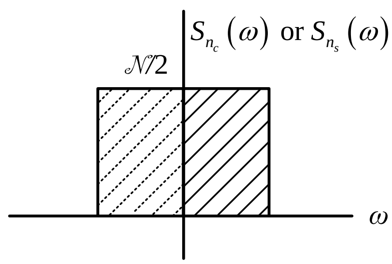


Fig. 5.7 PSD of quadrature components for SSB-SC channel noise

The power is given by

$$P_n = \overline{n^2} = 2\int_0^{\infty} S_{n_c}(\omega)df$$
$$= 2\int_0^{f_m} \frac{\mathcal{N}}{2} df = 2\times\frac{\mathcal{N}}{2}\times[f_m - 0] = \mathcal{N} f_m$$

So, $\overline{n_c^2} = \overline{n_s^2} = \mathcal{N} f_m$

5.4 Noise

Noise is an unpleasant and unwanted signal. In terms of the electrical signal, noise is defined as a signal or disturbance that is introduced with a baseband (or message) signal during the transmission process.

On the basis of sources of origin, noise is classified into the following two types:

1. External Noise
2. Internal Noise

Further categorization of noise is shown in Fig. 5.8.

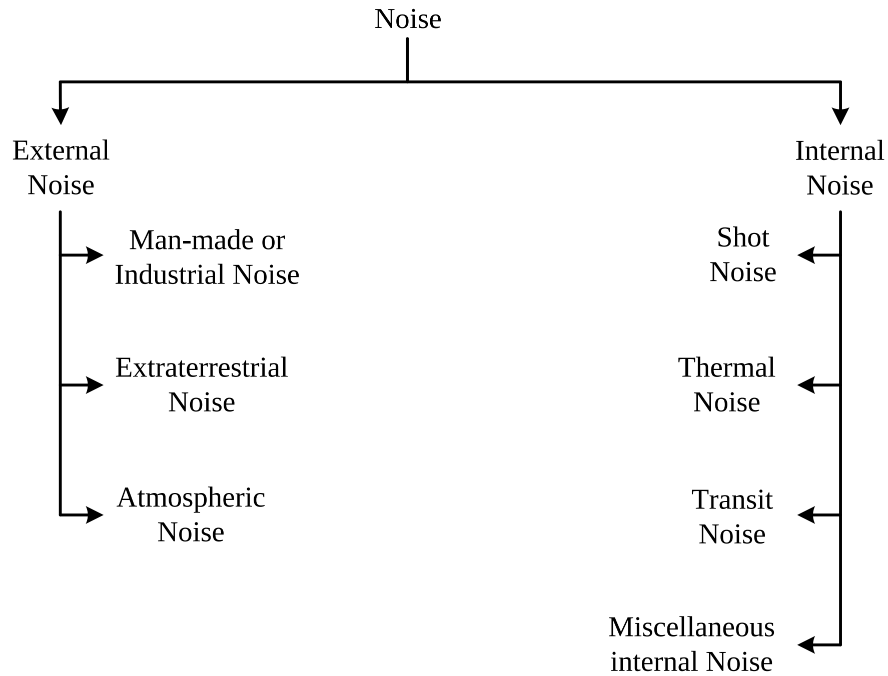


Fig. 5.8 Classification of noise

5.4.1 External Noise

The noise generated outside the circuit is called the external noise, which is basically divided into three types:

(a) Man-made Noise: Due to undesired pick-ups from electrical appliances.

(b) Extraterrestrial Noise

(c) Atmospheric Noise: Due to lightning and other atmospheric disturbances

5.4.2 Internal Noise

Internal noise is also called fluctuation noise. The cause of the generation of internal noise is the presence of active and passive components in the communication circuits. Internal noise is further divided into the following categories:

1. Shot Noise
2. Thermal Noise
3. Transit Noise
4. Miscellaneous Internal Noise

5.4.2.1 Shot Noise

Shot noise is the most common type of noise. The random nature of the charge carriers (holes and electrons) are responsible for shot noise. In electron tubes, random emission of electrons, whereas random generation and recombination of electron-hole pairs in semiconductors are the main cause of shot noise generation.

5.4.2.2 Thermal Noise

The noise that arises due to random thermal motion of free charged carriers (normally electrons) inside electrical conductors such as resistors is called thermal noise. This type of noise is also called resistor noise. Thermal noise is also termed as Johnson noise after its inventor J. B. Johnson.

The intensity of random motion of the charge carrier depends upon the thermal energy, i.e., temperature supplied to the conductor; therefore, this noise is termed as thermal noise. Thermal noise has zero value at zero absolute temperature. The power spectral density of the thermal noise is almost equal throughout the frequency spectrum and is given by

$$S_i(\omega) = \frac{2kTG}{1+\left(\frac{\omega}{\alpha}\right)^2} \tag{5.20}$$

where, α is the average number of collisions per second per electron (order of 10^{14}), k is the Boltzmann's constant, G is the conductance in mho (℧) and T is the ambient temperature in degree Kelvin.

If $\frac{\omega}{\alpha} \le 0.1$, the power spectral density of the thermal noise is

$$S_i(\omega) = 2kTG \tag{5.21}$$

Therefore, the power spectral density of the thermal noise is almost constant and independent of frequency for $\frac{\omega}{\alpha} \le 0.1$.

Power of Thermal Noise Voltage

The power density spectrum of a thermal noise voltage $v_n(t)$ is given by

$$S_v(\omega) = 2kTR \tag{5.22}$$

The relationship between noise power P_n and power density spectrum $S_v(\omega)$ is given as

$$P_n = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_v(\omega)\,d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} 2kTR\,d\omega \tag{5.23}$$

The above expression clearly shows that the noise power increases with increment in bandwidth, i.e., noise power tends to infinity for infinite bandwidth.

However, for finite bandwidth of $2\Delta f$ $(-\Delta f$ to $\Delta f)$, the RMS noise power and noise voltage are expressed as

$$P_n = S_v(\omega)\times 2\Delta f = 4kTR(\Delta f) \tag{5.24}$$

$$v_n = \sqrt{P_n} = \sqrt{4kTR(\Delta f)} \tag{5.25}$$

SE5.3 At room temperature of 27^oC, calculate the thermal noise generated by two resistors of 5 kΩ and 10 kΩ when the bandwidth is 20 kHz.

Sol: The RMS noise voltage is given as

$$v_n = \sqrt{P_n} = \sqrt{4kTR(\Delta f)}$$

Given Data:

Resistances	$R_1 = 5\Omega,\ R_2 = 10\Omega$
Bandwidth	$\Delta f = 20$ kHz
Ambient temperature	$T = 273 + 27 = 300\text{K}$
Boltzmann's constant	$k = 1.38\times10^{-23}$ J/°K

So, the RMS noise voltage individually generated by both resistances are

$$v_{n1} = \sqrt{P_{n1}} = \sqrt{4\times1.38\times10^{-23}\times300\times5\times10^3\times20\times10^3} = 1.287\times10^{-6} = 1.287\ \mu\text{V}$$

$$v_{n2} = \sqrt{P_{n2}} = \sqrt{4\times1.38\times10^{-23}\times300\times10\times10^3\times20\times10^3} = 1.819\times10^{-6} = 1.819\ \mu\text{V}$$

SE5.4 Repeat the above problem if both of the resistances are connected in series.

Sol: If the resistances are connected in series, the RMS noise voltage is given as

$$v_n = \sqrt{P_n} = \sqrt{4kT\left(\underbrace{R_1 + R_2}_{\text{Equivalent resistance}}\right)(\Delta f)}$$

So, the RMS noise voltage is

$$v_n = \sqrt{P_n} = \sqrt{4\times1.38\times10^{-23}\times300\times(5+10)\times10^3\times20\times10^3} = 2.289\times10^{-6} = 2.289\ \mu\text{V}$$

5.4.2.3 Transit Noise

The time taken by the charge carriers (holes or electrons) to move from input to output in the device (such as a transistor) is called transit time. Normally, the charge carriers move for a short distance, so the transit time is very less. At low frequency, the time period of the signal is very high compare to transit time, so it is negligible. But, if the frequency of operation is high, the transit time would be comparable to the time period and creates a problem. Therefore, transit time generates a random noise called transit noise.

5.4.2.4 Miscellaneous Internal Noise

1. **Flicker Noise**

Flicker noise (modulation noise) is the result of imperfection in surfaces (either around semiconductor junction or cathode of electron tubes). It is proportional to junction temperature and the emitter current.

The PSD of flicker noise is given as

$$S(\omega) \propto \frac{1}{f} \tag{5.26}$$

So, flicker noise is more significant at a very low frequency. Hence, flicker noise creates no problem and may be neglected at above 500 Hz frequency.

2. Partition Noise

Partition noise is similar to the shot noise as it is generated due to the random nature of the electrons. The only difference is that the emitted electrons (current) from the cathode are divided into two or more random paths among the various grids of multigrid tubes. This partition of electrons gives rise to a noise called partition noise.

5.5 White Noise

White noise is a random signal with equal intensity for all frequencies. The power density spectrum of all frequencies in white noise is equal, i.e. independent of frequency. If the probability of occurrence is identified by a Gaussian distribution function, the noise is recognized as white Gaussian Noise.

The power density of white noise is given by

$$S_n(\omega) = \frac{\mathcal{N}}{2} \tag{5.27}$$

5.6 Calculation of Noise in Linear System

Let a linear system with a single noise source is shown in Fig. 5.9. The input and output noise voltages are $v_{ni}(t)$ and $v_{no}(t)$ respectively.

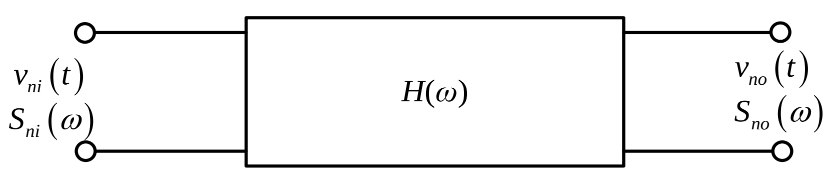


Fig. 5.9 Linear system with a single noise source

For a noiseless system, the power density spectrum of noise voltage at the output is:

$$S_{no}(\omega) = |H(\omega)|^2 S_{ni}(\omega) \tag{5.28}$$

Output noise power $P_o(\omega)$ is

$$P_o(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{ni}(\omega)|H(\omega)|^2 d\omega \tag{5.29}$$

So, the RMS noise voltage is

$$V_{rms} = \sqrt{\overline{v_{no}^2}} = \sqrt{P_o(\omega)} = \left[\frac{1}{2\pi}\int_{-\infty}^{\infty} S_{ni}(\omega)|H(\omega)|^2 d\omega\right]^{1/2} \tag{5.30}$$

Equivalent Circuit of Noise Resistor

The noisy resistor is represented as a noiseless conductance G in parallel with a thermal noise current source $i_n(t)$. The equivalent circuit of the noise resistor and its Thevenin equivalent is shown in Fig. 5.10.

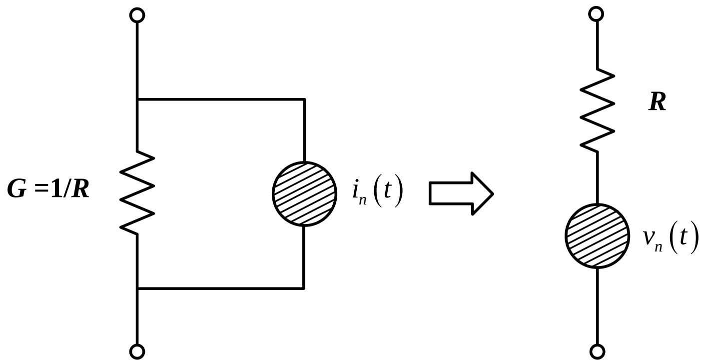


Fig. 5.10 Noisy resistor and its Thevenin equivalent

SE5.5 Determine the RMS noise voltage across capacitor for the circuit shown in Fig. 5.11 at 27°C. (k = 1.38 x 10^{-23} J/K).

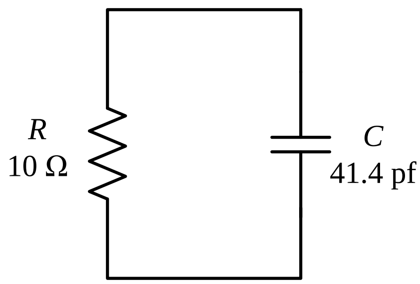


Fig. 5.11 Circuit of SE5.5

Sol: Resistance R is the only source of noise, so it can be replaced by a noiseless resistance R along with an input noise voltage source, as shown in Fig. 5.12.

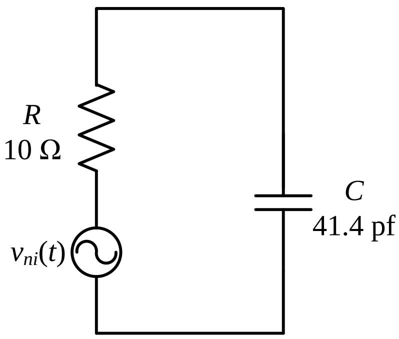


Fig. 5.12 The equivalent circuit of Fig. 5.11

The PSD of the input noise voltage source is given by

$$S_{ni}(\omega) = 2kTR$$

The transfer function of this circuit is given as

$$H(\omega) = \frac{1 - j\omega RC}{1 + (\omega RC)^2}$$

$$|H(\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

Therefore, the power spectrum density at the output is

$$S_{no}(\omega) = |H(\omega)|^2 S_{ni}(\omega) = \frac{2kTR}{\sqrt{1 + (\omega RC)^2}}$$

The RMS value of noise voltage $v_{no}(t)$ is

$$\overline{v_{no}^2} = P_o = \frac{2kTR}{\pi} \int_0^{\infty} \frac{1}{1 + (\omega RC)^2} d\omega = \frac{kT}{C}$$

$$\Rightarrow V_{rms} = \sqrt{\frac{kT}{C}}$$

Given T = 273 + 27 = 300K

Hence, output RMS voltage is

$$V_{rms} = \sqrt{\frac{kT}{C}} = \sqrt{\frac{1.38 \times 10^{-23} \times 300}{41.4 \times 10^{-12}}} = 10^{-5} V = 10 \mu V$$

5.7 Signal to Noise Ratio (*SNR* or *S/N*)

The ratio of signal power to noise power is defined as the signal-to-noise ratio. It is expressed in decibels (dB) and represent a degree of signal power level to the noise power level. The value of *S/N* greater than 1 (or > 0dB) indicates more signal power level in comparison with the noise power level. The SNR is expressed as

$$S/N = \frac{\text{Signal Power}}{\text{Noisel Power}} = \frac{P_s}{P_n} \quad (5.31)$$

where, P_s and P_n are signal and noise average powers measured within the same system bandwidth and at the same equivalent points.

The *S/N* values vs. *S/N* requirements are as follows:

(a) 5 dB to 10 dB: No connection establishment as noise and desired signal levels are comparable.

(b) 10 dB to 15 dB: Acceptable but unreliable connection.

(c) 15 dB to 25 dB: Minimally acceptable level but poor connectivity.

(d) 25 dB to 40 dB: It seems good connection.

(e) 41 dB or higher: Considered excellent connection.

Calculation of *S/N*

(i) For power,

$$\left(S/N\right)_{dB} = 10\log_{10}\left(\frac{P_s}{P_n}\right)$$

For voltage,

$$\left(S/N\right)_{dB} = 20\log_{10}\left(\frac{V_s}{V_n}\right).$$

(ii) If the measured signal and noise power are already in decibels form, then subtract signal power to noise power.

$$\left(S/N\right)_{dB} = \left(P_s\right)_{dB} - \left(P_n\right)_{dB}$$

Example: Let the measured signal power is 40 dB and noise power is 10 dB, then

$$\left(S/N\right)_{dB} = \left(P_s\right)_{dB} - \left(P_n\right)_{dB} = 40 - 10 = 30 \text{ dB}$$

5.8 Noise Factor and Noise Figure

The noise performance of an amplifier, radio receiver or any circuit block is specified by a number which is called noise factor and is given as

$$F = \frac{\left(S_i/N_i\right)}{\left(S_o/N_o\right)} \tag{5.32}$$

where, S_i/N_i and S_o/N_o are signal to noise ratio of input and output signals, respectively.

The noise figure $\left(NF\right)$ is the decibel (dB) equivalent of noise factor and is given by

$$NF = 10\log_{10}\left(F\right) = 10\log_{10}\frac{\left(S_i/N_i\right)}{\left(S_o/N_o\right)} = \left(S_i/N_i\right)_{dB} - \left(S_o/N_o\right)_{dB} \tag{5.33}$$

Noise figure and noise factor are interchangeable. Moreover, the noise figure is more popular than the term noise factor; therefore, in this chapter, the term noise factor (*F*) is used as noise figure.

A noise figure is the amount of noise that can be added by an element to the overall system without much affecting the quality of the received signal. The lower value of the noise figure is an indication of the better performance of the amplifier.

For a noiseless network, the power density at the output is due to the noise source at input only. So,

$$F = \frac{\text{power density of the total noise at the output of network}}{\text{power density at the output due to source only}} \tag{5.35}$$

The noise figure is related to the equivalent noise temperature of the amplifiers and is given by

$$F = 1 + \frac{T_e}{T}$$
$$\Rightarrow T_e = T(F-1) \quad (5.34)$$

where, T_e and T are noise temperature of network and source respectively.

5.8.1 Noise Figure of Cascaded Amplifiers

The noise figure for the cascaded structure of the several devices is calculated by Friis' formula. Let the n devices are connected in cascade with gains of G_1, G_2, ….., G_n and noise factors of F_1, F_2,…., F_n respectively. A schematic block diagram of three amplifiers is shown in Fig. 5.13.

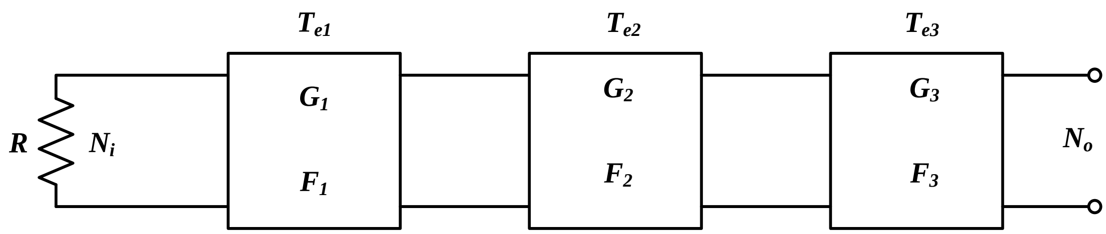


Fig. 5.13 A schematic block diagram of three amplifiers

Let N_i is the noise power at the input of the first amplifier. Therefore, amplified noise power at the output of the third amplifier due to input noise alone is

$$N_{oi} = G_1 G_2 G_3 N_i \quad (5.36)$$

The internal noise of the first amplifier is N_1. So, the contribution of this noise at the output is

$$N_{o1} = G_2 G_3 N_1 \quad (5.37)$$

Similarly, the contributions of second and third amplifiers' internal noises N_2 and N_3 at the output are N_{o2} and N_{o3}, respectively and are given as

$$N_{o2} = G_3 N_2 \quad (5.38)$$
$$N_{o3} = N_3 \quad (5.39)$$

Therefore, total noise power at the output is

$$N_o = N_{oi} + N_{o1} + N_{o2} + N_{o3}$$
$$= G_1 G_2 G_3 N_i + G_2 G_3 N_1 + G_3 N_2 + N_3 \quad (5.40)$$

If the input signal power is S_i, the amplified signal power at the output of the third amplifier is

$$S_o = G_1 G_2 G_3 S_i \quad (5.41)$$

So, the output S/N ratio is given as

$$\left(\frac{S}{N}\right)_{output} = \frac{S_o}{N_o} = \frac{G_1 G_2 G_3 S_i}{G_1 G_2 G_3 N_i + G_2 G_3 N_1 + G_3 N_2 + N_3} \quad (5.42)$$

Therefore, the noise factor is expressed as

$$F = \frac{(S/N)_{input}}{(S/N)_{output}} = \frac{\dfrac{S_i}{N_i}}{\dfrac{G_1G_2G_3S_i}{G_1G_2G_3N_i + G_2G_3N_1 + G_3N_2 + N_3}}$$

$$= \frac{S_i \times (G_1G_2G_3N_i + G_2G_3N_1 + G_3N_2 + N_3)}{G_1G_2G_3S_i \times N_i}$$

$$= 1 + \frac{N_1}{G_1N_i} + \frac{N_2}{G_1G_2N_i} + \frac{N_3}{G_1G_2G_3N_i} \qquad (5.43)$$

Since the noise factor for a single-stage amplifier is given as

$$F_1 = 1 + \frac{N_1}{G_1N_i} \qquad (5.44)$$

So, the equivalent noise factor of Fig. 5.13 is

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1G_2} \qquad (5.45)$$

Therefore, the generalized formula for total noise factor for *n*-cascaded stage structure is given as

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1G_2} + \frac{F_4 - 1}{G_1G_2G_3} + ... + \frac{F_n - 1}{G_1G_2....G_{(n-1)}} \qquad (5.46)$$

5.8.2 Equivalent Noise Temperature of Cascaded Stages

As we know, the relation between noise factor and equivalent noise temperature is given as

$$F - 1 = \frac{T_e}{T} \qquad (5.47)$$

where T_e is the overall noise temperature of the two cascaded stages.

Similarly,

$$F_1 - 1 = \frac{T_{e1}}{T},\ F_2 - 1 = \frac{T_{e2}}{T} \text{ and } F_3 - 1 = \frac{T_{e1}}{T}. \qquad (5.48)$$

Substituting the values of F, F_1, F_2 and F_3 in Eq. (5.46)

$$1 + \frac{T_e}{T} = 1 + \frac{T_{e1}}{T} + \frac{T_{e2}}{G_1T} \qquad (5.49)$$

$$T_e = T_{e1} + \frac{T_{e2}}{G_1} \qquad (5.50)$$

Therefore, the generalized formula for the equivalent noise temperature of the *n*-cascaded stage network is given by:

$$T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1G_2} + \frac{T_{e4}}{G_1G_2G_3} + ... + \frac{T_{en}}{G_1G_2....G_{(n-1)}} \qquad (5.51)$$

SE5.6 An amplifier has three stages with noise temperatures as $T_{e1} = 200$ K, $T_{e2} = 450$ K and $T_{e3} = 1000$ K. If the available power gain of the second stage is 5, what gain must the first stage have to guarantee an effective input noise temperature of 250 K?

Sol: The equivalent noise temperature is given as

$$T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2}$$

$$250 = 200 + \frac{450}{G_1} + \frac{1000}{5G_1}$$

$$\Rightarrow G_1 = \frac{650}{50} = 13$$

SE5.7 Three identical amplifiers (all with effective noise temperature 250 K and available power *G*) are cascaded. The overall spot effective input noise temperature of the cascade is 310 K. Determine the value of *G*.

Sol: The equivalent noise temperature is given as

$$T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2}$$

$$\Rightarrow 310 = 250 + \frac{250}{G} + \frac{250}{G^2}$$

$$60G^2 - 250G - 250 = 0$$

$$\Rightarrow 6G^2 - 25G - 25 = 0$$

$$\Rightarrow G = 5$$

5.9 Performance of Analog Systems in The Presence of Noise

A schematic diagram of a baseband communication system is presented in Fig. 5.14. The message signal with power S_T is transmitted over the channel. Since the channel noise is additive in nature, therefore, it may weaken and distort the signal. A mixed-signal (desired message signal + unwanted noise signal) is obtained at the receiver end. The quality of the received signal at the receiver is determined by the $\frac{S_o}{N_o}$ where, S_o and N_o are signal and noise powers at the output of the receiver, respectively.

However, the *S/N* ratio (i.e. signal power) can be improved by increasing the transmitted power S_T but it may result in increased cost of the transmission, channel capacity etc. Since the transmitted power is proportional to the received power at the receiver input, i.e. $S_T \propto S_i$, So, it is more convenient to deal with S_i instead of S_T.

Figure of merit

The figure of merit is described to compare the performance of the receivers for different continuous wave modulation. It is defined as ratio of output *S*/*N* to the input *S*/*N* and is given as

$$\text{Figure of merit} = \frac{(S_o / N_o)}{(S_i / N_i)} \tag{5.52}$$

5.10 Baseband System

In a baseband communication system, the message signal is transmitted directly over coaxial cable without modulation, as shown in Fig. 5.14. All filters are assumed as ideal filters with band-limited to ω_m (i.e. f_m Hz).

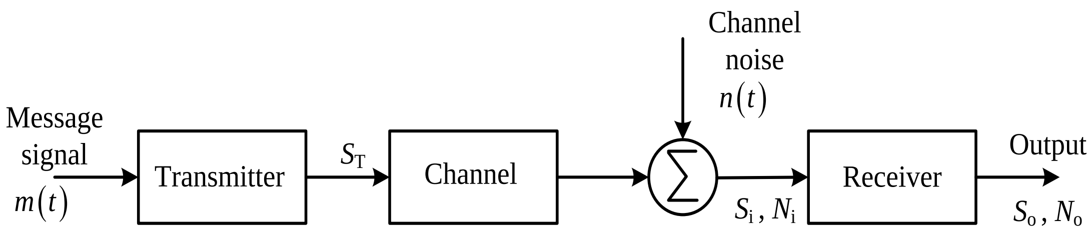


Fig. 5.14 A baseband communication system

Let the message signal $m(t)$ is supposed to be a zero-mean Then output signal power is

$$S_o = S_i \tag{5.53}$$

The noise power is given by

$$N_o = 2\int_0^{f_m} S_n(\omega)\, df \tag{5.54}$$

where, $S_n(\omega)$ is power spectral density and equal to $\mathcal{N}/2$. Hence,

$$N_o = 2\int_0^{f_m} \frac{\mathcal{N}}{2} df = 2 \times \frac{\mathcal{N}}{2} \times [f]_0^{f_m} \tag{5.55}$$

$$\Rightarrow N_o = \mathcal{N} f_m$$

Therefore, the *S/N* ratio is given by

$$\left(\frac{S_o}{N_o}\right)_{\text{Baseband}} = \frac{S_i}{\mathcal{N} f_m} = \gamma \tag{5.56}$$

where, γ is a parameter analogues to figure of merit to compare the performance of the system in the presence of noise. In this chapter, the term γ is used as an analogues to figure of merit to compare the performance of the receivers.

The required SNR

For voice signal:	5 to 10 dB
For telephone:	25 to 35 dB
For television:	45 to 55 dB

5.11 Amplitude Modulated System

The performance analysis of AM systems such as SSB-SC, DSB-SC etc. in the presence of noise are as follows:

(i) DSB-SC Systems

The schematic diagram of the DSB-SC system is presented in Fig. 5.15. At the transmitter side, DSB-SC modulated signal is generated as

$$\phi_{\text{DSB}}(t) = m(t)\cos(\omega_c t) \tag{5.57}$$

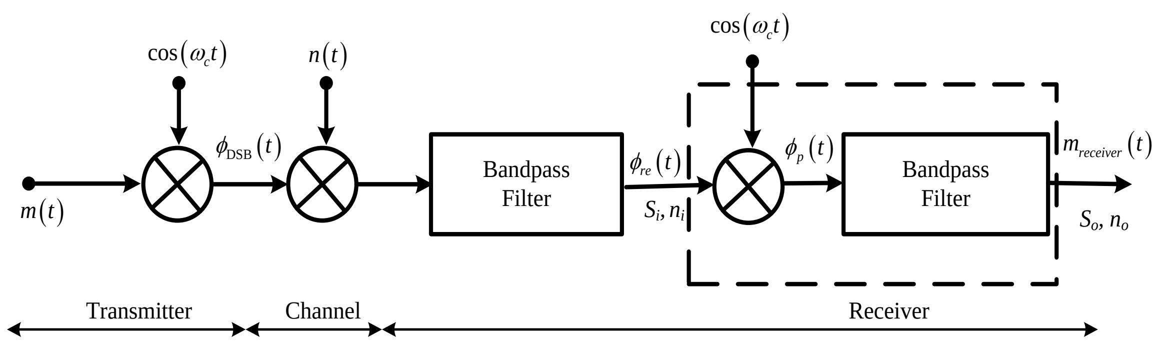


Fig. 5.15 The schematic diagram of the DSB-SC system

White noise $n(t)$ is added to this signal during transmission through the channel. The spectrum of this noise is centered on $\omega_c\left(\text{i.e. } f_c \text{ Hz}\right)$ with bandwidth of $2f_m$ Hz. So, the noise signal is represented as in-phase and quadrature component, i.e.

$$n(t) = n_c(t)\cos(\omega_c t) + n_s(t)\sin(\omega_c t) \tag{5.58}$$

Therefore, the signal at the input of the demodulator or receiver input is

$$\begin{aligned}\phi_{re}(t) &= \underbrace{m(t)\cos(\omega_c t)}_{\text{Signal component}} + \underbrace{n_c(t)\cos(\omega_c t) + n_s(t)\sin(\omega_c t)}_{\text{Noise component}} \\ &= \left[m(t) + n_c(t)\right]\cos(\omega_c t) + n_s(t)\sin(\omega_c t)\end{aligned} \tag{5.59}$$

Hence, the input signal power at the receiver is given by

$$S_i = \overline{\left[m(t)\cos(\omega_c t)\right]^2} = \frac{1}{2}\overline{m^2(t)} \tag{5.60}$$

The synchronous detection method is used for the recovery of the message signal. In this method, the received signal is multiplied by the carrier signal and passed through the LPF. Therefore, the output of the product modulator at the receiver end is

$$\begin{aligned}\phi_p(t) &= \left\{\left[m(t) + n_c(t)\right]\cos(\omega_c t) + n_s(t)\sin(\omega_c t)\right\} \times \cos(\omega_c t) \\ &= \left[m(t) + n_c(t)\right]\cos(\omega_c t) \times \cos(\omega_c t) + n_s(t)\sin(\omega_c t) \times \cos(\omega_c t)\end{aligned}$$

$$= \left[m(t) + n_c(t)\right]\cos^2(\omega_c t) + \frac{1}{2}n_s(t)\sin(2\omega_c t)$$

$$\phi_p(t) = \left[m(t) + n_c(t)\right]\left[\frac{1+\cos(2\omega_c t)}{2}\right] + \frac{1}{2}n_s(t)\sin(2\omega_c t) \tag{5.61}$$

This signal is passed through the LPF, which suppresses the terms $\cos(2\omega_c t)$ and $\sin(2\omega_c t)$. The output obtained at the receiver is

$$m_{receiver}(t) = \frac{1}{2}\left[\underset{\text{Message}}{m(t)} + \underset{\text{Noise}}{n_c(t)}\right] \tag{5.62}$$

So, the output signal power is given by

$$S_o = \overline{\left[\frac{m(t)}{2}\right]^2} = \frac{1}{4}\overline{m^2(t)} \tag{5.63}$$

The noise power is given by

$$N_o = \overline{\left[\frac{n_c(t)}{2}\right]^2} = \frac{1}{4}\overline{n_c^2(t)} \tag{5.64}$$

In the DSB-SC system, the noise is white noise with PSD of $\mathcal{N}/2$ and with a bandwidth of $2f_m$ Hz. Hence,

$$\begin{aligned}
\overline{n_c^2(t)} &= 2\int_{f_c-f_m}^{f_c+f_m}\frac{\mathcal{N}}{2}df \\
&= 2\times\frac{\mathcal{N}}{2}\times\left[f\right]_{f_c-f_m}^{f_c+f_m} \\
&= \mathcal{N}\times\left[(f_c+f_m)-(f_c-f_m)\right] \\
&= 2\mathcal{N}f_m
\end{aligned} \tag{5.65}$$

Substituting the value of $\overline{n_c^2(t)}$ from Eq. (5.65) into Eq. (5.64)

$$N_o = \frac{1}{4}\times 2\mathcal{N}f_m = \frac{\mathcal{N}f_m}{2} \tag{5.66}$$

Therefore, the output *S/N* is given by

$$\begin{aligned}
\left(\frac{S_0}{N_0}\right)_{\text{DSB}} &= \frac{\frac{1}{4}\overline{m^2(t)}}{\frac{\mathcal{N}f_m}{2}} = \frac{\frac{1}{2}\overline{m^2(t)}}{\mathcal{N}f_m} \\
\Rightarrow \left(\frac{S_0}{N_0}\right)_{\text{DSB}} &= \frac{S_i}{\mathcal{N}f_m} = \gamma
\end{aligned} \tag{5.67}$$

Therefore, it is clear that the *S/N* ratio remains the same at the demodulator for both the baseband system and the DSB-SC system if the transmitted power is the same. In other words, the DSB-SC system has the identical capability as the baseband system has.

(ii) SSB-SC Systems

The SSB-SC signal is obtained by the suppression of one of the sidebands of the DSB-SC signal. The schematic diagram of the SSB-SC system is presented in Fig. 5.16. At the transmitter end, the SSB-SC modulated signal is generated and given by

$$\phi_{\text{SSB}}(t) = \underbrace{m(t)\cos(\omega_c t)}_{\text{Power} = \frac{\overline{m^2(t)}}{2}} \mp \underbrace{m_h(t)\sin(\omega_c t)}_{\text{Power} = \frac{\overline{m^2(t)}}{2}} \tag{5.68}$$

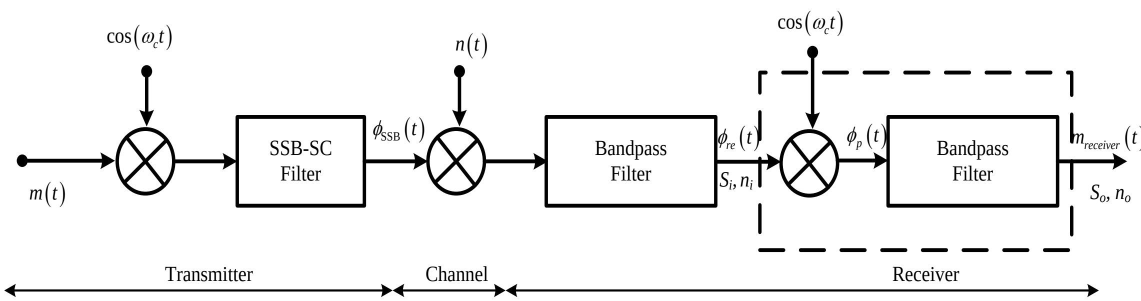


Fig. 5.16 The schematic diagram of the SSB-SC system

Hence, the power of SSB-SC at the receiver input is given by

$$S_i = \frac{\overline{m^2(t)}}{2} + \frac{\overline{m^2(t)}}{2} = \overline{m^2(t)} \tag{5.69}$$

The white noise is added in the channel, so the mixed signal at the receiver/demodulator input

$$\phi_{re}(t) = \left[m(t) + n_c(t)\right]\cos(\omega_c t) + \left[m_h(t) + n_s(t)\right]\sin(\omega_c t) \tag{5.70}$$

The signal is multiplied with $\cos(\omega_c t)$ in synchronous detection method. The output of the product modulator at the receiver end is

$$\begin{aligned}
\phi_p(t) &= \left\{\left[m(t)+n_c(t)\right]\cos(\omega_c t) + \left[m_h(t)+n_s(t)\right]\sin(\omega_c t)\right\}\times\cos(\omega_c t) \\
&= \left[m(t)+n_c(t)\right]\cos(\omega_c t)\times\cos(\omega_c t) + n_s(t)\sin(\omega_c t)\times\cos(\omega_c t) \\
&= \left[m(t)+n_c(t)\right]\cos^2(\omega_c t) + \frac{1}{2}n_s(t)\sin(2\omega_c t) \\
&= \left[m(t)+n_c(t)\right]\left[\frac{1+\cos(2\omega_c t)}{2}\right] + \frac{1}{2}m_h(t)\sin(2\omega_c t) + \frac{1}{2}n_s(t)\sin(2\omega_c t)
\end{aligned} \tag{5.71}$$

This signal is passed through the LPF, which suppresses the terms $\cos(2\omega_c t)$ and $\sin(2\omega_c t)$. The output obtained at the receiver is

$$m_{receiver}(t) = \frac{1}{2}\left[m(t) + n_c(t)\right] \tag{5.72}$$

So, the output signal power is given by

$$S_o = \overline{\left[\frac{m(t)}{2}\right]^2} = \frac{1}{4}\overline{m^2(t)} \tag{5.73}$$

The noise power is given by

$$N_o = \overline{\left[\frac{n_c(t)}{2}\right]^2} = \frac{1}{4}\overline{n_c^2(t)} \tag{5.74}$$

The noise is white noise with PSD of $\mathcal{N}/2$ and with the bandwidth of f_m Hz in the case of SSB-SC. If USB is transmitted in SSB-SC, the noise is band-limited from ω_c to ω_{c+m}. Hence,

$$\begin{aligned}\overline{n_c^2(t)} &= 2\int_{f_c}^{f_c+f_m} \frac{\mathcal{N}}{2} df \\ &= 2 \times \frac{\mathcal{N}}{2} \times \left[f\right]_{f_c}^{f_c+f_m} \\ &= \mathcal{N} \times \left[(f_c + f_m) - (f_c)\right] \\ &= \mathcal{N} f_m\end{aligned} \tag{5.75}$$

Substituting the value of $\overline{n_c^2(t)}$ From Eq. (5.75) into Eq. (5.74)

$$N_o = \frac{1}{4} \times \mathcal{N} f_m = \frac{\mathcal{N} f_m}{4} \tag{5.76}$$

Therefore, the output *S/N* is given by

$$\begin{aligned}\left(\frac{S_0}{N_0}\right)_{\text{SSB}} &= \frac{\frac{1}{4}\overline{m^2(t)}}{\frac{1}{4}\mathcal{N} f_m} = \frac{\overline{m^2(t)}}{\mathcal{N} f_m} \\ \Rightarrow \left(\frac{S_0}{N_0}\right)_{\text{SSB}} &= \frac{S_i}{\mathcal{N} f_m} = \gamma\end{aligned} \tag{5.77}$$

It can be concluded that the performances of the DSB-SC system and the SSB-SC are identical to the baseband system in terms of source utilization.

(iii)AM system

Following methods are used for the demodulation of the AM signal:

1. Coherent or synchronous detection
2. Envelope detection

1. **Coherent or synchronous detection:**

The AM signal is given as

$$\phi_{AM}(t) = \left[A_c + m(t)\right]\cos(\omega_c t) \tag{5.78}$$

The input signal power at the demodulator input is

$$S_i = \frac{1}{2}\overline{\left[A_c + m(t)\right]^2} = \frac{1}{2}\left[A_c^2 + 2A_c\overline{m(t)} + \overline{m^2(t)}\right]$$

$$S_i = \frac{1}{2}\left[A_c^2 + \overline{m^2(t)}\right] \tag{5.79}$$

White noise is added to this signal by channel. The added noise has a spectral density of $\mathcal{N}/2$ and bandwidth of $2f_m$, similar to DSB-SC. So, the noise power is

$$N_o = \frac{1}{2}\mathcal{N} f_m \tag{5.80}$$

The signal is multiplied with $\cos(\omega_c t)$ in synchronous detection method. So, the output of the product modulator at the receiver end is

$$\begin{aligned}
\phi_p(t) &= \left\{\left[A_c + m(t) + n_c(t)\right]\cos(\omega_c t) + n_s(t)\sin(\omega_c t)\right\}\times\cos(\omega_c t) \\
&= \left[A_c + m(t) + n_c(t)\right]\cos(\omega_c t)\times\cos(\omega_c t) + n_s(t)\sin(\omega_c t)\times\cos(\omega_c t) \\
&= \left[A_c + m(t) + n_c(t)\right]\cos^2(\omega_c t) + \frac{1}{2}n_s(t)\sin(2\omega_c t) \\
&= \left[A_c + m(t) + n_c(t)\right]\left[\frac{1+\cos(2\omega_c t)}{2}\right] + \frac{1}{2}n_s(t)\sin(2\omega_c t)
\end{aligned} \tag{5.81}$$

The high-frequency term $\pm 2\omega_c$ (i.e. $\pm 2f_c$ Hz) is suppressed by the LPF. Hence, the signal at the output of the receiver is given as

$$m_{receiver}(t) = \frac{1}{2}\left[A_c + m(t) + n_c(t)\right] \tag{5.82}$$

The AC term is eliminated by the DC blocker. Therefore, the power of the output signal is

$$S_o = \frac{1}{4}\overline{m^2(t)} \tag{5.83}$$

Hence, the *S/N* obtained at the receiver output is given by

$$\left(\frac{S_o}{N_o}\right)_{\text{AM,coh}} = \frac{\frac{1}{4}\overline{m^2(t)}}{\frac{1}{2}Nf_m} = \frac{\frac{1}{4}\overline{m^2(t)}}{\frac{1}{2}\left(A_c^2 + \overline{m^2(t)}\right)}\times\frac{\frac{1}{2}\left(A_c^2 + \overline{m^2(t)}\right)}{\frac{1}{2}\mathcal{N} f_m}$$

$$\Rightarrow \left(\frac{S_o}{N_o}\right)_{\text{AM,coh}} = \frac{\overline{m^2(t)}}{\left(A_c^2 + \overline{m^2(t)}\right)}\times\frac{S_i}{\mathcal{N} f_m}$$

$$\Rightarrow\left(\frac{S_o}{N_o}\right)_{\text{AM,coh}} = \frac{\overline{m^2(t)}}{\left(A_c^2+\overline{m^2(t)}\right)}\times\gamma \tag{5.84}$$

If $m(t)_{\max} = m_p$ and $A_c \geq m_p$, then, the maximum value of output SNR is obtained for $A_c = m_p$.

So,

$$\left(\frac{S_o}{N_o}\right)_{\max} = \frac{\overline{m^2(t)}}{\left(m_p^2+\overline{m^2(t)}\right)}\gamma = \frac{\overline{m^2(t)}}{\left(m_p^2\Big/\overline{m^2(t)}+1\right)}\gamma$$

Since, $\quad \dfrac{m_p^2}{\overline{m^2(t)}} \geq 1$

So, $$\left(\frac{S_o}{N_o}\right) \leq \frac{\gamma}{2} \tag{5.85}$$

It is clear that the *S/N* is at least 3 dB more worse than the *S/N* in DSB-SC and SSB-SC systems.

2. AM Envelope Detector

The mixed AM signal at the receiver input is given by

$$\phi_{re}(t) = \left[A_c + m(t) + n_c(t)\right]\cos(\omega_c t) + n_s(t)\sin(\omega_c t) \tag{5.86}$$

The desired term at the receiver is $\left[A_c + m(t)\right]\cos(\omega_c t)$. The input signal power is given by

$$S_i = \frac{1}{2}\left(A_c^2 + \overline{m^2(t)}\right) \tag{5.87}$$

The above mixed signal in polar form is represented as

$$\phi_{re}(t) = E_{AM}(t)\cos(\omega_c t + \varphi) \tag{5.88}$$

where, the envelope $E_{AM}(t)$ is given as

$$E_{AM}(t) = \sqrt{\left[A_c + m(t) + n_c(t)\right]^2 + n_s^2(t)} \tag{5.89}$$

Now two cases are considered: 1) Small noise and 2) Large noise

(i) Small noise:

If $\left[A_c + m(t)\right] \gg n(t)$, for all values of *t*, then $\left[A_c + m(t)\right] \gg n_c(t)$ or $n_s(t)$ for all *t*. in this case, the envelope is approximated as

$$E_{AM}(t) \approx A_c + m(t) + n_c(t) \tag{5.90}$$

The DC component is blocked by the envelope detector. Therefore, the remaining signal consists of desired signal $m(t)$ and noise signal $n(t)$. Hence, output signal power and noise power are given by:

$$S_o = \overline{m^2(t)}$$

$$N_o = \overline{n_c^2(t)} = 2\mathcal{N} f_m \tag{5.91}$$

The output SNR is given by

$$\left(\frac{S_o}{N_o}\right)_{\text{AM,env}} = \frac{\overline{m^2(t)}}{2\mathcal{N} f_m} = \frac{\overline{m^2(t)}}{A_c^2 + \overline{m^2(t)}} \times \frac{A_c^2 + \overline{m^2(t)}}{\mathcal{N} f_m}$$
$$\Rightarrow \frac{\overline{m^2(t)}}{A_c^2 + \overline{m^2(t)}} \times \frac{S_i}{\mathcal{N} f_m} \tag{5.92}$$

The Eq. (5.92) is similar to the synchronous detector approach. It is concluded that in the presence of small noise, the performance of the envelope detector is identical to the synchronous detector.

(ii) Large noise

If $n(t) \gg \left[A_c + m(t)\right]$ $\left(\text{or } n_s(t) \text{ and } n_c(t) \gg \left[A_c + m(t)\right]\right)$, for all values of *t*, then

$$E_{AM}(t) \approx \sqrt{n_c^2(t) + 2n_c(t)\left[A_c + m(t)\right] + n_s^2(t)} \tag{5.93}$$

which is further given as

$$E_{AM}(t) \approx E_N(t)\sqrt{1 + \frac{2\left[A_c + m(t)\right]}{E_N(t)}\cos\varphi_n(t)} \tag{5.94}$$

where $\quad E_N(t) \approx \sqrt{n_c^2(t) + n_s^2(t)} \quad$ & $\quad \varphi_n(t) = -\tan^{-1}\left[\frac{n_c(t)}{n_s(t)}\right]$

Since, $E_n(t) \gg \left[A_c + m(t)\right]$, again the above term is approximated as

$$E_{\text{AM}}(t) \approx E_N(t)\left(1 + \frac{\left[A_c + m(t)\right]}{E_N(t)}\cos\varphi_n(t)\right) \approx E_N(t) + \left[A_c + m(t)\right]\cos\varphi_n(t) \tag{5.95}$$

Since, no term is proportional to $m(t)$ in Eq. (5.95), there is no use of envelope detector for detection of message signal $m(t)$ in the presence of large noise. The term $\cos\varphi_n(t)$ is a time-varying function that distorts the message signal $m(t)$.

5.12 Angle Modulated Systems

The schematic diagram of the angle modulated system is presented in Fig. 5.17. The angle modulated signal is represented as

$$\phi_{\text{EM}}(t) = A_c \cos\left[\omega_c t + \varphi(t)\right]$$

$$\text{where, } \varphi(t) = \begin{cases} k_p m(t) & \text{for PM} \\ k_f \int_{-\infty}^{t} m(\tau)\,d\tau & \text{for FM} \end{cases} \tag{5.96}$$

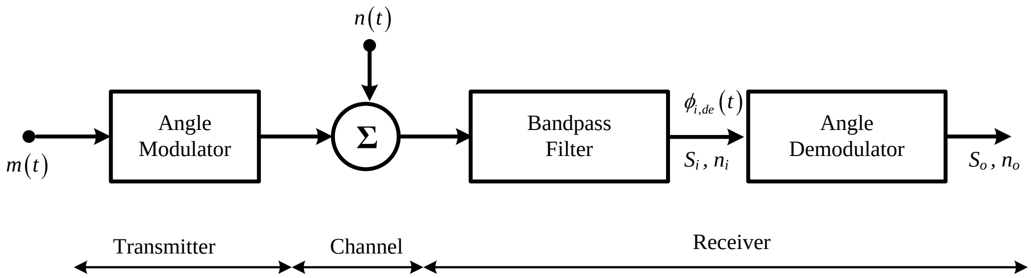


Fig. 5.17 The schematic diagram of the angle modulated system

The channel noise is

$$\mathrm{n}(t) = A_n \cos\left[\omega_c t + \psi(t)\right] \tag{5.97a}$$

The channel noise $\mathrm{n}(t)$ is represented in terms of quadrature component as

$$\mathrm{n}(t) = \mathrm{n}_c(t)\cos\omega_c t + \mathrm{n}_s(t)\sin\omega_c t \tag{5.97b}$$

The angle modulation is nonlinear; therefore, the signal power and the noise power are calculated individually to determine the *S/N* ratio. The performance of PM in the presence of noise is considered first and the same result is extended for FM modulation.

Phase Modulation

The frequency variations of the modulated carrier are slower than that of the noise. Therefore, the unmodulated carrier is considered as constant for several cycles and moreover, the noise power is calculated by considering the signal power zero.

Let the input signal at the demodulator input of the Fig. 5.17 is given as

$$\begin{aligned}\phi_{i,de}(t) &= A_c \cos\left[\omega_c t + \varphi(t)\right] + \mathrm{n}(t)\\ &= A_c \cos\left[\omega_c t + \varphi(t)\right] + A_n \cos\left[\omega_c t + \psi(t)\right]\\ &= E(t)\cos\left[\omega_c t + \underbrace{\varphi(t) + \Delta\varphi(t)}_{\text{Detected phase } \phi_o(t)}\right] \quad \text{For phase modulation } \varphi(t) = k_p m(t)\end{aligned} \tag{5.98}$$

The demodulator detects the phase of the Eq. (5.98). The phasor diagram of Eq. (5.98) is shown in Fig. 5.18.

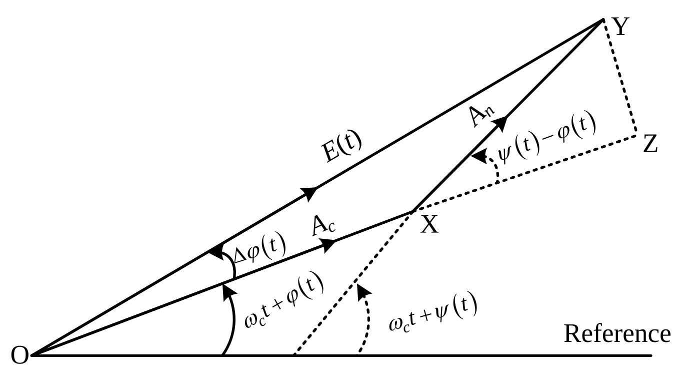


Fig. 5.18 The phasor diagram of Eq. (5.98)

In ΔXYZ

$$\mathrm{YZ} = A_n \sin\left[\psi(t) - \varphi(t)\right] \tag{5.99}$$

For small noise, $A_n << A_c$, In ΔOXY

$$\sin\left(\Delta\varphi(t)\right) \approx \Delta\varphi(t) = \frac{\mathrm{YZ}}{\mathrm{OY}} \;\; \& \qquad \mathrm{OY} \approx \mathrm{OX} = A_c \tag{5.100}$$

Hence, $$\Delta\varphi(t) = \frac{A_n}{A_c}\sin\left[\psi(t) - \varphi(t)\right] \tag{5.101}$$

Therefore, the detected phase of the PM demodulator $\phi_o(t)$ is obtained by substituting the value of $\Delta\varphi(t)$ from Eq. (5.101) into detected phase $\phi_o(t)$ of Eq. (5.98)

$$\begin{aligned}\phi_o(t) &= \varphi(t) + \Delta\varphi(t) \\ &= k_p m(t) + \frac{A_n}{A_c}\sin\left[\psi(t) - \varphi(t)\right]\end{aligned} \tag{5.102}$$

As we know, the modulated carrier varies slowly in comparison with noise; therefore, $\varphi(t)$ is considered as constant. Hence,

$$\phi_o(t) = k_p m(t) + \underbrace{\frac{A_n}{A_c}\sin\left[\psi(t) - \varphi\right]}_{\Delta\varphi(t)}$$

$$\begin{aligned}\Delta\varphi(t) &= \frac{A_n}{A_c}\sin\psi(t)\cos\varphi - \frac{A_n}{A_c}\cos\psi(t)\sin\varphi \\ &= \frac{\mathrm{n}_s(t)}{A_c}\cos\varphi - \frac{\mathrm{n}_c(t)}{A_c}\sin\varphi\end{aligned} \tag{5.103}$$

The PSD is given as

$$S_{\phi_o}(\omega) = \frac{\cos^2\varphi}{A_c^2}S_{\mathrm{n}_s}(\omega) + \frac{\sin^2\varphi}{A_c^2}S_{\mathrm{n}_c}(\omega) \tag{5.104}$$

As we know, $S_{\mathrm{n}_s}(\omega) = S_{\mathrm{n}_c}(\omega)$

$$\Rightarrow S_{\phi_o}(\omega) = \frac{S_{\mathrm{n}_s}(\omega)}{A_c^2} \tag{5.105}$$

For white channel noise, the PSD of white noise is $\mathcal{N}/2$. So,

$$\Rightarrow S_{\phi_o}(\omega) = \begin{cases}\dfrac{\mathcal{N}}{A_c^2} & |f| \le \Delta f + f_m \\ 0 & \text{otherwise}\end{cases} \tag{5.106}$$

Since, the message signal bandwidth is only f_m. Therefore, the noise power is given as

$$P_n = 2\int_0^{f_m} S_{\phi_o}(\omega)\,df = 2\int_0^{f_m} \frac{\mathcal{N}}{A_c^2}\,df = \frac{2\mathcal{N} f_m}{A_c^2} \tag{5.107}$$

The signal power is given as

$$P_s = k_p^2 \overline{m^2} \tag{5.108}$$

Therefore, the output signal to noise ratio is given as

$$\left(\frac{S_o}{N_o}\right)_{\text{PM}} = \frac{P_s}{P_n} = \frac{k_p^2 \overline{m^2}}{2\mathcal{N} f_m / A_c^2} = \left(A_c k_p\right)^2 \frac{\overline{m^2}}{2\mathcal{N} f_m} \tag{5.109}$$

The figure of merit γ is

$$\gamma = \frac{S_i}{\mathcal{N} f_m} = \frac{A_c^2/2}{\mathcal{N} f_m} = \frac{A_c^2}{2\mathcal{N} f_m} \tag{5.110}$$

Substituting the value of γ from Eq. (5.110) into Eq. (5.109)

$$\left(\frac{S_o}{N_o}\right)_{\text{PM}} = k_p^2 \overline{m^2} \gamma \tag{5.111}$$

Since, $\Delta\omega = k_p m'_p$, So

$$\left(\frac{S_o}{N_o}\right)_{\text{PM}} = \left(\Delta\omega\right)^2 \frac{\overline{m^2}}{m'^2_p}\gamma = \left(2\pi\Delta f\right)^2 \frac{\overline{m^2}}{m'^2_p}\gamma \tag{5.112}$$

Frequency Modulation

The output of the FM modulator is $k_f m(t)$. The output signal power is given as:

$$S_o = k_f^2 \overline{m^2} \tag{5.113}$$

The FM output is obtained by the differentiating the output of the phase demodulator as shown in Fig. 5.19.

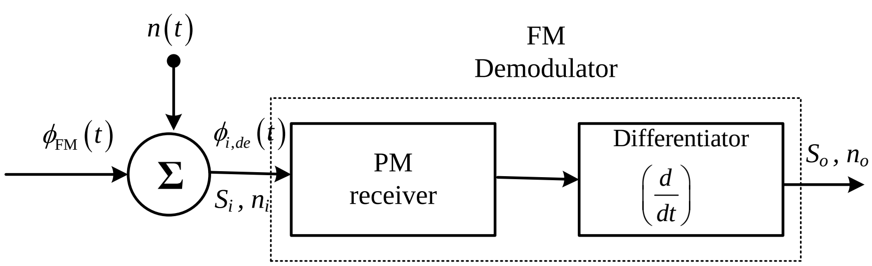


Fig. 5.19 FM demodulator by PM demodulator

The transfer function of the differentiator is $(j\omega)$. Therefore, the PSD of the output noise is

$S_{n_o}(\omega) = |j\omega|^2 \frac{\mathcal{N}}{A_c^2} = \frac{\mathcal{N}}{A_c^2}\omega^2$ which is bandlimited to f_m, i.e.

$$S_{n_o} = \begin{cases} \frac{\mathcal{N}}{A_c^2}\omega^2 & |\omega| \le 2\pi f_m \\ 0 & |\omega| > 2\pi f_m \end{cases} \tag{5.114}$$

The output noise power is given as

$$N_o = 2\int_0^{f_m} \frac{\mathcal{N}}{A_c^2}(2\pi f)^2 df = \frac{8\pi^2 \mathcal{N} f_m^3}{3A_c^2} \tag{5.115}$$

Hence, output $\frac{S_o}{N_o}$ is obtained by Eq. (5.113) and Eq. (5.115)

$$\left(\frac{S_o}{N_o}\right)_{\text{FM}} = 3\left(\frac{k_f^2 \overline{m^2}}{(2\pi f_m)^2}\right)\left(\frac{A_c^2}{\mathcal{N} f_m}\right) = 3\left(\frac{k_f^2 \overline{m^2}}{(2\pi f_m)^2}\right)\gamma \tag{5.116}$$

Since, $\Delta\omega = k_f m_p$

$$\left(\frac{S_o}{N_o}\right)_{\text{FM}} = 3\left(\frac{\Delta f}{f_m}\right)\left(\frac{\overline{m^2}}{m_p^2}\right)\gamma \tag{5.117}$$

From Eq. (5.112) and Eq. (5.117)

$$\frac{(S_o/N_o)_{\text{PM}}}{(S_o/N_o)_{\text{FM}}} = \frac{(2\pi f_m)^2 m_p^2}{3m'^2_p} \tag{5.118}$$

It is concluded from Eq. (5.118)

If $(2\pi f_m)^2 m_p^2 > 3m'^2_p$ PM is superior to FM (i.e. if PSD of $m(t)$ is concentrated at lower frequencies)

If $(2\pi f_m)^2 m_p^2 < 3m'^2_p$ FM is superior to PM (i.e. if PSD of $m(t)$ is concentrated at higher frequencies)

Since, in tone modulation, the signal power is concentrated at the highest frequency; therefore, FM is superior to PM for tone modulation.

For small noise, i.e. $A_n << A_c$, the message signal dominates over noise and further the faithful recovery of the original message signal is possible at the receiver end. Let, $A_n \gg A_c$, in this case, noise power is much greater than the carrier power and therefore, noise dominates and message contained in angle modulated signal is corrupted. Therefore, a desirable S_i/N_i ratio is required for detection of message signal. Experimental studies shows that the effect of noise is almost negligible for the $S_i/N_i \geq 10$, therefore, threshold point is defined to be at

$$\frac{S_i}{N_i} = 10 \tag{5.119}$$

ADDITIONAL SOLVED EXAMPLES

SE5.8 A typical microwave receiver as shown in Fig. 5.20 is used in satellite communication. Evaluate:

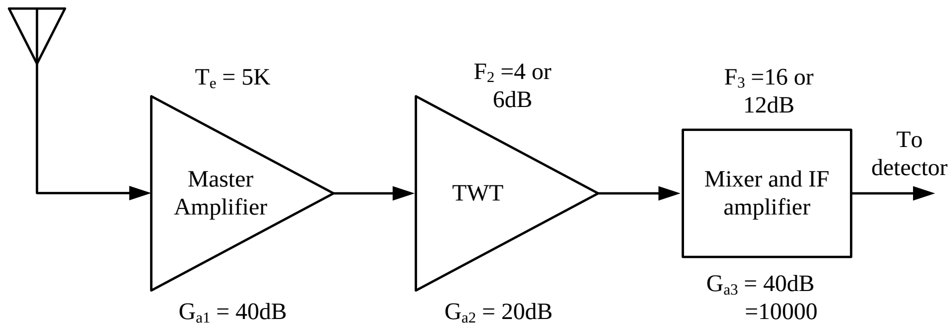


Fig. 5.20 A typical microwave receiver

(a) The overall noise figure of the receiver

(b) The overall equivalent temperature of the receiver.

Assume that ambient temperature T = 17°C.

Sol: The noise figure of the first stage is

$$F_1 = 1 + \frac{T_e}{T} = 1 + \frac{5}{273+17}$$
$$= 1 + \frac{5}{290}$$

(a) Similarly, the overall noise figure is

$$F = \left(1 + \frac{5}{290}\right) + \frac{4-1}{10000} + \frac{16-1}{1000000} \simeq 1.02$$

(b) The overall noise temperature is

$$T_e = T\left(F-1\right) = 290*(0.02) = 5.8\text{K}$$

SE5.9 The noise figure of the individual stages of a two-stage amplifier is 2.03 and 1.54, respectively. The available power gain of the first stage is 62. Evaluate the overall noise figure.

Sol: The overall noise figure is

$$F = F_1 + \frac{F_2 - 1}{G_1}$$
$$= 2.03 + \frac{1.54-1}{62}$$
$$= 2.03 + 0.0247 = 2.0547$$

SE5.10 An amplifier operating over the frequency range from 18 to 20 MHz has a 10 kΩ input resistor. Calculate the RMS noise voltage at the input to this amplifier if the ambient temperature is 27^oC.

Sol: The RMS noise voltage is given as

$$v_n = \sqrt{P_n} = \sqrt{4kTR(\Delta f)}$$

Given Data:

Input resistance	$R = 10\Omega$
Bandwidth	$\Delta f = 20 - 18 = 2$ MHz
Ambient temperature	$T = 273 + 27 = 300\text{K}$
Boltzmann's constant	$k = 1.38\times10^{-23}$ J/K

So, the RMS noise voltage is

$$v_n = \sqrt{P_n} = \sqrt{4\times1.38\times10^{-23}\times300\times10\times10^3\times2\times10^6} = 1.82\times10^{-5} = 18.2\ \mu\text{V}$$

SE5.11 A radio receiver with 10 kHz bandwidth has a noise figure of 30 dB. Calculate the noise power at the input. (k = 1.38 x 10^{-23}J/K and room temperature of 27^oC)

Sol: Given Data:

Noise figure	$F = 30$ dB
Bandwidth	$\Delta f = 10$ kHz
Ambient temperature	$T = 273 + 27 = 300\text{K}$
Boltzmann's constant	$k = 1.38\times10^{-23}$ J/°K

The noise figure is

$$(F)_{dB} = 10\log(F)$$
$$30 = 10\log(F)$$
$$\Rightarrow F = 1000$$

So, the noise power is

$$P_n = FkT(\Delta f) = 1000\times1.38\times10^{-23}\times300\times10\times10^3 = 4.14\times10^{-14}\ \text{W}$$

SE5.12 A receiver with noise resistance of 100Ω is connected to an antenna with an input resistance of 100Ω. Calculate the noise figure.

Sol: Noise figure is given as

$$F = 1 + \frac{R_{eq}}{R_s}$$
$$= 1 + \frac{100}{100} = 2$$

SE5.13 The input SNR is 37 dB for a device. Calculate the output SNR if the noise figure of the device is 2.

Sol: As we know,

$$F = \frac{(S/N)_{\text{input}}}{(S/N)_{\text{output}}}$$

So,

$$(F)_{\text{dB}} = \left((S/N)_{\text{input}}\right)_{\text{dB}} - \left((S/N)_{\text{output}}\right)_{\text{dB}}$$

$$\Rightarrow \left((S/N)_{\text{output}}\right)_{\text{dB}} = \left((S/N)_{\text{input}}\right)_{\text{dB}} - (F)_{\text{dB}} = 37 - 10\log 2$$

$$\Rightarrow \left((S/N)_{\text{output}}\right)_{\text{dB}} = 37 - 3 = 34 \text{ dB}$$

PROBLEMS

P5.1 Explain the following terms:

(i) Thermal Noise

(ii) Shot noise

(iii) Noise Figure

(iv) Signal to Noise Ratio

(v) Equivalent Noise Temperature

P5.2 Define the figure of merit and explain its importance.

P5.3 Explain the bandpass noise model.

P5.4 Determine the expressions for the figure of merit in DSB-SC.

P5.5 Derive an expression for the signal to noise power ratio for the SSB-SC amplitude modulation system in the presence of white Gaussian noise of power spectral density.

P5.6 Explain the performance of FM in the presence of noise.

P5.7 Explain the performance of AM in the presence of noise.

P5.8 Determine the output SNR in a PM system for the modulation.

NUMERICALS PROBLEMS

P5.9 A receiver with noise resistance of 150Ω is connected to an antenna with an input resistance of 300Ω. Calculate the noise figure.

P5.10 The input SNR is 30 dB for a device. Calculate the output SNR if the noise figure of the device is 1.5.

P5.11 An amplifier operating over the frequency range from 10 to 15 MHz has a 20 kΩ input resistor. Calculate the RMS noise voltage at the input to this amplifier if the ambient temperature is 300K.

P5.12 Three amplifiers have following characteristics: F_1 = 9 dB, F_2 = 6 dB, F_3 = 4 dB, G_1 = 48 dB, G_2 = 35 dB, G_3 = 20 dB. The amplifiers are connected in random. Determine the sequence of combination which gives best noise figure referred to the input. Calculate overall noise figure and equivalent noise temperature in this case.

P5.13 The equivalent noise temperature of a parametric amplifier is 55 K. Calculate its noise figure if the ambient temperature is 27°C.

P5.14 Two resistors each of 2 kΩ are at temperature 300 K and 400 K respectively. Find the power density spectrum of noise voltage at the terminal formed by (a) series combination (b) parallel combination.

MULTIPLE-CHOICE QUESTIONS

MCQ5.1 The Gaussian process is a

(a) Wide sense stationary process
(b) Strict sense stationary process
(c) Both (a) and (b)
(d) None of the above

MCQ5.2 One of the following types of noise assumes importance at high frequencies

(a) Transit time noise (c) Thermal noise
(b) Shot noise (d) Flicker noise

MCQ5.3 The noise caused by random variations in the arrival of electrons or holes at the output electrode of an amplifying device

(a) Flicker noise (c) Transit time noise
(b) White noise (d)Shot noise

MCQ5.4 Spectral density of white noise

(a) Varies with frequency
(b) Varies with amplitude of the signal
(c) is constant
(d) varies with frequency

MCQ5.5 Noise figure of a device is 2. If input SNR is 37 dB, the output SNR would be

(a) 34 dB (c) 14 dB
(b) 74 dB (d) 39 dB

MCQ5.6 A system has a receiver noise resistance of 50 Ω. It is connected to an antenna with an input resistance of 50 Ω. the noise figure of the system is

(a) 50 (c) 1
(b) 100 (d) 2

MCQ5.7 Gain and NF of a single-stage amplifier are 10 dB and 3 dB, respectively. When two such amplifiers are cascaded, then gain and NF of the cascaded amplifier will be

(a) 20 dB, 6 dB (c) 3.2 dB, 20 dB
(b) 20 dB, 3.2 dB (d) 6 dB, 20 dB

MCQ5.8 Two resistors R_1 and R_2 (in ohms) at temperatures T_1 K and T_2 K, respectively are connected in series. Their equivalent noise temperature

(a) $(R_1T_1 + R_2T_2)/(R_1 + R_2)$ (c) $R_1T_1 + R_2T_2$
(b) $(R_1T_1 + R_2T_2)/(R_1R_2)$ (d) $T_1 + T_2$

MCQ5.9 The noise figure of merit in SSB modulated signal is.

(a) 1 (c) Greater than 1
(b) Less than 1 (d) None of the above

MCQ5.10 Matched filter is

(a) Linear
(b) Non-linear
(c) Both (a) and (b)
(d) None of the above

MCQ ANSWERS

MCQ5.1	(c)	MCQ5.6	(d)
MCQ5.2	(a)	MCQ5.7	(b)
MCQ5.3	(d)	MCQ5.8	(a)
MCQ5.4	(c)	MCQ5.9	(a)
MCQ5.5	(a)	MCQ5.10	(a)

CHAPTER 6

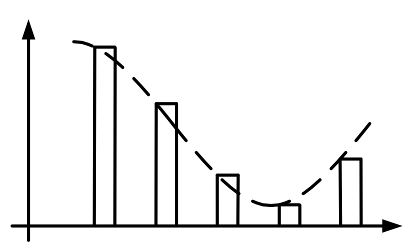
PULSE MODULATION

Definition

Unlike continuous-wave modulation, the pulse shaped carrier wave is used for the modulation in pulse modulation.

Highlights

Representation



6.1 Introduction

In previous chapters, the continuous wave modulations are discussed. There is another type of modulation scheme, which is termed as pulse modulation. Unlike continuous-wave modulation, the pulse shaped carrier wave is used for the modulation in pulse modulation.

6.2 Pulse Modulation

Like continuous wave modulation scheme, pulse modulation is further divided into different categories, as shown in Fig. 6.1.

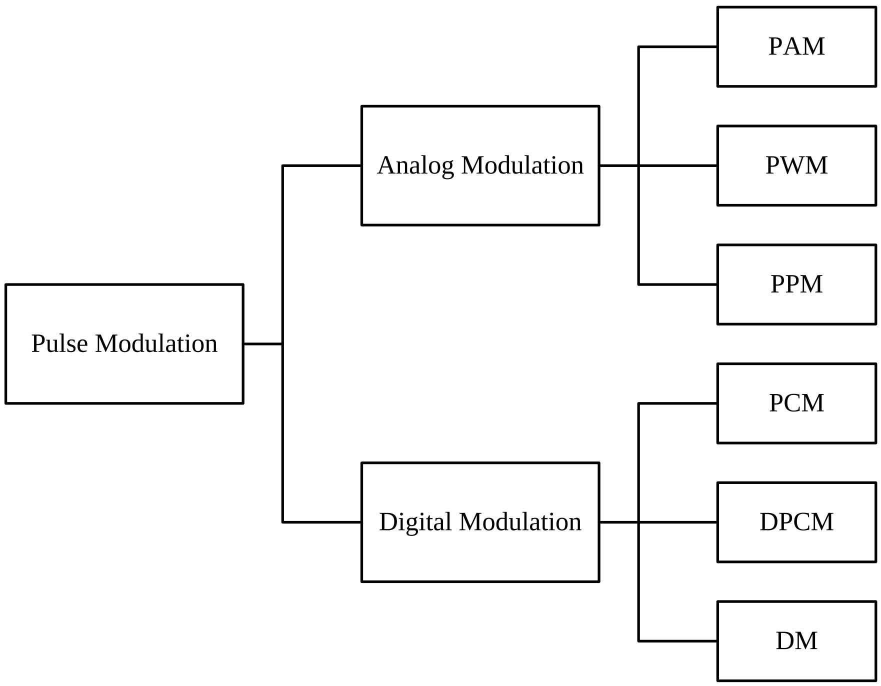
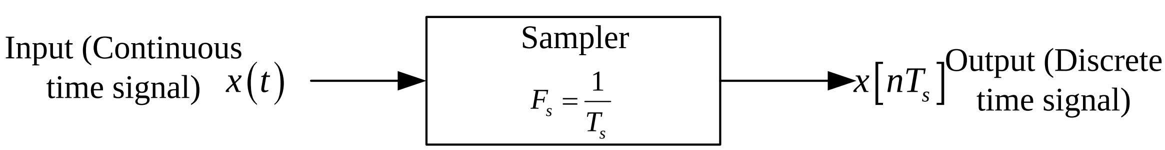


Fig. 6.1 Classification of pulse modulation

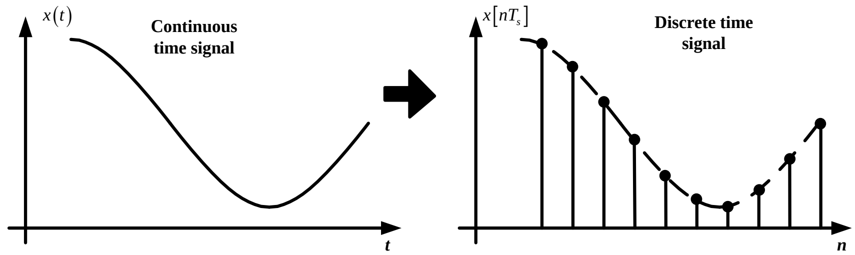
The pulse modulation scheme mainly deals with discrete-time signals. Therefore, it is important to understand the conversion process of continuous-time signals into the discrete-time signal. This conversion process is called sampling.

6.3 Sampling

The whole process to convert the continuous-time signal into its equivalent discrete-time signal, to process this signal using a discrete-time system and to convert the sampled signal back to the continuous time signal (Fig. 6.2(a)-(b)) is defined as the sampling. The continuous time signal is continuously sampled at discrete-time intervals in the sampling process.



(a)



(b)

Fig. 6.2 (a) Block diagram of sampler (b) Periodic sampling of a continuous-time signal

6.3.1 Sampling Theorem for Continuous-Time Signal

For the reconstruction of the continuous-time signal from its samples, the sampling rate or sampling frequency should follow the given rule

Sampling rate ≥ Nyquist rate (6.1)

Nyquist rate is defined as the minimum sampling frequency required for the reconstruction of the continuous-time signal from its samples. The Nyquist rate depends upon the highest frequency component presents in the message signal and it is expressed as

Nyquist rate = 2 x Highest frequency component of the message signal

$$= 2f_{max} \tag{6.2}$$

Let consider a continuous-time signal

$$x(t) = A\cos(2\pi Ft + \theta) \tag{6.3}$$

Signal $x(t)$ given in Eq. (6.3) is sampled at a sampling rate of F_s samples/second or Sampling period of $T_s = \frac{1}{F_s}$. Thus, the obtained discrete signal is expressed as

$$x(nT_s) = x(t)\big|_{t=nT_s} \tag{6.4}$$

$$x(nT_s) = A\cos(2\pi Ft + \theta)\big|_{t=nT_s} = A\cos(2\pi FnT_s + \theta)$$

$$= A\cos\left(2\pi \frac{F}{F_s} n + \theta\right) \tag{6.5}$$

$$x(nT_s) = A\cos(2\pi fn + \theta) \tag{6.6}$$

where, $x(n) = x(nT_s)$ = Discrete-time signal

F = Frequency of continuous-time signal, F_s = Sampling frequency in samples/second

6.3.2 Sampling Rate

The sampling rate is also called sampling frequency. It is defined as the average number of samples per second for the reconstruction of the original signal from its sampled form. The relationship between sampling rate F_s and sampling interval T_s is given as

$$F_s = \frac{1}{T_s} \tag{6.7}$$

SE6.1 Find the Nyquist rate and the Nyquist interval for the signal

$$\phi(t) = \frac{1}{2\pi}\cos(2000\pi t)\cos(500\pi t)$$

Sol: The signal is

$$\phi(t) = \frac{1}{2\pi}\cos(2000\pi t)\cos(500\pi t)$$

$$= \frac{1}{4\pi}\left[2\cos(2000\pi t)\cos(500\pi t)\right] \qquad \left[\because 2\cos A\cos B = \cos(A+B)+\cos(A-B)\right]$$

$$= \frac{1}{4\pi}\left[\cos(2000\pi t + 500\pi t) + \cos(2000\pi t - 500\pi t)\right]$$

$$= \frac{1}{4\pi}\left[\cos(2500\pi t) + \cos(1500\pi t)\right]$$

Therefore, the frequency components are

$$\omega_1 = 2\pi f_{m1} = 2500\pi \qquad \omega_2 = 2\pi f_{m2} = 1500\pi$$

$$\Rightarrow f_{m1} = 1250 \text{ Hz} \qquad \Rightarrow f_{m2} = 750 \text{ Hz}$$

The highest frequency component is $f_{m1} = 1250$ Hz. So, Nyquist rate is

$$f_{Nyquist} = 2 f_{m1} = 2500 \text{ Hz}$$

The Nyquist interval is

$$T_{Nyquist} = \frac{1}{f_{Nyquist}} = \frac{1}{2500} = 0.0004 \text{ sec} = 0.4 \text{ msec}$$

SE6.2 Find the Nyquist rate and the Nyquist interval for the signal

$$\phi(t) = 2\cos(20\pi t) + 10\sin(200\pi t) - 20\sin(400\pi t)$$

Sol: The frequency components are

$$\omega_1 = 2\pi f_{m1} = 20\pi \qquad \omega_2 = 2\pi f_{m2} = 200\pi \qquad \omega_3 = 2\pi f_{m3} = 400\pi$$

$$\Rightarrow f_{m1} = 10 \text{ Hz} \qquad \Rightarrow f_{m2} = 100 \text{ Hz} \qquad \Rightarrow f_{m3} = 200 \text{ Hz}$$

The highest frequency component is $f_{m3} = 200$ Hz. So, Nyquist rate is

$$f_{Nyquist} = 2 f_{m3} = 400 \text{ Hz}$$

The Nyquist interval is

$$T_{Nyquist} = \frac{1}{f_{Nyquist}} = \frac{1}{400} = 0.0025 \text{ sec} = 2.5 \text{ msec}$$

6.3.3 Impulse Train Sampling of Continuous Time signals

The sampled signal $x_p(t)$ of the continuous-time signal $x(t)$ is obtained by the multiplication of a periodic impulse train $p(t)$ with the continuous-time signal $x(t)$, as shown in Fig. 6.3.

The sampled signal in the time domain is given by

$$x_p(t) = x(t)\,p(t)$$

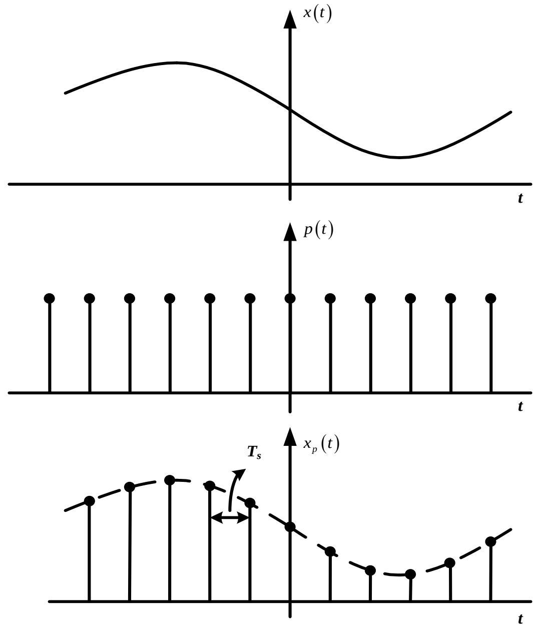


Fig. 6.3 Impulse train sampling of a continuous-time signal

The expression of the periodic impulse train $p(t)$ is given by

$$p(t)=\sum_{n=-\infty}^{\infty}\delta(t-nT_s) \tag{6.8}$$

Since, $x(t)\delta(t-t_o)=x(t_o)\delta(t-t_o)$ (6.9)

Therefore, the $x_p(t)$ is given as

$$x_p(t)=\sum_{n=-\infty}^{\infty}x(t)\delta(t-nT_s)=\sum_{n=-\infty}^{\infty}x(nT_s)\delta(t-nT_s) \tag{6.10}$$

From the multiplication property of CTFT

$$x_p(t)=x(t)\,p(t)\xleftrightarrow{\text{CTFT}}\frac{1}{2\pi}\left[X(\omega)*P(\omega)\right]$$

Or (6.11)

$$X_p(\omega)=\frac{1}{2\pi}\left[X(\omega)*P(\omega)\right]$$

Taking the CTFT of Eq. (6.8)

$$\text{CTFT}\{p(t)\}=\text{CTFT}\left\{\sum_{n=-\infty}^{\infty}\delta(t-nT_s)\right\}$$

Or (6.12)

$$P(\omega)=\frac{2\pi}{T_s}\left\{\sum_{k=-\infty}^{\infty}\delta(\omega-k\omega_s)\right\}$$

Therefore, $X_p(\omega)=\frac{1}{2\pi}\left[X(\omega)*P(\omega)\right]$

$$=\frac{1}{T_s}\sum_{k=-\infty}^{\infty}X(\omega-k\omega_s) \tag{6.13}$$

The frequency-domain representation of the sampling process is shown in Fig. 6.4. It is cleared from Figs. 6.4(c)-(e) that the original signal is recovered from the sampled signal only when $\omega_s \geq 2\omega_m$

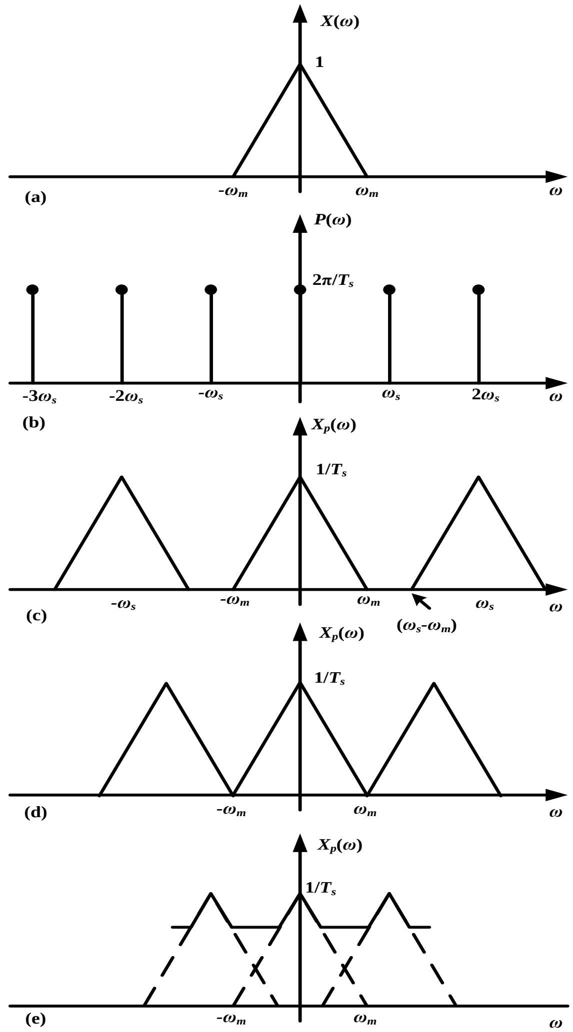


Fig. 6.4 Spectrum of (a) Continuous-time original signal (b) Periodic impulse train (c) Sampled version of continuous-time signal $x_p(t)$ with $\omega_s > 2\omega_m$ (d) Sampled version of continuous time signal $x_p(t)$ with $\omega_s = 2\omega_m$ (e) Sampled version of continuous time signal $x_p(t)$ with $\omega_s < 2\omega_m$

SE6.3 The condition that should be satisfied in order to recover the original signal back?

(a) $\omega_m <= \omega_c >= \omega_s - \omega_m$

(b) $\omega_m > \omega_c > \omega_s - \omega_m$

(c) $\omega_m > \omega_c < \omega_s$

(d) $\omega_m <= \omega_c <= \omega_s - \omega_m$

Sol: From Fig. 6.4, The condition to recover the original signal back is

$$\omega_m <= \omega_c <= \omega_s - \omega_m$$

Hence, option (d) is correct.

6.4 Pulse Amplitude Modulation

Like continuous amplitude modulation, the amplitude of the pulse carrier varies in accordance with the instantaneous value of the modulating signal in pulse amplitude modulation (PAM). In the PAM scheme, the modulated waveform follows the original signal's amplitude, as shown in Fig. 6.5.

There are two methods of PAM signal generation:

1. Natural sampling or shaped to sampling
2. Flat-top sampling

Among both above methods, Flat-top sampling is very popular and widely used because noise affects the top of the pulses during the transmission and can easily be removed if the pulses are Flat-top.

6.4.1 Natural Sampling

The process of natural sampling is shown in Fig. 6.5. Natural sampling is described as the process of multiplication of the original signal with the unit amplitude rectangular pulses. In natural sampling, the amplitude of the PAM signal is similar to the modulating signal.

In natural sampling, if the signal is sampled at the Nyquist rate, the signal is reconstructed back by passing the signal through an efficient low pass filter.

6.4.2 Flat Top Sampling

Although the signal is reconstructed back by using LPF, still it is not possible to recover the signal without distortion. Therefore, flat-top sampling is used to avoid noise distortion. In flat-top sampling, the amplitude of the samples is equal to the instantaneous value of the message signal at the instant of sampling and remains constant until the next sample is obtained, as shown in Fig. 6.6.

A flat-top sampled signal is generated by a sample and hold circuit shown in Fig. 6.7(a). The working of the sample and hold circuit is as follows:

The sampling switch S_1 is closed for short time duration and the capacitor charges quickly up to the instantaneous value of signal voltage. Now the switch is opened and the capacitor holds the charge until discharge switch S_2 is closed. Further, the capacitor discharges to the zero voltage through the discharge switch. Again, S_1 is closed and S_2 is opened, the capacitor again charged to the instantaneous value of signal voltage. This process continues and a Flat-top sampled PAM signal equivalent to the input signal as shown in Fig. 6.7(b) is obtained.

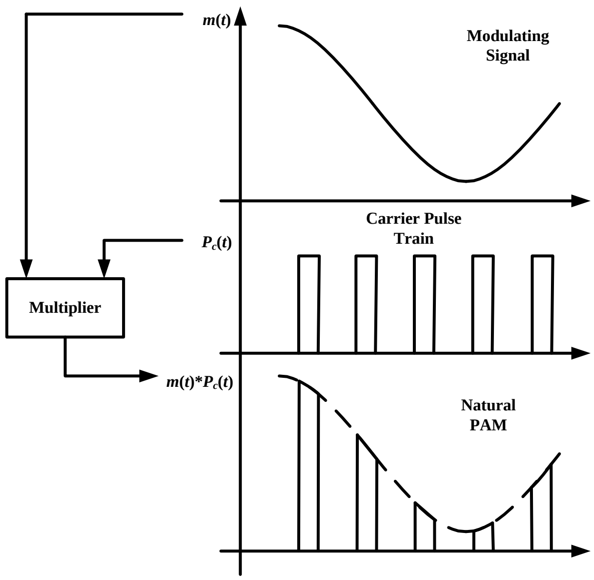


Fig. 6.5 Pulse amplitude modulation

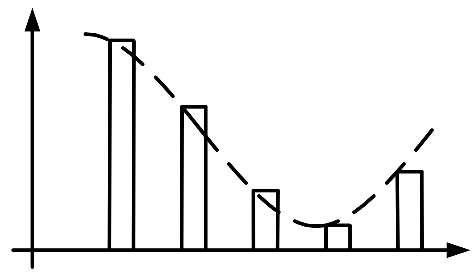
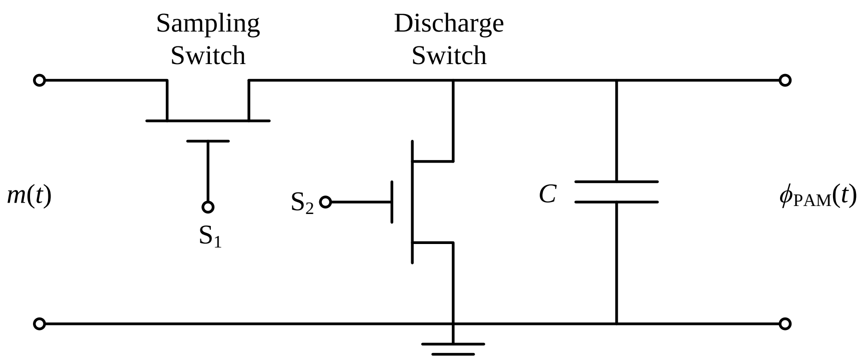
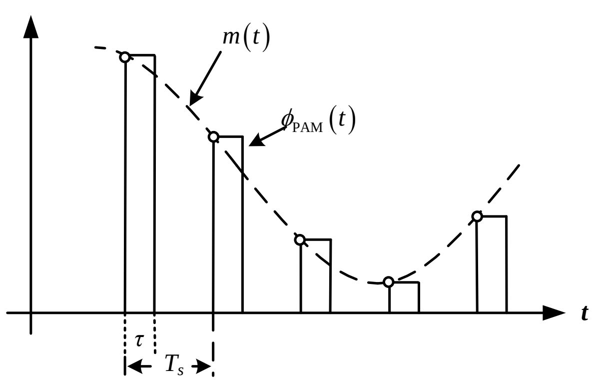


Fig. 6.6 Flat-top sampling



(a)



(b)

Fig. 6.7 (a) Sample and hold circuit (b) Flat-top PAM signal

6.4.3 Transmission Bandwidth in PAM

Let the pulse width or duration in PAM signal is τ which is very less compared to sampling time T_s i.e.

$$\tau \ll T_s \tag{6.14}$$

According to Nyquist theorem, the sampling frequency must be greater or equal to the Nyquist rate i.e.

$$f_s \geq 2f_m$$
$$\frac{1}{T_s} \geq 2f_m \Rightarrow T_s \leq \frac{1}{2f_m} \tag{6.15}$$

where, f_m is the maximum frequency component of the modulating signal $m(t)$. Therefore,

$$\tau \ll T_s \leq \frac{1}{2f_m} \tag{6.16}$$

If ON and OFF time duration of the PAM pulses are equal, then the maximum frequency is

$$f_{\max} = \frac{1}{\tau + \tau} = \frac{1}{2\tau} \tag{6.17}$$

Therefore, transmission bandwidth should be

$$(\text{BW})_{\text{PAM}} \geq f_{\max} \Rightarrow (\text{BW})_{\text{PAM}} \geq \frac{1}{2\tau} \tag{6.18}$$

Since, $\tau \ll \frac{1}{2f_m} \Rightarrow \frac{1}{\tau} \gg 2f_m$, So

$$(\text{BW})_{\text{PAM}} \geq \frac{1}{2\tau} \gg f_m$$
$$\Rightarrow (\text{BW})_{\text{PAM}} \gg f_m \tag{6.19}$$

6.4.4 Demodulation of PAM

The message signal $m(t)$ is recovered from the PAM signal by a demodulator shown in Fig. 6.8. Here, the holding circuit is a zero-order hold circuit. In this circuit, the switch is closed at the arrival of the pulse and the capacitor starts charging. Further, the switch is opened at the end of the pulse and the capacitor holds the charge until the next pulse arrives. The low pass filter is used to remove the distortion and to smooth the pulses.

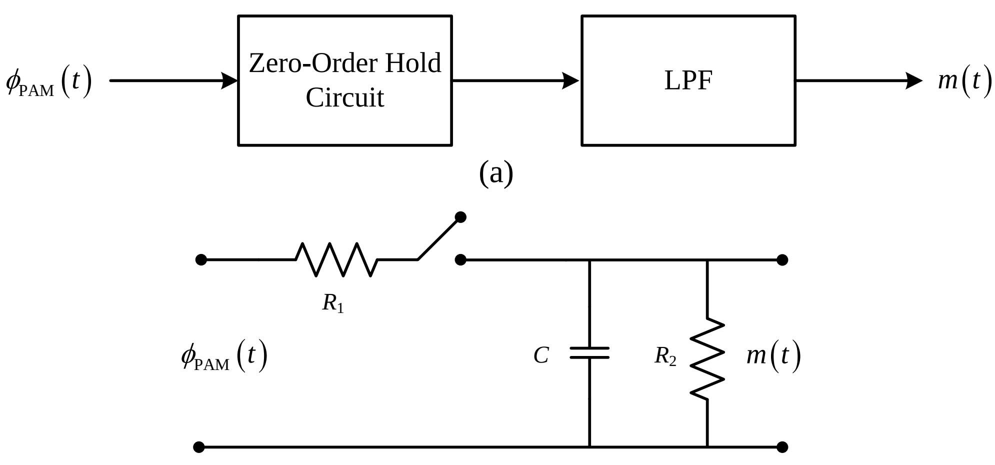


Fig. 6.8 (a) Demodulation of PAM (b) Zero-order hold circuit

6.5 Pulse Width Modulation

Other names of pulse width modulation (PWM) are pulse time modulation (PTM) and pulse duration modulation (PDM). In PWM (PTM or PDM), the pulse width (time or duration) of the carrier signal varies in accordance with the instantaneous amplitude of the baseband signal. The PWM is an analog modulating scheme, as shown in Fig. 6.9.

Similar to continuous angle modulation, the amplitude of the carrier pulse remains constant, whereas the width of the pulse varies in PWM. An amplitude limiter circuit is used in PWM to keep the amplitude at the desired level.

On the basis of the pulse position, there are three types of PWM which are as follows:

(i) The leading and the trailing edge of the pulse varies in accordance with the baseband signal, whereas the center of the pulse remains fixed (Fig. 6.9a)

(ii) The leading edge of the pulse remains fixed, whereas the trailing edge varies in accordance with the baseband signal (Fig. 6.9b).

(iii)The trailing edge of the pulse remains fixed, whereas the leading edge varies in accordance with the baseband signal (Fig. 6.9c).

All of the above PWM types with their timing slots are shown in Fig. 6.9.

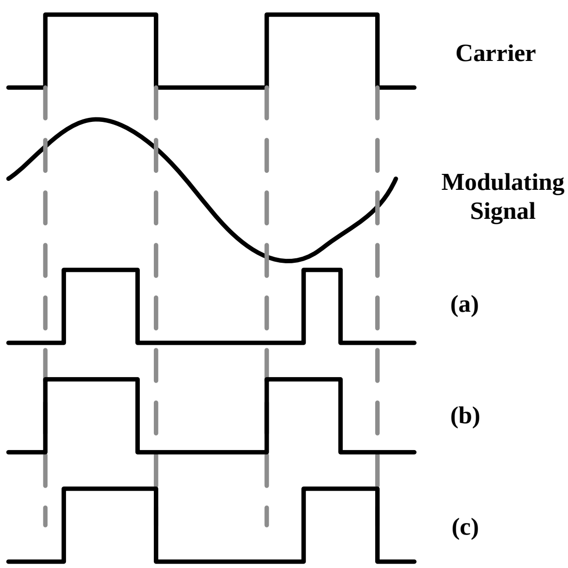


Fig. 6.9 Various types of PWM signals

6.6 Pulse Position Modulation

Another type of pulse modulation scheme is pulse position modulation (PPM), where the position of each pulse varies in accordance with the instantaneous sampled value of the baseband signal. In PPM, the amplitude and the width of the pulses remain constant. The starting point of the pulses in PPM is the trailing edge of the pulses in PWM, as shown in Fig. 6.10.

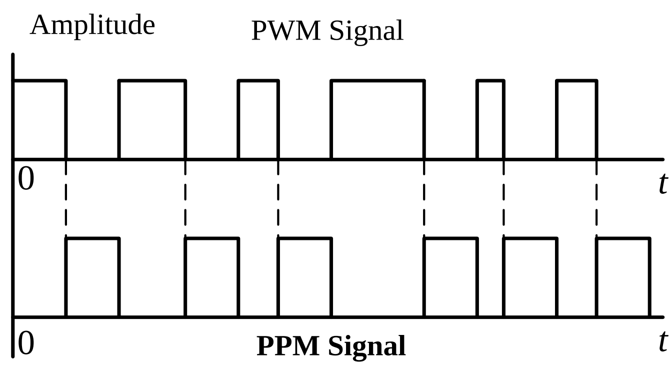


Fig. 6.10 PWM and PPM signals

Advantage

The main advantage of the PPM is that the power handled by the modulation remains constant as the amplitude and width of the pulses are constant.

Disadvantage

The complexity of the structure increases since PPM needs synchronization between the transmitter and receiver.

The comparison between PAM, PWM and PPM is shown in Table 6.1.

Table 6.1 Comparison of PAM, PWM and PPM

S. N.	Parameters	PAM	PWM	PPM
1.	**Similarity**	Same as AM	Same as FM	Same as PM
2.	**Variable parameters**	Amplitude	Width	Position
3.	**Bandwidth dependency**	width of the pulse	the rise time of the pulse	the rise time of the pulse
4.	**Noise interference**	High	Low	Low
5.	**System complexity**	High	Low	Low
6.	**Instantaneous transmitted power**	varies with the amplitude of the pulses	varies with the amplitude and width of the pulses	remains constant with the width of the pulses

6.7 Pulse Code Modulation

The pulse code modulation is the representation of analog signal into the corresponding binary sequence, i.e., combinations of 0s and 1s. The PCM output for the instantaneous values of analog signal is shown in Fig. 6.11.

Unlike different types of pulse modulated signals, the output of the PCM signal is represented by the digits. Therefore, pulse code modulation is digital process. Each binary sequence in PCM output is the approximate amplitude value of the sampled signal at that instant. The encoded message signal of the PCM is obtained by the discretization in the time and the amplitude both.

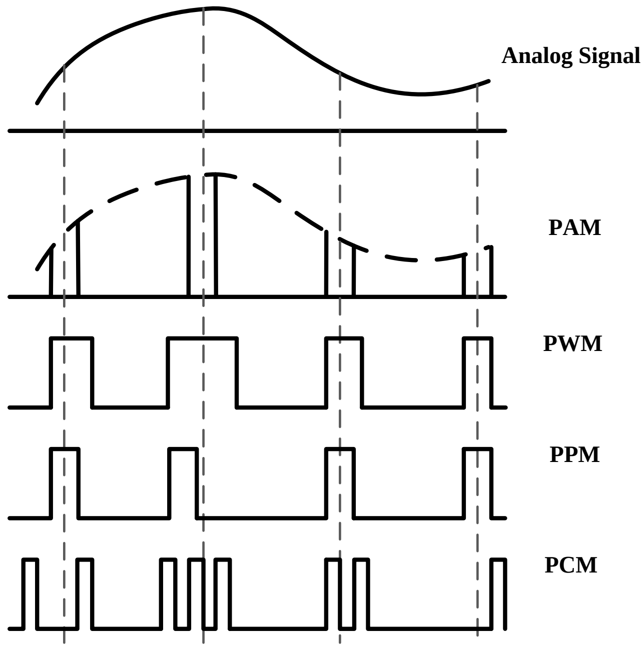


Fig. 6.11 PAM, PWM, PPM and PCM outputs w.r.t. instantaneous values of an analog signal

6.7.1 Discrete in Time and Amplitude

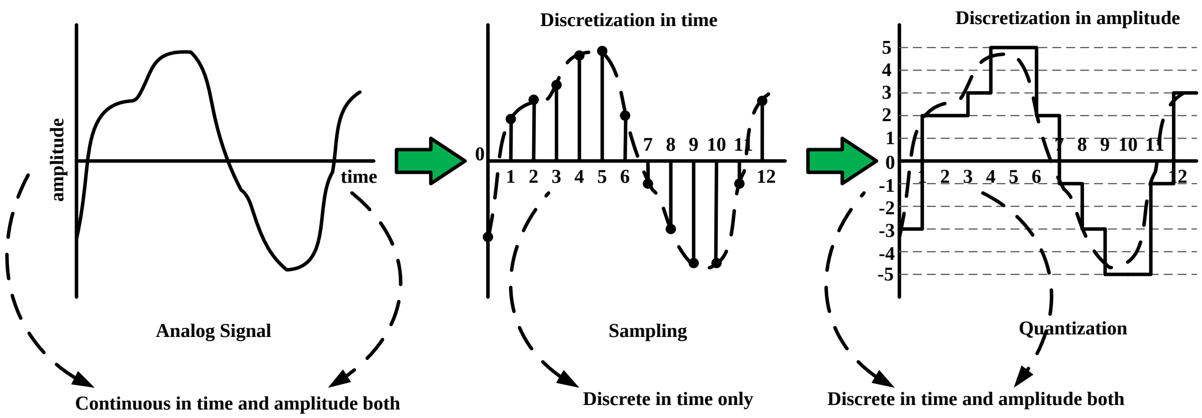
A discrete-time signal (or discrete signal) is defined as a time series or sequences of a quantity. If any signal is sampled and converted into a discrete-time domain from a continuous time domain, the process is called time discretization. On the other hand, if the signal's amplitude is discretised, then the process is called amplitude discretization. In short, discretization in time and amplitude is the conversion of amplitude and time of the signal from the continuous domain into the discrete domain (Fig. 6.12(a)).

As shown in Fig 6.12(b), the analog signal is defined for every instant of the time. i.e. continuous-time, whereas the digital signal is discrete amplitude and discrete-time signal. In other words, the digital signal is a quantized discrete signal. So,

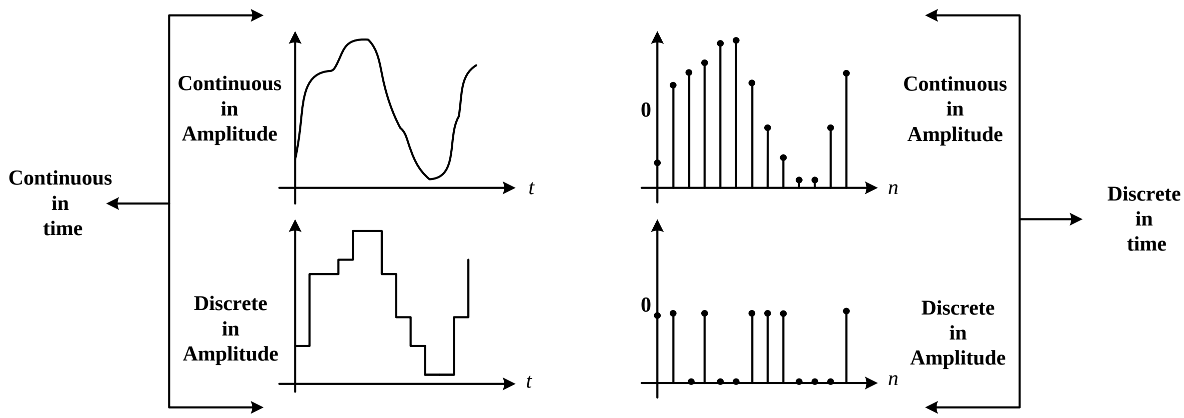
1. **The amplitude discretization- Continuous amplitude signal to discrete amplitude signal conversion.**
2. **The time discretization- Continuous time signal to discrete time signal conversion.**

6.7.2 Quantization

The digitization process of an analog signal consists of the rounding off the values which are approximately equal to the analog values. As shown in Fig. 6.13, the analog signals are sampled at a different instant and further, the values of the signal at these instants are round off to a near stabilized point. Such a process of digitization of analog signal is called **Quantization.**



(a)



(b)

Fig. 6.12 (a) Discretization in time and amplitude (b) Discrete in time and amplitude

Therefore, quantization is defined as a process that changes the continuous amplitude signal into a discrete amplitude signal, or quantization represents the sampled values of the amplitude by a finite set of levels.

In the quantization process, the continuous amplitude signal is discretized with a number of quantization levels or steps. In the PCM process, the quantization step is done to represent each sample of the signal by a fixed number of bits.

The quantization of analog signal is shown in Fig. 6.13. The analog signal is sampled with discrete amplitude. Further, this discretised signal is quantized at the nearest quantization levels.

The distance or gap between two adjacent representation levels is defined as step-size Δ. Let the analog signal is quantized within an amplitude ranges from $-m_p$ to $+m_p$ with L steps, then the step size Δ is given by

$$\Delta = \frac{2m_p}{L} \qquad (\Delta = \text{step size}) \qquad (6.20)$$

The categories of quantization are shown in Fig. 6.14.

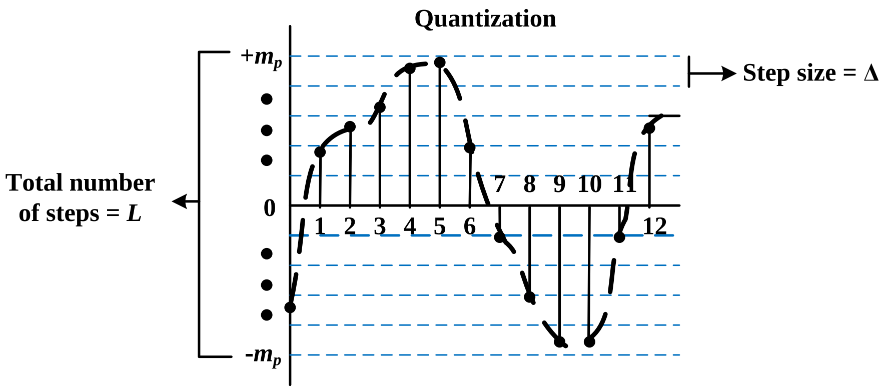


Fig. 6.13 Quantization levels

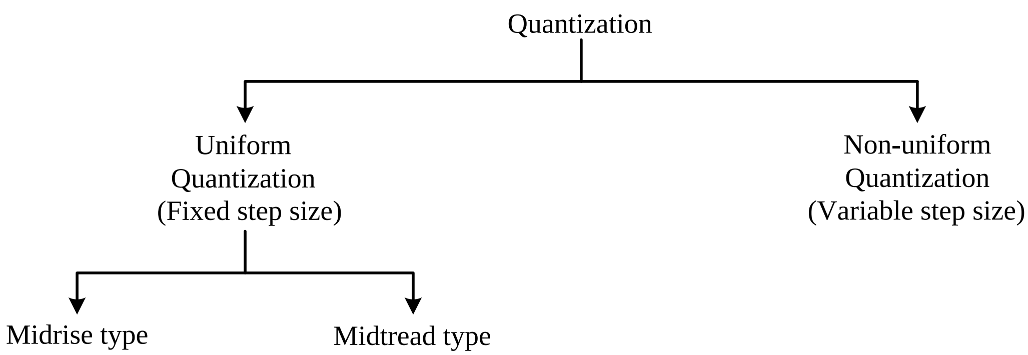


Fig. 6.14 Types of quantization

6.7.2.1 Uniform Quantizer

It is a type of quantizer where the step size remains fixed for the entire input range. Further, uniform quantizer is divided into two categories:

1. **Midrise Type Uniform Quantizer**

The characteristics of the midrise type quantizer is shown in Fig. 6.15. This is called so because the origin lies between the rising parts of the staircase graph.

2. **Midtread Type Uniform Quantizer**

It is named so because the origin lies between the tread parts of the staircase graph, as shown in Fig. 6.16.

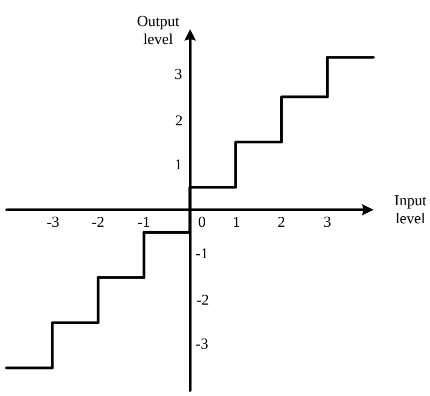


Fig. 6.15 Midrise type uniform quantizer

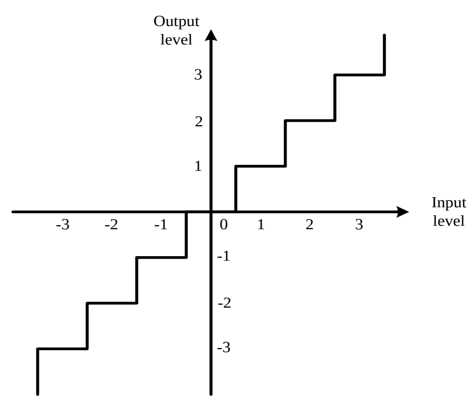


Fig. 6.16 Midtread type uniform quantizer

The number of levels in Midrise Quantizer is $L = 2^N$ whereas in Mid tread Quantizer $L = 2^N - 1$
Working of the uniform quantizer is as follows:

Let take the midrise type quantizer as shown in Fig. 6.17(a). Here, the sampled input can take any value from -3Δ to $+3\Delta$. Because of the quantizer characteristics, only fixed digital levels such as $\pm\frac{\Delta}{2}$, $\pm\frac{3\Delta}{2}$ and $\pm\frac{5\Delta}{2}$ are available. Therefore,

$$\begin{aligned} &\text{If} \quad x(nT_s) = +3\Delta, \quad \text{then} \quad x_q(nT_s) = +\frac{5\Delta}{2}; \\ &\text{If} \quad x(nT_s) = -3\Delta, \quad \text{then} \quad x_q(nT_s) = -\frac{5\Delta}{2} \end{aligned} \tag{6.21}$$

Therefore, the quantization error is

$$x_q(nT_s) - x(nT_s) = +\frac{5\Delta}{2} - (+3\Delta) = -\frac{\Delta}{2};$$

$$x_q(nT_s) - x(nT_s) = -\frac{5\Delta}{2} - (-3\Delta) = \frac{\Delta}{2}$$

Hence, the maximum quantization error is given as $\pm\frac{\Delta}{2}$.

Similarly, if $x(nT_s) = +2\Delta$ or -2Δ, then

$$\begin{aligned} &\text{If} \quad x(nT_s) = +2\Delta, \quad \text{then} \quad x_q(nT_s) = +\frac{3\Delta}{2}; \\ &\text{If} \quad x(nT_s) = -2\Delta, \quad \text{then} \quad x_q(nT_s) = -\frac{3\Delta}{2} \end{aligned} \tag{6.22}$$

Therefore, the quantization error is

$$x_q(nT_s) - x(nT_s) = +\frac{3\Delta}{2} - (+2\Delta) = -\frac{\Delta}{2};$$

$$x_q(nT_s) - x(nT_s) = -\frac{3\Delta}{2} - (-2\Delta) = \frac{\Delta}{2}$$

Hence, the maximum quantization error is given as $\pm\Delta/2$. So, the maximum quantization error is

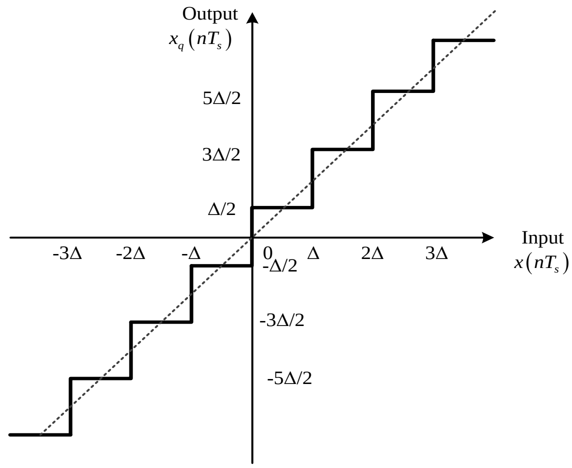
$$|N_q| = \left|\frac{\Delta}{2}\right| \tag{6.23}$$

The plot of quantizer error w.r.t. sampled input is shown in Fig. 6.17(b).

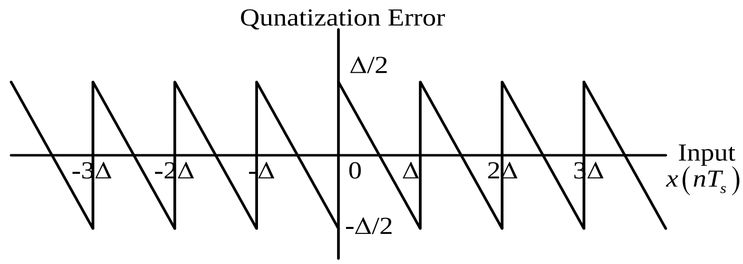
6.7.2.2 Non-uniform Quantizer

In this type of quantizer, step size, i.e. quantization levels, are unequal and varies in accordance with the input signal. The non-uniform quantizer is shown in Fig. 6.18.

In non-uniform quantization, the step size is not fixed. It varies according to certain laws or as per input signal amplitude.



(a)



(b)

Fig. 6.17 (a) Transfer characteristics of midrise type quantizer (b) Quantization error vs. input

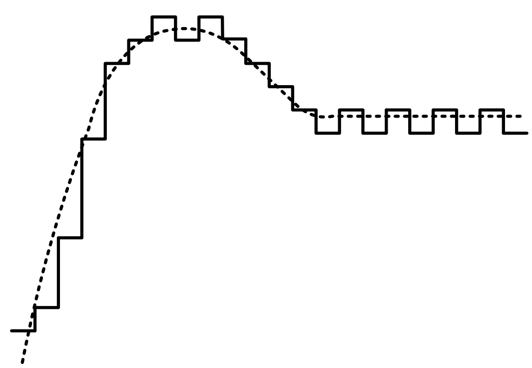


Fig. 6.18 Non-uniform quantizer

6.7.3 Basic Elements of PCM

A PCM system consists of the transmitter and receiver parts. The block diagram of the PCM system is shown in Fig. 6.19. The transmitter section of the PCM system consists of the following elements:

1. **Sampler:** It is an electronic circuit device that is used to convert the instantaneous value of analog signal into a corresponding discrete signal at a different instant. The sampling frequency must be greater than or equal to the Nyquist frequency for the reconstruction of the original signal.

2. **Quantizer:** The sampled data obtained by the sampler are further rounded off to a near stabilized point by the quantizer.
3. **Encoder:** Encoding is the last process of digitization of analog signals. The quantized signal after quantizer is converted into corresponding binary codes by the encoder.
4. **Low Pass Filter (LPF):** To avoid the aliasing effect, it is necessary to eliminate the high-frequency component, which is greater than the highest frequency component of the message signal. Therefore, the LPF is used at the input side of the PCM system.
5. **Regenerative Repeater:** To increase the signal strength and to compensate the signal loss, the regenerative repeater is used at the output of the channel.

At the PCM receiver side, this binary encoded signal is further converted into the original analog message signal. The PCM receiver system consists of a decoder and reconstruction circuit.

6.7.4 Noise in PCM Systems

There are two major sources of noise in a PCM system

(i) Transmission noise

(ii) Quantization noise

The decision boundary at the receiver for the pulse detection is selected as half of the pulse size. Therefore, the transmission noise has no effect so long as the peak noise amplitude is less than the selected decision boundary.

Instead of the actual value of the signal, the quantized value of the signal is transmitted by the PCM transmitter. Therefore, the difference between the actual value and quantized value comes into the form of an error, which is called quantization noise, as shown in Fig. 6.20. The quantization error is given by

$$q(t) = m(t) - m_q(t) \tag{6.24}$$

where, $m(t)$ and $m_q(t)$ are the actual and the quantized values of the message signal.

The power or mean square value of the quantization noise is given as

$$N_q = \frac{\Delta^2}{12} \tag{6.25}$$

Substituting the value of Δ from Eq. (6.20) into Eq. (6.25)

$$N_q = \frac{(2m_p / L)^2}{12} = \frac{m_p^2}{3L^2} \tag{6.26}$$

As we know, the power of the baseband signal $m(t)$ is given by

$$S_o = \overline{m^2(t)} \tag{6.27}$$

So, signal to noise (quantization noise) ratio (SNR) is given by

$$\frac{S_o}{N_o} = \frac{S_o}{N_q} = \frac{\overline{m^2(t)}}{(m_p^2/3L^2)} = 3L^2\,\frac{\overline{m^2(t)}}{m_p^2} \tag{6.28}$$

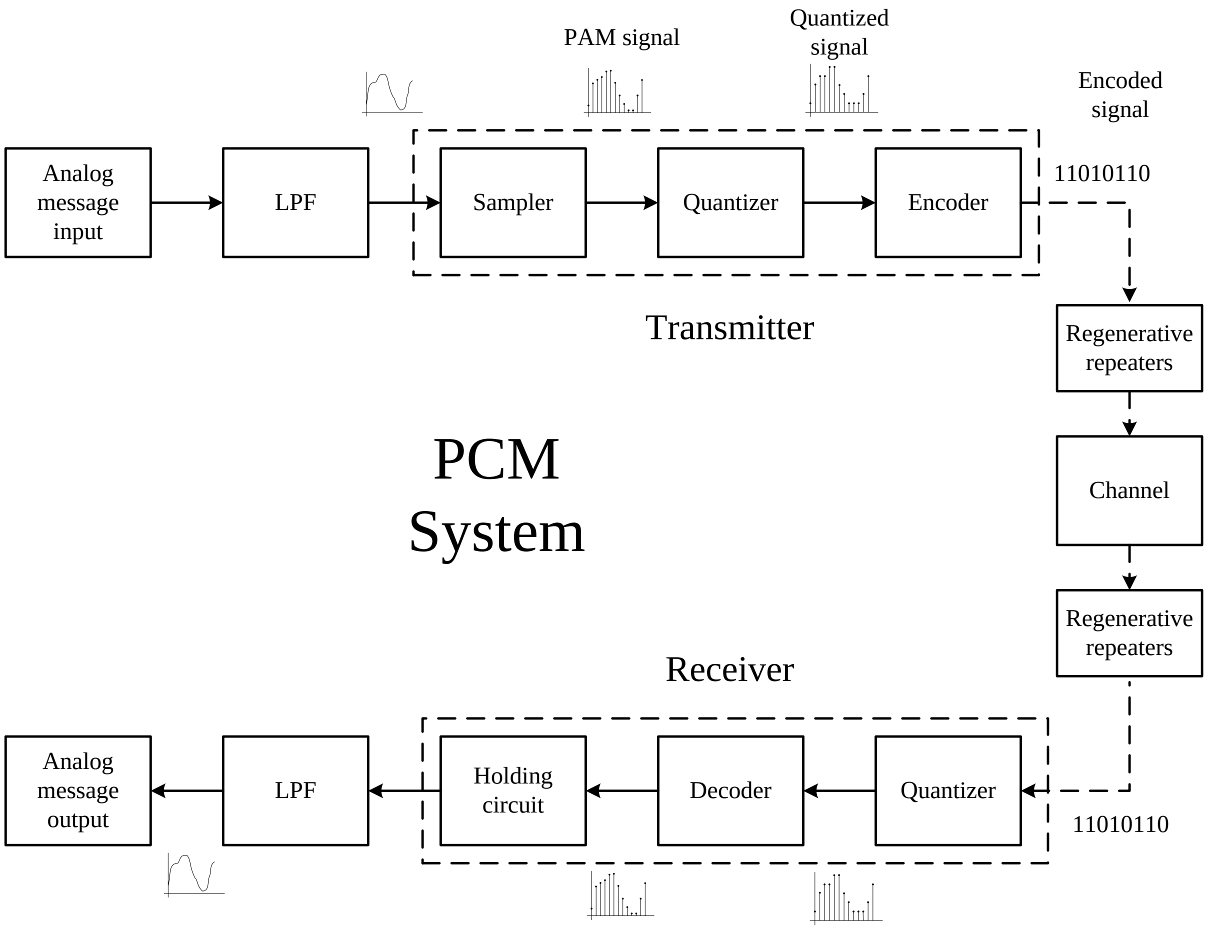


Fig. 6.19 Block diagram of a PCM system

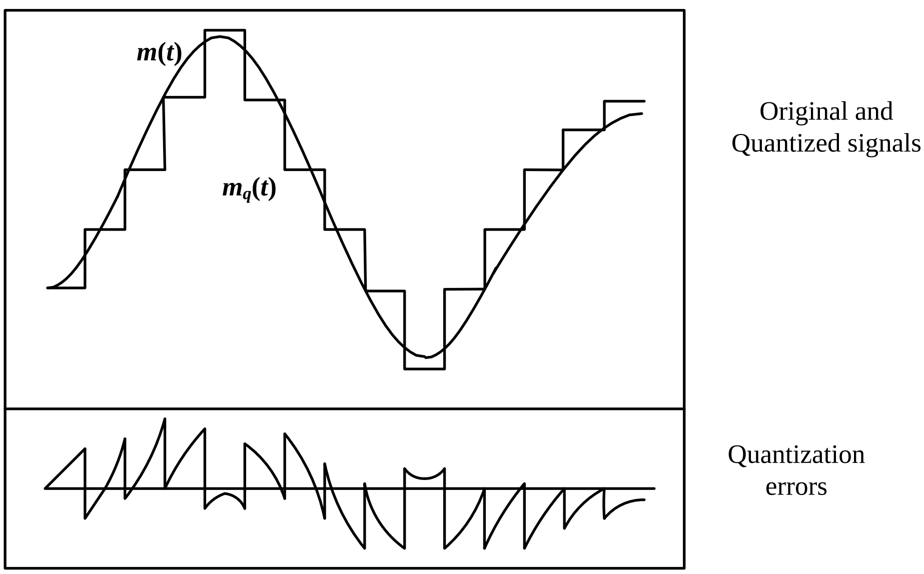


Fig. 6.20 Quantization error

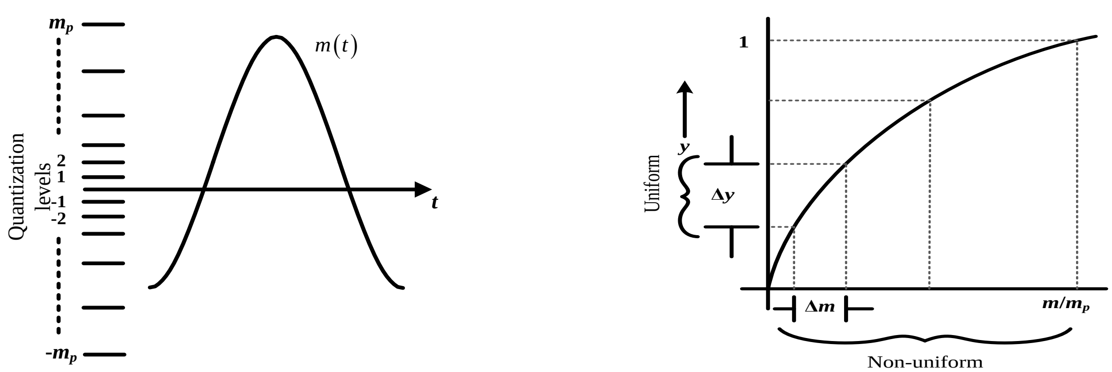
The following few points are concluded from the Eq. (6.28)

(i) The output SNR is linearly dependent upon the baseband signal power $\overline{m^2(t)}$.

(ii) It is clear from Eq. (6.25) that the noise power is proportional to the square of the step size. Therefore, the quality of the signal deteriorates for smaller amplitude signals compared to larger amplitude signals for the same step size.

(iii) This problem can be resolved by using variable step sizes for quantization. The smaller step size for smaller amplitude (non-uniform quantizing) makes the SNR effective (Fig. 6.21a).

The above objective of the variable step size for the quantization is obtained by the compander, which performs companding- a combination of **compressing** and **expanding** both operations. The compander performs the function of expanding/compressing, i.e. mapping of input signal increments Δm into corresponding larger/smaller increments Δy for small/large input signals. This non-linear characteristic of compander is shown in Fig. 6.21(b). This approach is used to reduce the effects of noise and crosstalk.



(a) (b)

Fig. 6.21 (a) Nonuniform quantization (b) Input-output characteristics of compander

Following two compression laws are adopted to perform the companding operation:

(i) ***μ*-Law:** For the positive amplitude, μ-law is given as

$$y = \frac{1}{\ln(1+\mu)} \ln\left(1 + \frac{\mu m}{m_p}\right) \qquad 0 \le \frac{m}{m_p} \le 1 \tag{6.29}$$

The μ-Law characteristics for the normalized amplitude is shown in Fig. 6.22 (a). The characteristic curve is linear for $\mu = 0$ and therefore, uniform quantization and no compression is achieved for $\mu = 0$. The μ-law companding is used for music and speech signals. The μ-law companding is used in North America.

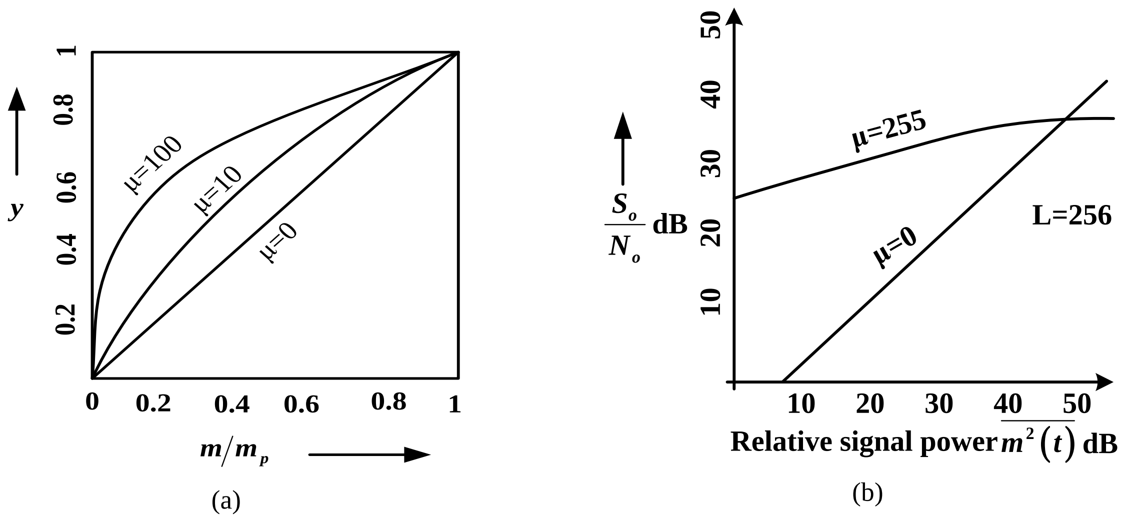


Fig. 6.22 (a) μ-Law characteristics (b) Ratio of signal to quantization noise ratio in PCM system with and without compression (b)

(ii) ***A*-Law:** For the positive amplitude, A-law is given as

$$y = \begin{cases} \dfrac{A}{1+\ln(A)}\left(\dfrac{m}{m_p}\right) & 0 \le \dfrac{m}{m_p} \le \dfrac{1}{A} \\ \dfrac{1}{1+\ln(A)}\left(1+\ln\dfrac{Am}{m_p}\right) & \dfrac{1}{A} \le \dfrac{m}{m_p} \le 1 \end{cases} \tag{6.30}$$

The *A*-Law characteristics for the normalized amplitude is shown in Fig. 6.23.

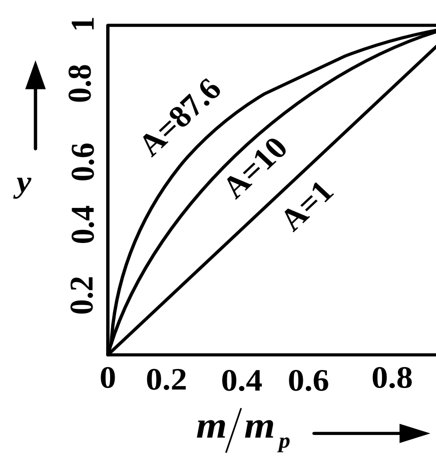


Fig. 6.23 A-Law characteristics

The *A*-law contains a non-zero value as it has a mid-rise at the origin. The *A*-law characteristic curve is linear for *A* = 1 and therefore, uniform quantization and no compression is achieved. The *A*-law companding is normally used in the PCM telephone system. The *A*-law companding is used in Europe and the rest of the world.

6.7.5 Transmission Bandwidth of a PCM System

Let the length of PCM code is N, i.e. N bits (binary digits) are required to represent a level of PCM code. If the total number of quantization levels is L, then

$$L = 2^N$$
$$\Rightarrow N = \log_2 L \tag{6.31}$$

The maximum frequency component present in the input signal is f_m. So, the minimum sampling frequency or Nyquist frequency is given as

$$f_s = 2 f_m \tag{6.32}$$

The sampling frequency represents samples per second, whereas the signaling rate or bit rate is given as bits per second. Therefore, the signaling rate of the PCM system is given as

$$\text{Number of bits/sec} = \frac{\text{bits}}{\text{sample}} \times \frac{\text{sample}}{\text{sec}}$$

$$r = N \times f_s \tag{6.33}$$

The transmission bandwidth of the PCM system is given as

$$(\text{BW})_{\text{PCM}} \geq \frac{1}{2} r$$

$$\geq \frac{1}{2} N f_s$$

$$\geq N f_m \qquad (\because f_s = 2 f_m) \tag{6.34}$$

Therefore, the minimum transmission bandwidth of the PCM system is

$$(\text{BW}_{\text{PCM}})_{\min} = N f_m \tag{6.35}$$

6.7.6 Output SNR of The PCM System

The output SNR of the PCM system is given by

$$\frac{S_o}{N_o} = k(2)^{2N} \tag{6.36}$$

where,

$$k = \begin{cases} 3\dfrac{\overline{m^2(t)}}{m_p^2} & \text{for uncompressed signal} \\ \dfrac{3}{\left[\ln(1+\mu)\right]^2} & \text{for compressed signal} \end{cases} \tag{6.37}$$

Since the transmission bandwidth of the PCM system is given as

$$(\text{BW})_{\text{PCM}} = N f_m \tag{6.38}$$

So, $$\frac{S_o}{N_o} = k\left(2\right)^{2\text{BW}_{\text{PCM}}/f_m} \quad (6.39)$$

$$\Rightarrow \left(\frac{S_o}{N_o}\right)_{dB} = 10\log_{10}\left(k\left(2\right)^{2N}\right) \quad (6.40)$$

$$\left(\frac{S_o}{N_o}\right)_{dB} = 10\log_{10} k + 10 \times 2N \times \log_{10} 2$$

$$= \left(P + 6N\right)_{dB} \quad (6.41a)$$

Therefore, increment by 1 bit in codeword results in the quadruple of SNR or increment in SNR by 6 dB.

For uncompressed signal, if $\overline{m^2\left(t\right)} = m_p^2$

$$10\log_{10} k = 10\log_{10}\left(3\frac{\overline{m^2\left(t\right)}}{m_p^2}\right) = 10\log_{10} 3 = 4.77$$

So, $$\left(\frac{S_o}{N_o}\right)_{dB} = \left(4.77 + 6N\right)_{dB} \quad (6.41b)$$

Points to Remember:

1. *Like PAM, PWM and PPM, PCM is also a pulse modulation method. The main difference is that PCM is a digital modulation method, whereas others are analog modulation methods*.
2. *Since all the messages are transmitted in coded form, therefore, a PCM encoder in the transmitter and a PCM decoder in the receiver are required*.
3. *PCM is not a modulation as none of the properties of the carrier signal is changing with the instantaneous value of modulating signal*.
4. *Sampling, quantizing and encoding are the main operations of the PCM system.*
5. *The minimum transmission bandwidth of the PCM system is* Nf_m .
6. *Increment by 1 bit in codeword results into the quadruple of SNR or increment in SNR by 6 dB.*

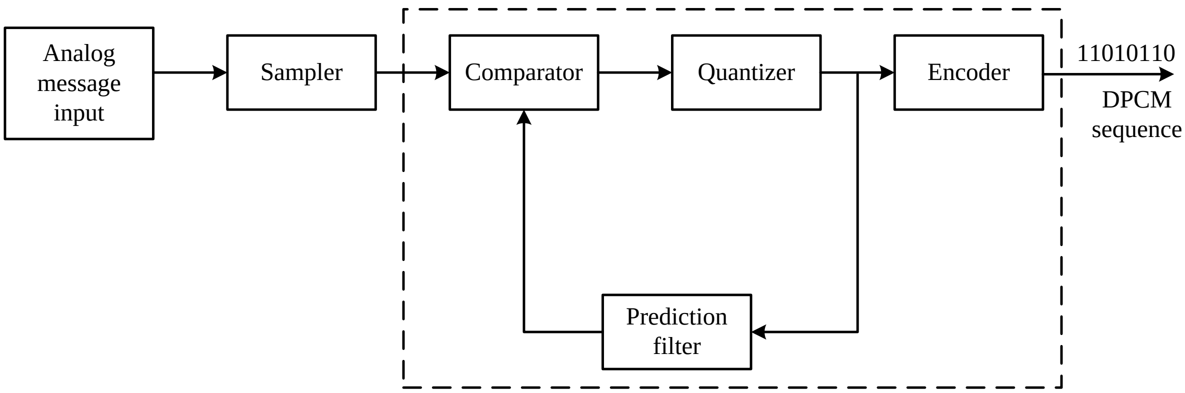
6.8 Differential Pulse-Code Modulation (DPCM)

Instead of codeword representation for each sample as in PCM, the DPCM represents the difference between each sample. Therefore, DPCM is called differential pulse code modulation. It was invented by C. Chapin at Bell Labs in 1950.

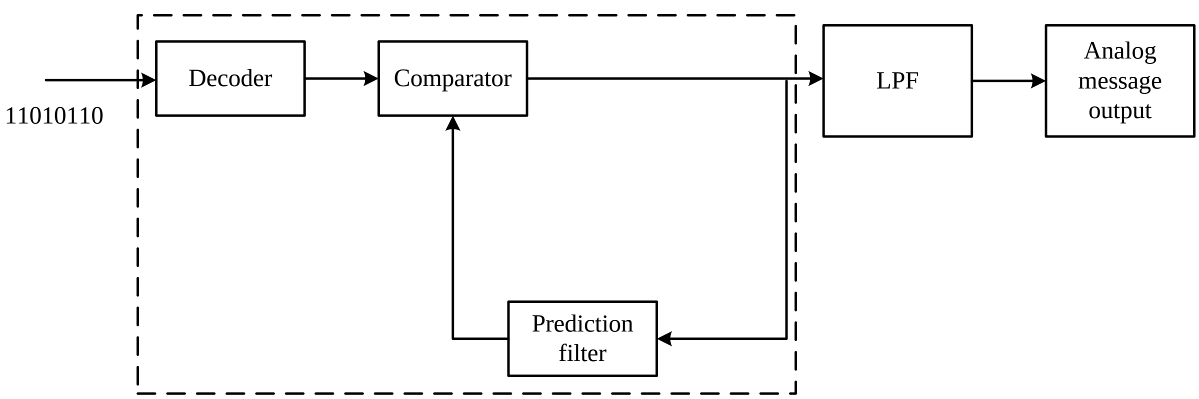
DPCM works similar to the PCM, but it uses some functionality for the prediction of the next samples. The following two methods are used to predict the next sample

1. **Method 1:** The analog signal is sampled and quantized. The output is taken as the difference between the two consecutive samples.
2. **Method 2:** Unlike option 1, the output is taken as a difference between the sampled signal and the signal obtained from a local model of the decoder process. This difference signal is quantized, which provide a better option to incorporate the controlled loss in encoding.

Since the difference signal is always less in the amplitude than the actual sample, therefore, less quantization level is needed for the difference signal. Hence, DPCM has reduced bits/sec or reduced bit rate. The basic block diagram of the DPCM system with DPCM transmitter and DPCM receiver is shown in Fig. 6.24, respectively.



(a) Transmitter



(b) Receiver

Fig. 6.24 Block diagram of differential pulse-code modulation (DPCM) system

6.8.1 DPCM Transmitter

The main functional block diagram of the DPCM circuit consists of the quantizer and predictor circuit. The other elements of the DPCM transmitter are comparator or summer circuits and one encoder circuit.

The sampled data at the input of the first comparator is subtracted from the output of the predictor circuit. The output of the predictor circuit is nothing but the previously sampled data. Therefore, the difference between the present and the previous data is quantized by the quantizer and further, this difference is encoded by the encoder for transmission. The quantized difference data is added to previous data to obtain the next data for the first comparator input.

6.8.2 DPCM Receiver

The DPCM receiver circuit consists of a decoder, a comparator or summer and a predictor circuit, as shown in Fig. 6.24. The encoded signal transmitted by the transmitter is decoded at the receiver end. This decoded signal is the difference quantized signal which is added into the output of the predictor output.

Limitations

- More complicated transmitter and receiver circuits than those in the PCM system.

6.9 Delta Modulation

For the reconstruction of the original signal, the sampling rate should be equal to or higher than the Nyquist rate. The delta modulation is a type of modulation where the sampling rate is very high, whereas the step size after quantization is very small, like Δ. Since the step size remains fixed in delta modulation, it is also called linear delta modulation.

Delta modulation is like DPCM with a 1-bit data stream as transmitted data. The block diagram of delta modulation is shown in Fig. 6.25. The pulse generator produces the positive pulse $P_i(t)$. The modulator multiplies the outputs of the pulse generator $P_i(t)$ and difference amplifier $\Delta(t)$ where,

$$\Delta(t) = m(t) - m_q(t) = \begin{cases} \text{positive} & if \quad m(t) > m_q(t) \\ \text{negative} & if \quad m(t) < m_q(t) \end{cases} \tag{6.42}$$

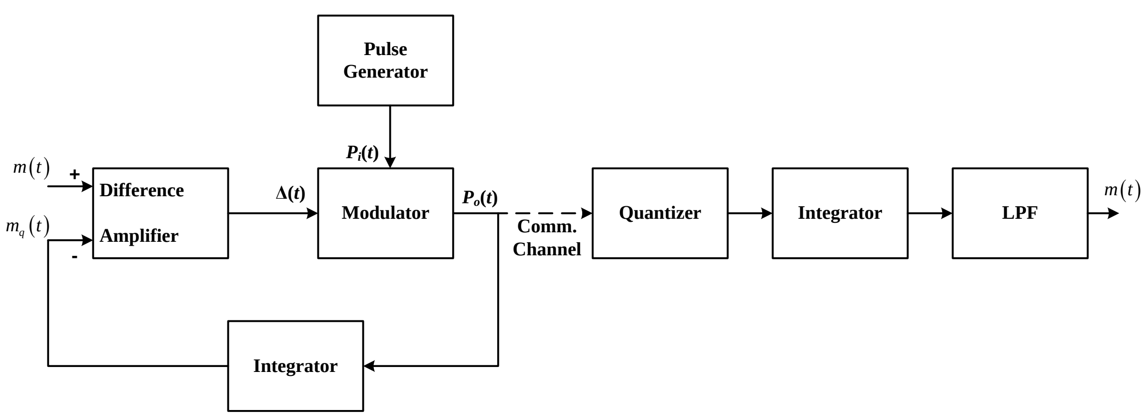


Fig. 6.25 Block diagram of a delta modulation system

The initial value is assumed arbitrary. $P_o(t)$ is either positive or negative depending upon whether $\Delta(t)$ is positive or negative. Therefore, the analog signal is approximated with a set of segments. Each segment is compared with the original signal to determine the increment or decrement in the relative amplitude of the next segment.

The approximation approach of segmented signal $m_q(t)$ towards the original signal $m(t)$ through the delta modulation is shown in Fig. 6.26.

Delta modulation is used in voice transmission where quality is not of primary importance.

6.9.1 Advantages of Delta Modulation

1) Since delta modulation uses only one bit/sample for the transmission, therefore, the signaling rate as well as transmission bandwidth are quite low compare to that of the PCM.
2) Analog to digital converter is not required.

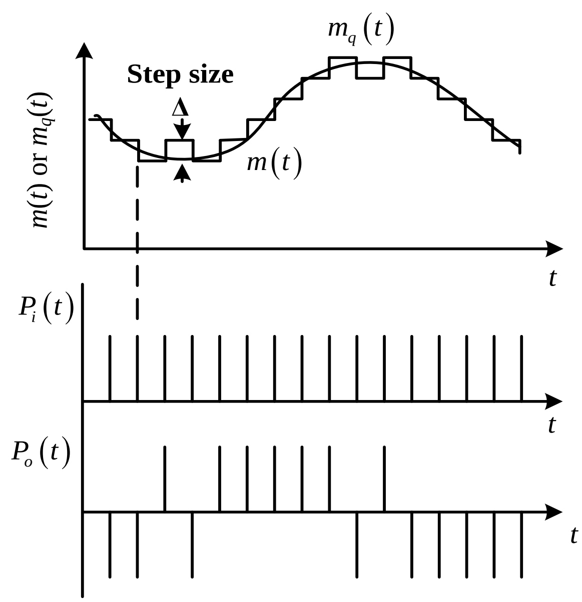


Fig. 6.26 Waveform in delta modulation

6.9.2 Limitations of Delta Modulation

Following two types of drawbacks limit the use of delta modulation (Fig. 6.27)

1) Slope overload distortion
2) Granular or idle noise

6.9.2.1 Slope Overload Distortion

Slope overload distortion occurs when the rate of rising of the original signal $m(t)$ is so high that the segmented signal $m_q(t)$ cannot approximate it, i.e., the step size 'Δ' is too small to approximate the original analog signal as shown in Fig. 6.27. This large error/noise between original signal $m(t)$ and approximated signal $m_q(t)$ is called slope over distortion.

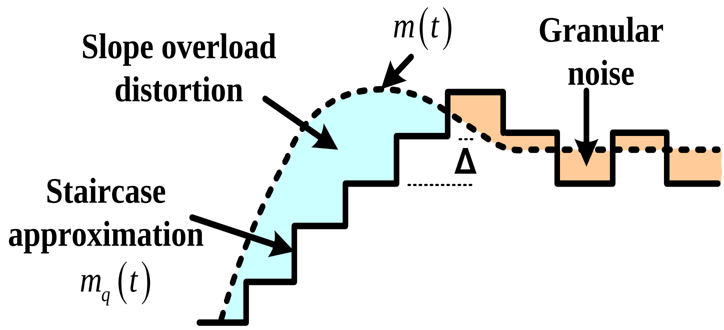


Fig. 6.27 Quantization errors (slope overload & granular noise) in delta modulation

If the sampling period and step size are T_s and Δ respectively, then the slope overload can be avoided if

$$\frac{\Delta}{T_s} \geq \left|\frac{dm(t)}{dt}\right|_{\max} \quad \text{(i.e. greater than the maximum slope of the signal } m(t)\text{)} \tag{6.43}$$

Let the signal $m(t)$ is sinusoidal i.e. $m(t) = A_m \cos(2\pi f_m t)$
The slope of the signal is

$$\left|\frac{dm(t)}{dt}\right| = A_m 2\pi f_m \sin 2\pi f_m t$$

$$\Rightarrow \left|\frac{dm(t)}{dt}\right|_{\max} = 2\pi A_m f_m$$

To avoid the slope overload

$$\left|\frac{dm(t)}{dt}\right|_{\max} \leq \frac{\Delta}{T_s} \Rightarrow 2\pi A_m f_m \leq \frac{\Delta}{T_s}$$
$$\Rightarrow A_m \leq \frac{\Delta}{2\pi f_m T_s} \tag{6.44}$$

So, the maximum amplitude and maximum allowable power of the signal are

$$(A_m)_{\max} = \frac{\Delta}{2\pi f_m T_s} \tag{6.45a}$$

$$(P_s)_{\max} = \frac{(A_m)_{\max}^2}{2} = \frac{(\Delta)^2}{8\pi^2 f_m^2 T_s^2} \tag{6.45b}$$

Another solution to reduce this type of error is the use of adaptive step size, i.e. large step size for high slope and small step size for low slope.

6.9.2.2 Granular or Idle Noise

Granular noise is also called idle noise. This type of error occurs if the step size is too large for the small variation of the original signal.

As shown in Fig. 6.27, the original signal is almost flat, but the approximated signal is an oscillating signal with ±Δ around the signal. This error/noise between original signal *m*(*t*) and approximated signal $m_q(t)$ is called **granular noise**. The only solution to this type of error is the small step size. The power or mean square value of the quantization or granular noise in delta modulation is given as

$$N_q = \frac{\Delta^2}{3} \tag{6.46}$$

6.10 Adaptive Delta Modulation

To reduce the quantization errors of the delta modulation, the step size kept adaptive, i.e. large step size for high slope and small step size for low slope. This process developed another type of modulation technique called adaptive delta modulation (ADM). Therefore, adaptive delta modulation is a modification of delta modulation.

The functional block diagram of the ADM modulator is shown in Fig. 6.28. Unlike the delta modulator, there is a level adjuster in the ADM modulator to control the gain of the integrator.

A comparative diagram of DM and ADM to approximate the analog signal is shown in Fig. 6.29. In this case, the rate of rising of the signal is very high. Since the step size of the delta modulation is fixed, therefore, the delta modulation is unable to track the signal effectively, whereas due to the adaptive nature of step size, ADM follows the signal effectively.

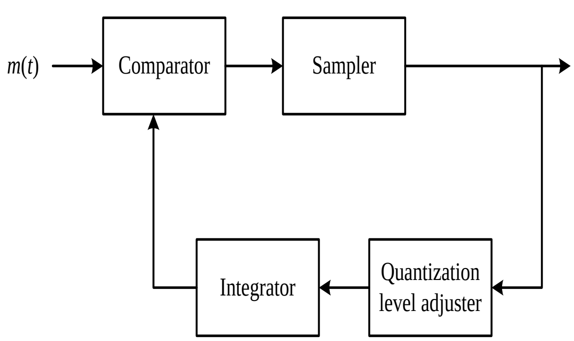


Fig. 6.28 Block diagram of ADM

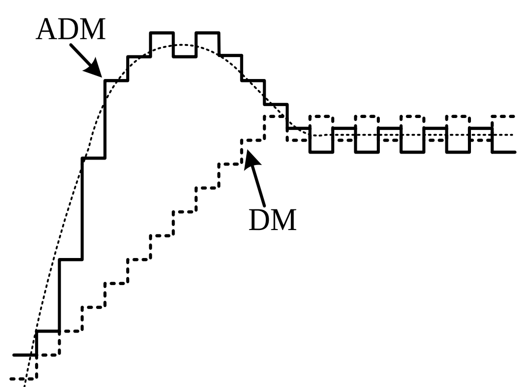


Fig. 6.29 Comparison of ADM and DM

The comparative analysis of PCM vs. DM and DM vs. DPCM are presented in Table 6.2 and Table 6.3, respectively.

Table 6.2 Comparison between pulse code modulation (PCM) and delta modulation (DM*)*

S. N.	Parameters	PCM	DM
1.	**Full form**	Pulse Code Modulation.	Delta Modulation.
2.	**Feedback**	No feedback neither in transmitter nor receiver.	Feedback exists in the transmitter.
3.	**Bits/sample**	4, 8, or 16 bits are used.	One bit /sample.
4.	**Transmission bandwidth**	Highest	Lowest
5.	**S/N ratio**	Good	Poor
6.	**Complexity in implementation**	Complex	Simple
7.	**Cost**	Costly	Cheap
8.	**Applications**	Audio and video telephony.	Image and voice transmission

Table 6.3 Comparison between DM and DPCM

S. N.	Parameters	DM	DPCM
1.	**Full form**	Delta Modulation.	Differential pulse code modulation.
2.	**Feedback**	Feedback exists in the transmitter.	Feedback in both transmitter as well as receiver
3.	**Bits/sample**	One bit/sample.	More than one but less than PCM bits/sample
4.	**Transmission bandwidth**	Lowest	Less than PCM
5.	**S/N ratio**	Poor	Good
6.	**Step size/Levels**	Step size is fixed	The numbers of levels are fixed
7.	**Error/distortion**	Slope overload distortion and granular noise.	Slope overload distortion and quantization noise.
8.	**Efficiency**	Less efficient than DPCM	More efficient.
9.	**Applications**	Image and voice transmission.	Audio and video transmission

ADDITIONAL SOLVED EXAMPLES

SE6.4 The bandwidth of an input signal to the PCM is restricted to 4 kHz. The input signal varies in amplitude from -3.8V to +3.8V and has an average power of 30 mW. The required signal to noise ratio is given as 20 dB. The PCM modulator produces binary output. Assuming uniform quantization

(i) Find the number of bits required per sample.

(ii) Outputs of 30 such PCM coders are time-multiplexed. What would be the minimum required transmission bandwidth for this multiplexed signal? **(UPTU: 2006-07)**

Sol: The signal to noise ratio is

$$\left(\frac{S_o}{N_o}\right)_{dB} = 10\log_{10}\left(\frac{S_o}{N_o}\right)$$

$$20 = 10\log_{10}\left(\frac{S_o}{N_o}\right)$$

$$\Rightarrow \left(\frac{S_o}{N_o}\right) = 100$$

(i) The signal to noise (quantization noise) ratio (SNR) is given by

$$\frac{S_o}{N_o} = \frac{\overline{m^2(t)}}{\left(m_p^2/3L^2\right)} = 3L^2\frac{\overline{m^2(t)}}{m_p^2}$$

If there are N bits for PCM code, then the quantization level is given as

$$L = 2^N$$

So,

$$\frac{S_o}{N_o} = 3\times 2^{2N}\frac{\overline{m^2(t)}}{m_p^2}$$

Given, $\overline{m^2(t)} = 30$ mW, $m_p = 3.8$

Hence,

$$100 = 3\times 2^{2N}\frac{30\times 10^{-3}}{3.8^2}$$

$$\Rightarrow 2^{2N} = \frac{100\times 3.8^2}{3\times 30\times 10^{-3}} = 16\times 10^3$$

$$\Rightarrow 2N\log 2 = \log\left(16\times 10^3\right)$$

$$\Rightarrow 2N = \frac{4.204}{0.3010} = 13.97$$

$$\Rightarrow N = \frac{13.97}{2} = 6.99 \approx 7 \text{ bits}$$

(ii) The transmission bandwidth of 30 time-multiplexed PCM coders is $(n = 30)$

$$\text{BW} \geq nNf_m$$
$$\geq (30 \times 7 \times 4) \text{ kHz}$$
$$\geq 840 \text{ kHz}$$

Hence, the minimum required bandwidth is

$$\text{BW} = 840 \text{ kHz}$$

SE6.5 The bandwidth of TV video plus audio signal is 4.5 MHz. If this signal is converted into a PCM bitstream with 1024 quantization levels, determine the number of bits/sec of the resulting signal. Assume the signal is sampled at a rate 20% above the Nyquist rate.

Sol: Number of bits N to encode 1024 PCM quantization levels L is

$$L = 2^N$$
$$\Rightarrow N = \frac{\log L}{\log 2} = \frac{\log 1024}{\log 2} = 10$$

The Nyquist rate $f_{Nyquist} = 2 f_m = 2 \times 4.5 = 9$ MHz

Since the sampling rate is 20% above Nyquist rate, So,

$$f_s = 20\% \text{ above } f_{Nyquist} = 1.2 f_{Nyquist} = 1.2 \times 9 = 10.8 \text{ MHz}$$

So, number of bits/sec = $10.8 \times 10 = 108$ Mbps

SE6.6 Twenty-four channels, each band-limited to 3.4 kHz, are to be time-division multiplexed by using PCM. Calculate the bandwidth of the PCM system for 128 quantization levels and an 8 kHz sampling frequency.

Sol: Given data:

Number of channel n = 24

Length of code N is obtained as 2^N = 128

$$N = \log_2(128) = 7$$

Sampling frequency ($2f_m$) = 8 kHz

Therefore, bandwidth is given as BW = $[n(N+1)+1]2 f_m$ = [24(7+1) + 1]*8000

= 1.544 x 10^6 = 1.544 MHz

Approximated BW = $2f_m nN$ = 8000*24*7 = 1.299 MHz.

Note: If the same number of channels are frequency division multiplexed by using an SSB modulation, the required bandwidth, assuming 4 kHz per channel

BW = 24 x 4 kHz = 96 kHz

Therefore, bandwidth requirement is more in PCM.

SE6.7 One kilohertz (1kHz) signal $m(t)$ is sampled at 8 kHz with 12-bit encoding for PCM transmission.

(a) How many bits are transmitted per second in PCM? What is the bandwidth required in this case?

(b) Now switch to using DM with 8 kHz sampling. How many bits are transmitted per second using DM? What is the bandwidth required in using DM?

Sol: The signal frequency is f_m = 1 kHz and the sampling rate is 8 kHz.

(a) For PCM we have 8,000 samples per second and 12 bits per sample; which equals 8000 x 12 = 96,000 bits/second. The bandwidth is one-half of this, giving 48,000 Hz.

(b) Now for DM, we have 1 bit per sample at 8,000 samples per second. Thus, we have 8,000 bits per second and a bandwidth of 4,000 Hz.

SE6.8 The ramp signal $m(t) = at$ is applied to a delta modulator with sampling periods T_s and step size δ. Slope overload distortion would occur if

(a) $\delta < a$

(b) $\delta > a$

(c) $\delta < aT_s$

(d) $\delta > aT_s$

Sol: The slope overload distortion occurs if the slope of the signal at sampling instant $t = T_s$ is greater than step size, i.e.

$$aT_s > \delta$$
$$\Rightarrow \delta < aT_s$$

SE6.9 Determine the Nyquist rate of the signal $x(t) = 2\cos(2000\pi t)\cos(5000\pi t)$. **(IES: 2010)**

Sol: Since,

$$2\cos(A)\cos(B) = \cos(A+B) + \cos(A-B)$$

So,

$$\begin{aligned} x(t) &= 2\cos(2000\pi t)\cos(5000\pi t) \\ &= \cos(2000\pi t + 5000\pi t) + \cos(5000\pi t - 2000\pi t) \\ &= \cos(7000\pi t) + \cos(3000\pi t) \end{aligned}$$

The maximum frequency component present in the signal $x(t)$ is

$$f_{\max} = \frac{7000\pi}{2\pi} = 3500$$

Therefore, the Nyquist rate is given by

$$f_{nyquist} = 2 \times f_{\max} = 2 \times 3500 = 7000 = 7\text{kHz}$$

SE6.10 A television signal with a bandwidth of 4.2 MHz is transmitted using binary PCM. The number of quantization levels is 512. Calculate (a) Codeword length (b) Transmission bandwidth (c) Final bit rate (d) Output signal to quantization ratio.

Sol: Given data:

The bandwidth of the signal = 4.2 MHz

Quantization levels L = 512

(a) Codeword length $N = \log_2(L) = \log_2(512) = 9$

(b) Transmission bandwidth = $N \times f_m = 9 \times 4.2 = 37.8$ MHz

(c) Final bit rate $R_b = n \times f_s$

$$f_s = 2 \times f_m = 2 \times 4.2 = 8.4\text{MHz}$$

Bit rate $R_b = 9 \times 8.4 = 75.6$ Mbits/sec

(d) $$(\text{SNR})_Q = \frac{A^2/2}{(2A)^2/\left(12 \times (512)^2\right)}$$

$$= \frac{12 \times (512)^2}{8} = 55.94\text{dB}$$

SE6.11 Determine the Nyquist rate and the Nyquist interval for the signal

$x(t) = -10\sin(20\pi t)\cos(400\pi t)$.

Sol: The given signal

$$x(t) = -10\sin(20\pi t)\cos(400\pi t)$$
$$= -5\left[\sin(400\pi + 20\pi)t - \sin(400\pi - 20\pi)t\right]$$
$$= -5\left[\sin(420\pi)t - \sin(380\pi)t\right]$$

The maximum frequency component of the message signal is

$$f_{\max} = \frac{420\pi}{2\pi} = 210 \text{ Hz}$$

Therefore, the Nyquist rate is

$$f_{nyquist} = 2f_{\max} = 2 \times 210 \text{ Hz=420 Hz}$$

The Nyquist interval is

$$T_{nyquist} = \frac{1}{f_{nyquist}} = \frac{1}{420} \text{ sec}$$

SE6.12 Suppose the maximum frequency in a band-limited signal $x(t)$ is 5 kHz. Then determine the minimum sampling frequency to recover the signal $x(t)\cos 2000\pi \text{t}$ from its sampled signal.

Sol: Let the Fourier Transform of the signal $x(t)$ is $X(\omega)$, i.e.

$$x(t) \xleftrightarrow{\text{FT}} X(\omega)$$

The Fourier Transform of $\cos\omega_c t$ is

$$\cos\omega_c t \xleftrightarrow{FT} \pi\left[\delta(\omega+\omega_c)+\delta(\omega-\omega_c)\right]$$

According to the multiplication property of the Fourier Transform

$$x(t)y(t) \xleftrightarrow{FT} \frac{1}{2\pi}\left[X(\omega)Y(\omega)\right]$$

So, $$x(t)\cos\omega_c t \xleftrightarrow{FT} \frac{1}{2\pi}X(\omega)\times\pi\left[\delta(\omega+\omega_c)+\delta(\omega-\omega_c)\right]$$

$$\xleftrightarrow{FT} \frac{1}{2}\left[X(\omega)\delta(\omega+\omega_c)+X(\omega)\delta(\omega-\omega_c)\right]$$

Since $x(t)$ is band-limited to ω_m. So,

$$\omega_m\cos\omega_c t \xleftrightarrow{FT} \frac{1}{2}\left[X(\omega_m+\omega_c)+X(\omega_m-\omega_c)\right]$$

Therefore, the maximum frequency component in the signal is

$$f_{\max} = f_m + f_c = \frac{1}{2\pi}(\omega_m+\omega_c) = 5+1 = 6 \text{ kHz}$$

Hence, the Nyquist rate is

$$f_{nyquist} = 2f_{\max} = 2\times 6 \text{ kHz=12 kHz}$$

SE6.13 A PCM system uses a uniform quantizer followed by a 7-bit binary encoder. The bit rate of the system is equal to 50 x 10^6 bits/sec. What is the maximum message signal bandwidth for which the system operates satisfactorily? **(UPTU: 2005-06)**

Sol: Given data

Bit rate	$r = 50\times10^6$ bps
Number of bits	$N = 7$

(i) Let the message signal bandwidth is f_m. The sampling frequency is

$$f_s = 2f_m$$

The bit rate is

$$r \geq Nf_s$$
$$\geq 2Nf_m$$

Hence, message bandwidth is

$$f_m \leq \frac{r}{2N}$$

So, maximum message bandwidth is

$$(f_m)_{\max} = \frac{r}{2N} = \frac{50\times10^6}{2\times7} = 3.57\times10^6 = 3.57 \text{ MHz}$$

SE6.14 A sinusoidal signal of 2 kHz frequency is applied to a delta modulator. The sampling rate and step-size Δ of the delta modulator are 20,000 samples per second and 0.1V, respectively. To prevent slope overload, the maximum amplitude of the sinusoidal signal (in Volts) is **(GATE: 2015)**

(a) $\frac{1}{2\pi}$ (c) $\frac{2}{\pi}$

(b) $\frac{1}{\pi}$ (d) π

Sol: The slope overload distortion occurs if the slope of the signal at sampling instant $t = T_s$ is greater than step size, i.e. to avoid the slope overload

$$aT_s < \Delta \Rightarrow \frac{a}{f_s} < \Delta \Rightarrow a < \Delta * f_s$$
$$a < 0.1 * 20000$$
$$a < 2000$$

Therefore, the maximum slope is 2000. Let the signal is sinusoidal with amplitude A_m i.e.

$$m(t) = A_m \sin 2\pi f_m t$$

The slope of the signal is

$$\frac{dm(t)}{dt} = A_m 2\pi f_m \sin 2\pi f_m t$$

To avoid the slope overload

$$a = A_m 2\pi f_m < 2000$$
$$\Rightarrow A_m < \frac{2000}{2\pi f_m}$$
$$\Rightarrow A_m < \frac{2000}{2\pi \times 2 \times 10^3}$$
$$\Rightarrow A_m < \frac{1}{2\pi}$$

Hence, the maximum amplitude of the signal is

$$(A_m)_{\max} = \frac{1}{2\pi}$$

Hence, option (a) is correct.

SE6.15 The Nyquist sampling rate for the signal $s(t) = \frac{\sin(500\pi t)}{\pi t} \times \frac{\sin(700\pi t)}{\pi t}$ is given by

(a) 400 Hz (c) 1200 Hz

(b) 600 Hz (d) 1400 Hz

Sol: Since,

$$2\sin(A)\sin(B) = \cos(A-B) - \cos(A+B)$$

So,

$$s(t) = \frac{\sin(500\pi t)}{\pi t} \times \frac{\sin(700\pi t)}{\pi t}$$

$$= \frac{1}{2(\pi t)^2}\left(2\sin(500\pi t)\sin(700\pi t)\right)$$

$$= \frac{1}{2(\pi t)^2}\left[\cos(700\pi t - 500\pi t) - \cos(700\pi t + 500\pi t)\right]$$

$$= \frac{1}{2(\pi t)^2}\left[\cos(200\pi t) - \cos(1200\pi t)\right]$$

The maximum frequency component present in the signal $s(t)$ is

$$f_{\max} = \frac{1200\pi}{2\pi} = 600$$

Therefore, the Nyquist rate is given by

$$f_{nyquist} = 2 \times f_{\max} = 2 \times 600 = 1200 \text{ Hz}$$

Hence, option (c) is correct.

SE6.16 The amplitude of a random signal is uniformly distributed between -5V and 5V. If the positive values of the signal are uniformly quantized with a step size of 0.05 V, and the negative values are uniformly quantized with a step size of 0.1 V, the resulting signal to quantization noise ratio is approximately **(GATE: 2009)**

(a) 46 dB (c) 42 dB

(b) 43.8 dB (d) 40 dB

Sol: For positive value, total steps are

$$L_1 = \frac{5}{0.05} = 100$$

Therefore, the number of required bits for PCM encoding is

$$2^{N_1} = L_1 = 100$$
$$\Rightarrow N_1 \approx 7$$

The signal to quantization noise ratio for the positive side is

$$\left(\frac{S_o}{N_o}\right)_{dB} = \left(10\log_{10} k + 6N_1\right)_{dB}$$

$$10\log_{10} k = 10\log_{10}\left(3\frac{\overline{m^2(t)}}{m_p^2}\right) = 10\log_{10}\left(\frac{3*5^2/2}{5^2}\right) = 10\log_{10} 1.5 = 1.76$$

So, $\frac{S_o}{N_o} = 1.76 + 6N_1 = 1.76 + 6 \times 7 = 43.76$ dB

Similarly, for negative value, total steps are

$$L_2 = \frac{5}{0.1} = 50$$

Therefore, the number of required bits for PCM encoding is

$$2^{N_2} = L_2 = 50 \Rightarrow N_1 \approx 6$$

The signal to quantization noise ratio for the positive side is

$$\frac{S_o}{N_o} = 1.76 + 6N_2 = 1.76 + 6 \times 6 = 37.76 \text{ dB}$$

The best signal to quantization noise ratio is 43.76 dB.

Hence, option (b) is correct.

SE6.17 A sinusoidal signal of amplitude A is quantized by a uniform quantizer. Assume that the signal utilizes all the representation levels of the quantizer. If the signal to quantization noise ratio is 31.8 dB, the number of levels in the quantizer is ……………... **(GATE: 2015)**

Sol: The signal to noise ratio is

$$\frac{S_o}{N_o} = 1.76 + 6N$$

$$31.8 = 1.76 + 6N \Rightarrow N = \frac{31.8 - 1.76}{6} = 5$$

So, the number of levels is

$$L = 2^N = 2^5 = 32$$

SE6.18 The amplitude of a random signal is uniformly distributed between -5V and 5V. If the signal to quantization noise ratio required in uniformly quantizing the signal is 43.5dB, the step size of the quantization is approximately

(a) 0.0333 V
(b) 0.05 V
(c) 0.0667 V
(d) 0.10 V

Sol: The signal to noise ratio is

$$\frac{S_o}{N_o} = 1.76 + 6N$$

$$43.5 = 1.76 + 6N$$

$$N = \frac{43.5 - 1.76}{6} = 6.96 \approx 7$$

So, the number of levels is $L = 2^N = 2^7 = 128$

Approximated step size

$$\Delta = \frac{2m_p}{L} = \frac{10}{128} = 0.078125 \approx 0.0667 \text{ V}$$

Hence, option (c) is correct.

PROBLEMS

P6.1 State and prove the sampling theorem.

P6.2 Explain how to generate PAM signals for various types of sampling techniques.

P6.3 Explain the scheme to generate PWM and PPM.

P6.4 Explain the term discretization in the time and the amplitude.

P6.5 What is pulse code modulation (PCM)? Explain with a suitable diagram?

P6.6 Derive the expression for the signal to quantization noise ratio of PCM system that employs linear quantization technique. Assume that input to the PCM system is a sinusoidal signal.

P6.7 Describe DM and ADM with a proper block diagram.

P6.8 What is the use of quantization and companding in PCM systems?

P6.9 Discuss in brief PCM and DPCM.

P6.10 What is delta modulation? Discuss the errors in the Delta modulation technique.

P6.11 Explain the disadvantages of delta modulation. How can it be overcome?

OR

What are the slope overload distortion and the granular noise in delta modulation and how it is removed in ADM?

P6.12 Explain the use of prediction in DPCM.

NUMERICAL PROBLEMS

P6.13 Find the Nyquist rate and Nyquist interval of $\sin(2\pi t)$.

P6.14 Find the Nyquist rate and Nyquist interval for the signal $x(t) = \frac{\sin 500\pi t}{\pi t}$.

P6.15 Find the Nyquist rate and Nyquist interval for the signal $x(t) = \left[\frac{\sin 500\pi t}{\pi t}\right]^2$.

P6.16 The sampling frequency of a signal is F_s = 2000 samples per second. Find its Nyquist interval.

P6.17 The spectral range of a bandpass signal extends from 10.0 MHz to 10.04 MHz. Find the minimum sampling rate.

P6.18 Determine the Nyquist rate of the signal $x(t) = 1 + \cos(2000\pi t) + \sin(4000\pi t)$.

P6.19 Find the Nyquist rate and Nyquist interval for the signal $x(t) = -10\sin 40\pi t \cos 300\pi t$.

P6.20 We are given a continuous-time signal $x(t)$ with a Nyquist rate ω_o. Find the Nyquist rate for the continuous-time signal $y(t) = x(t)\cos\omega_0 t$.

P6.21 We are given a continuous-time signal $x(t)$ with a Nyquist rate ω_o. Find the Nyquist rate for the continuous-time signal $x^2(t)$.

P6.22 A signal having bandwidth equal to 3.5 kHz is sampled, quantized and coded by a PCM system. The coded signal is then transmitted over a transmission channel supporting a transmission rate of 50 kbits/sec. Determine the maximum signal to noise ratio that can be obtained by this system. The input signal has a peak-to-peak value of 4V and an RMS value of 0.2V.

P6.23 The bandwidth of signal input to the PCM is restricted to 4 kHz. The input varies from -3.8V to +3.8V and has an average power of 30 mW. The required signal to noise ratio is 20 dB. The modulator produces binary output. Assume uniform quantization.

(i) Calculate the number of bits required per sample.

(ii) Outputs of 30 such PCM coders are time-multiplexed. What is the minimum required transmission bandwidth for the multiplexed signal?

P6.24 A PCM system uses a uniform quantizer followed by a 7-bit binary encoder. The bit rate of the system is equal to 50×10^6 bits/sec.

(i) What is the maximum message bandwidth for which the system operates satisfactorily?

(ii) Determine the output signal to quantization noise ratio when a full load sinusoidal modulating wave of frequency 1MHz is applied to the input.

P6.25 A message signal band-limited to 5 kHz is sampled at the minimum rate as dictated by the sampling theorem. The samples are quantized and encoded into six binary bits. Calculate the bit transmission rate and the maximum signal to quantization noise ratio.

P6.26 Twenty-four voice signals are sampled uniformly and then time division multiplexed. The highest frequency component for each voice signal is 3.4 kHz.

(i) If the signals are pulse amplitude modulated using Nyquist rate sampling, what is the minimum channel bandwidth?

(ii) If the signals are pulse code modulated with an 8-bit encoder, what is the sampling rate? The bit rate of the system is 1.5 x 10^6 bits/sec.

P6.27 The peak-to-peak input to an 8-bit PCM coder is 2 volts. Calculate the signal power to quantization noise power ratio (in dB) for input of $0.5\cos(\omega_m t)$.

MULTIPLE-CHOICE QUESTIONS

MCQ6.1 Which FSK has no phase discontinuity?

(a) Uniform FSK
(b) Discrete FSK
(c) Continuous FSK
(d) None of the above

MCQ6.2 The digital modulation scheme in which the step size is not fixed is

(a) DM (c) DPCM
(b) ADM (d) PCM

MCQ6.3 Granular noise occurs when

(a) Step size is too small
(b) Step size is too large
(c) There is interference from the adjacent channel
(d) Bandwidth is too large

MCQ6.4 DPCM suffers from

(a) Slope overload distortion
(b) Quantization noise
(c) Both (a) and (b)
(d) None of the above

MCQ6.5 In delta modulation, the bit rate is

(a) *N* times the sampling frequency
(b) *N* times the modulating frequency
(c) *N* times the Nyquist criteria
(d) None of the above

MCQ6.6 In digital transmission, the modulation technique that requires minimum bandwidth is

(a) PCM (c) DM
(b) PAM (d) PWM

MCQ6.7 In delta modulation

(a) One bit per sample is transmitted
(b) All the coded bits used for sampling are transmitted
(c) The step size is fixed
(d) Both (a) and (c) are correct

MCQ6.8 The characteristics of compressor in law companding are

(a) Continuous in nature
(b) Logarithmic in nature
(c) Linear in nature
(d) Discrete in nature

MCQ6.9 What are the three steps in generating PCM in the correct sequence

(a) Sampling, quantizing and encoding
(b) Encoding, sampling and quantizing
(c) Sampling, encoding and quantizing
(d) Quantizing, sampling and encoding

MCQ6.10 For a 10-bit PCM system, the signal to quantization noise ratio is 62 dB. If the number of bits increased by 2, then how would the signal quantization noise ratio change?

(a) Increase by 6 dB
(b) Decrease by 6 dB
(c) Increase by 12 dB
(d) Decrease by 12 dB

MCQ6.11 Quantization noise occurs in

(a) PAM (c) PCM

(b) PWM (d) PPM

MCQ6.12 The condition that should be satisfied in order to recover the original signal back?

(a) $\omega_m <= \omega_c <= \omega_s - \omega_m$

(b) $\omega_m > \omega_c > \omega_s - \omega_m$

(c) $\omega_m <= \omega_c >= \omega_s - \omega_m$

(d) $\omega_m > \omega_c < \omega_s$

MCQ6.13 The Nyquist rate and the Nyquist interval for the signal $x(t) = -10\sin(20\pi t)\cos(400\pi t)$ are

(a) 380 Hz, 1/380 sec

(b) 420 Hz, 1/420 sec

(c) 400 Hz, 1/400 sec

(d) 800 Hz, 1/800 sec

MCQ6.14 In a PCM system, a five bit-encoder is used. If the difference between two consecutive levels is 1V, then the range of the encoder is

(a) 0-30 V (c) 1-30 V

(b) 0-31 V (d) 1-32 V

MCQ6.15 The number of bits per sample in a PCM system is increased from 8 to 16; then, the bandwidth will be increased

(a) 4 times (c) 16 times

(b) 2 times (d) 8 times

MCQ6.16 Good voice reproduction via PCM requires 128 quantization levels. If the bandwidth of the voice channel is 4 kHz, then the data rate is

(a) 128 kbps (c) 28 kbps

(b) 250 kbps (d) 56 kbps

MCQ6.17 The demodulation of a delta modulated signal is achieved by

(a) Differentiation (c) Integration

(b) BPF (d) Sampling

MCQ6.18 For which of the following system, the signal to noise ratio is the highest

(a) PAM (c) PAM and PWM

(b) PWM (d) PPM

MCQ6.19 A 3000 Hz bandwidth channel has a capacity of 30 kbps. The signal to noise ratio of the channel is

(a) 25 dB (c) 40 dB

(b) 20 dB (d) 30 dB

MCQ6.20 Which one of the following pulse communications systems is digital

(a) PAM (c) PPM

(b) PCM (d) PWM

MCQ ANSWERS

MCQ6.1	(c)	MCQ6.11	(c)
MCQ6.2	(b)	MCQ6.12	(a)
MCQ6.3	(b)	MCQ6.13	(b)
MCQ6.4	(c)	MCQ6.14	(b)
MCQ6.5	(a)	MCQ6.15	(b)
MCQ6.6	(c)	MCQ6.16	(d)
MCQ6.7	(d)	MCQ6.17	(c)
MCQ6.8	(a)	MCQ6.18	(d)
MCQ6.9	(a)	MCQ6.19	(d)
MCQ6.10	(c)	MCQ6.20	(b)

CHAPTER 7

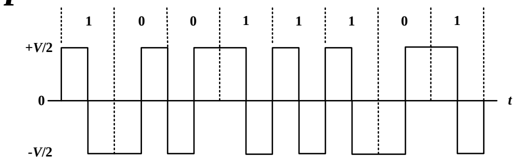
LINE CODING

Definition

The process of code conversion from digital codes to corresponding electrical pulses for data transmission over a transmission line is called transmission coding or line coding

Highlights

Representation



7.1 Introduction

In a digital transmission system, the digitalized signal is transmitted over the transmission line through a digital multiplexer using the process of interleaving to utilize the channel capacity effectively. The multiplexer's output is coded into electrical pulses before the transmission to make it suitable for the transmission over the communication channel. In this chapter, the introduction of different approaches for the above context is presented with their pros and cons.

7.2 Line Codes

The encoding process of digital signals into electrical pulses is called line coding or transmission coding. In other words,

"The process of code conversion from digital codes to corresponding electrical pulses for data transmission over a transmission line is called transmission coding or line coding."

However, there is a number of line coding, but an effective line code should have the following desirable properties:

1. **Transmission bandwidth:** The transmission bandwidth of a line code should be as small as possible.
2. **Power efficiency:** Required transmission power should be as small as possible for a given bandwidth and error probability.
3. **Error detection and correction capability:** It is desirable to have error detection as well as error correction capability.
4. **Favourable power spectral density:** It is required to have zero DC power content (PSD at $\omega = 0$) to enable AC coupling.
5. **Self-synchronization:** It could be possible to extract the clock pulse from the received information.
6. **Transparency:** Avoid long strings of 0s and 1s so that any sequence of 0s and 1s have only one inference.

A number of methods are possible to assign the electrical waveform (pulses) to digital data. For example, the simplest possible line code for the binary data is on-off line code where '1' is transmitted by pulse $p(t)$ and '0' is transmitted by no pulse or zero pulses.

Generally, the line codes are divided into the following three categories:

1. Unipolar
2. Polar
3. Bi-polar

Further classifications are shown in Fig. 7.1.

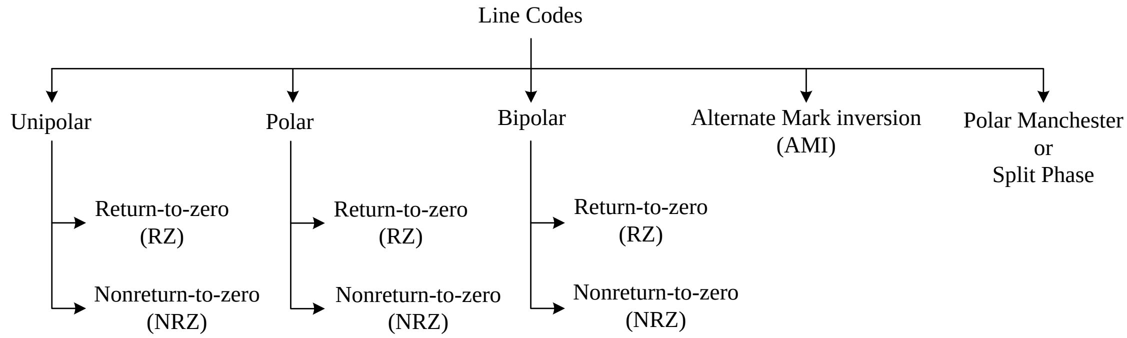


Fig. 7.1 Classifications of line codes

7.2.1 Unipolar Line Coding

In unipolar line coding, bit '1' is encoded by pulse $p(t)$ and '0' is encoded by no pulse. Therefore, unipolar coding is also called on-off keying (OOK). Different types of unipolar line coding are as follows:

(a) Unipolar return-to-zero (Unipolar RZ)

(b) Unipolar nonreturn-to-zero (Unipolar NRZ)

7.2.1.1 Unipolar Return-to-Zero (Unipolar RZ)

In unipolar RZ coding, the electrical pulse pattern has zero value or no pulse when bit '0'is transmitted and has high magnitude ($+V$ volt) when bit '1'is transmitted. If the pulse duration is considered as T_b and bit '1' is transmitted, then $+V$ volt magnitude will present only for $T_b/2$ duration and for the remaining duration, there will be no pulse in RZ line coding i.e.

$$p(t)=\begin{cases} +V & \text{for} \quad 0<t<\dfrac{T_b}{2} \\ 0 & \text{for} \quad \dfrac{T_b}{2}<t<T_b \end{cases} \quad (\textit{If '1' is transmitted}) \text{ and}$$

$$p(t)=0 \quad \text{for} \quad 0<t<T_b \quad (\textit{If '0' is transmitted}) \tag{7.1}$$

The RZ coding for bit pattern '10011101' is shown in Fig. 7.2(a)

Advantages

The advantages of unipolar RZ line coding are as follows:

1. It is simple.

Disadvantages

The unipolar RZ coding shows the following limitations:

1. No error detection and correction capability.

2. Less immune to noise.
3. Required twice as much power as a polar signaling needs.
4. Excessive bandwidth is required.
5. Non zero PSD at $\omega = 0$, therefore, it rules out the use of AC coupling.

7.2.1.2 Unipolar nonreturn-to-zero (unipolar NRZ)

Unlike unipolar RZ line coding, bit '1' is encoded by $+V$ volt for the complete bit duration of T_b in unipolar NRZ line coding. Still, there will be no pulse for bit '0' transmission, i.e.

$$\begin{aligned} p(t) &= +V \quad \text{for} \quad 0 < t < T_b \quad (\textit{If '1' is transmitted}) \\ p(t) &= 0 \quad \text{for} \quad 0 < t < T_b \quad (\textit{If '0' is transmitted}) \end{aligned} \tag{7.2}$$

The Unipolar NRZ coding for bit pattern '10011101' is shown in Fig. 7.2(b).

Advantages

The advantages of unipolar NRZ line coding are as follows:

1. It is simple.
2. A lesser bandwidth is required than unipolar RZ.

Disadvantages

The unipolar NRZ coding shows the following limitations:

1. No error detection and correction capability.
2. Less immune to noise.
3. Required twice as much power as a polar signaling needs.
4. Non zero PSD at $\omega = 0$, therefore, it rules out the use of AC coupling.

7.2.2 Polar

In polar line coding, bit '1' is encoded by pulse $p(t)$ and '0' is encoded by $-p(t)$. Different types of polar line coding are as follows:

(a) Polar return-to-zero (Polar RZ)
(b) Polar nonreturn-to-zero (Polar NRZ)

7.2.2.1 Polar Return-to-Zero (Polar RZ)

In polar RZ, the electrical pulse pattern has positive voltage polarity ($+V$ volt) for bit '1', whereas bit '0' is encoded by negative voltage polarity ($-V$ volt). For RZ format, the pulse is transmitted only for half duration, i.e. only for $T_b / 2$ duration in both cases of '0' and '1' bits transmission. Mathematically,

$$p(t)=\begin{cases}+V & \text{for} \quad 0\le t<\dfrac{T_b}{2} \\ 0 & \text{for} \quad \dfrac{T_b}{2}\le t<T_b\end{cases} \qquad (\textit{If '1' is transmitted})$$

$$p(t)=\begin{cases}-V & \text{for} \quad 0\le t<\dfrac{T_b}{2} \\ 0 & \text{for} \quad \dfrac{T_b}{2}\le t<T_b\end{cases} \qquad (\textit{If '0' is transmitted}) \tag{7.3}$$

The polar RZ coding for bit pattern '10011101' is shown in Fig. 7.2(c).

Advantages

The advantages of unipolar NRZ line coding are as follows:

1. It is transparent and straightforward, i.e. there is always a pulse, either positive or negative.

Disadvantages

The unipolar NRZ coding shows the following limitations:

1. Always need excessive bandwidth, i.e. no bandwidth-efficient.
2. Occupies twice the bandwidth of Polar NRZ
3. No error detection and correction capability.
4. Non zero PSD at $\omega=0$, therefore, it rules out the use of AC coupling.

7.2.2.2 Polar Nonreturn-to-Zero (Polar NRZ)

Unlike polar RZ line coding, bits '1' and '0' are represented by $+V$ volt and $-V$ volt pulses respectively for the complete T_b duration in polar NRZ line coding, i.e.

$$\begin{aligned} p(t)&=+V \quad \text{for} \quad 0<t<T_b \quad (\textit{If '1' is transmitted}) \\ p(t)&=-V \quad \text{for} \quad 0<t<T_b \quad (\textit{If '0' is transmitted}) \end{aligned} \tag{7.4}$$

The polar NRZ coding for bit pattern '10011101' is shown in Fig. 7.2(d).

Advantages

The advantages of unipolar NRZ line coding are as follows:

1. A lesser bandwidth is required in comparison with polar RZ.

Disadvantages

The unipolar NRZ coding shows the following limitations:

1. Still not bandwidth efficient line coding.
2. No error detection and correction capability.
3. No clock is present.
4. .Non zero PSD at $\omega=0$, therefore, it rules out the use of AC coupling.

7.2.3 Bipolar

It is also called pseudoternary or alternate mark inversion (AMI), where, '0' is transmitted by no pulse, but '1' is transmitted as $p(t)$ or $-p(t)$ depending upon whether previous '1' is encoded as $-p(t)$ or $p(t)$. So, there are three electrical pulses representation as 0, $p(t)$ and $-p(t)$. The bipolar RZ coding for bit pattern '10011101' is shown in Fig. 7.2(e). The pulse duration is symbol bit duration in NRZ type, whereas the pulse duration is half of symbol bit duration in bipolar RZ type line coding.

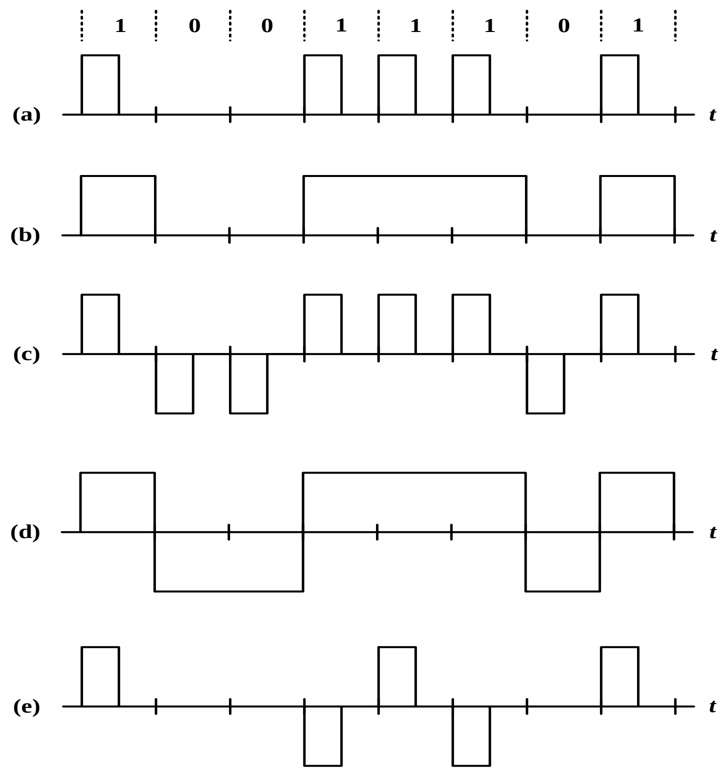


Fig. 7.2 Coding for bit pattern '10011101' (a) Unipolar RZ (b) Unipolar NRZ (c) Polar RZ (d) Polar NRZ (e) Bipolar RZ

Advantages

The advantages of bipolar line coding are as follows:

1. Single error detection capability.
2. Required low bandwidth than polar and unipolar schemes.

Disadvantages

The unipolar NRZ coding shows the following limitations:

1. No clock is present.
2. Required twice as much power as a polar signaling needs.

7.2.4 Split Phase Manchester

In split-phase Manchester coding, symbol '1' is encoded as a positive pulse for half duration followed by a negative half pulse for the other half duration of the signal.

On the other hand, symbol '0' is encoded as a negative pulse for half duration followed by a positive pulse for the other half duration of the signal. Mathematically,

$$p(t)=\begin{cases}+\frac{V}{2} & \text{for} \quad 0\le t<\frac{T_b}{2}\\ -\frac{V}{2} & \text{for} \quad \frac{T_b}{2}\le t<T_b\end{cases} \qquad (\textit{If '1' is transmitted})$$

$$p(t)=\begin{cases}-\frac{V}{2} & \text{for} \quad 0\le t<\frac{T_b}{2}\\ +\frac{V}{2} & \text{for} \quad \frac{T_b}{2}\le t<T_b\end{cases} \qquad (\textit{If '0' is transmitted}) \tag{7.5}$$

The split phase Manchester coding for bit pattern '10011101' is shown in Fig. 7.3.

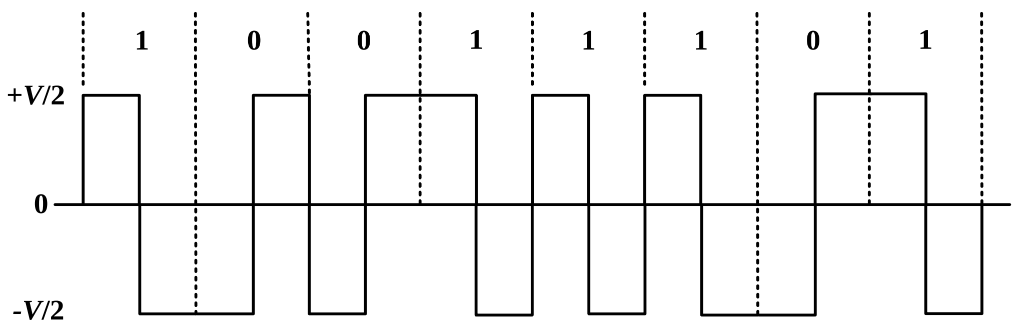


Fig. 7.3 Split phase Manchester coding for bit pattern '10011101'

7.2.5 High-Density Bipolar (HDB) Signaling

When a long bit stream of 0 is transmitted in bipolar signaling, it arises the problem of synchronization at the receiver. This non-transparency problem of bipolar signaling is eliminated by adding pulses when a long stream of consecutive 0s exceeds a number *N* (any value 1, 2, 3 and so on). This scheme is called high-density bipolar (HDB) coding and is denoted by HDBN.

Example: HDB3 coding

HDB3 uses 000V and B00V sequence with both B = 1(Bipolar) and V = 1(Violate).

(i) The sequence 000V is used when there is an odd number of 1s following the last special sequence.

(ii) B00V is used when there is an even number of 1s following the last special sequence.

Note: Substitution of 000V or B00V makes the number of non-zero pulses even.

The waveform of HDB3 coding for sequence 0100001011010000000000101100001 is shown in Fig. 7.4.

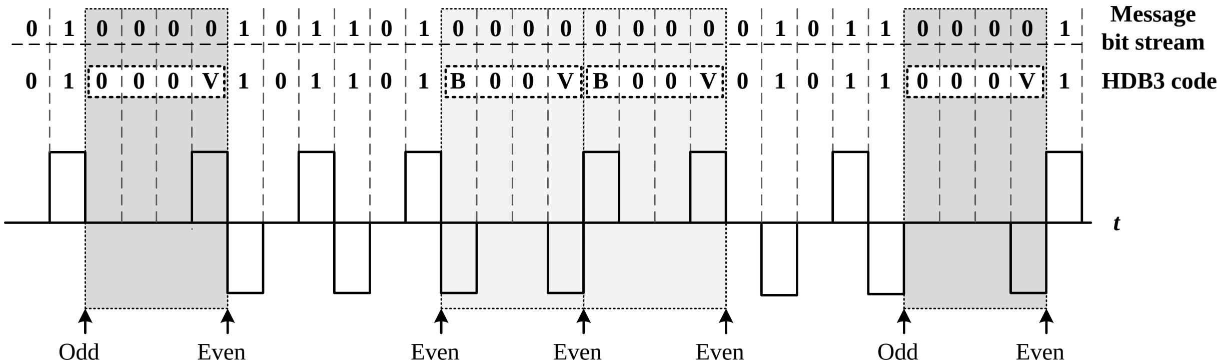


Fig. 7.4 HDB3 coding for bit pattern ‘0100001011010000000000101100001’

7.2.6 Binary with *N* Zero Substitution (BNZS) Signaling

The BNZS is similar to the HDBN line code, where one of the two special sequences containing some 1s to increase timing content is used for *N* successive zeros. Binary with eight-zero substitution (B8ZS) is an example of BNZS, which is used in DS1 signals. In B8ZS, the string of eight 0s is replaced by pattern **000VB0VB**. Similarly, the string of six 0s is replaced by pattern **0VB0VB** in B6ZS code used in DS2 signals, the string of three 0s is replaced by pattern **00V or B0V** in B3ZS code used in DS3 signals. The waveforms of B8ZS and HDB3 coding for sequence 1100000000110000010 is shown in Fig. 7.5.

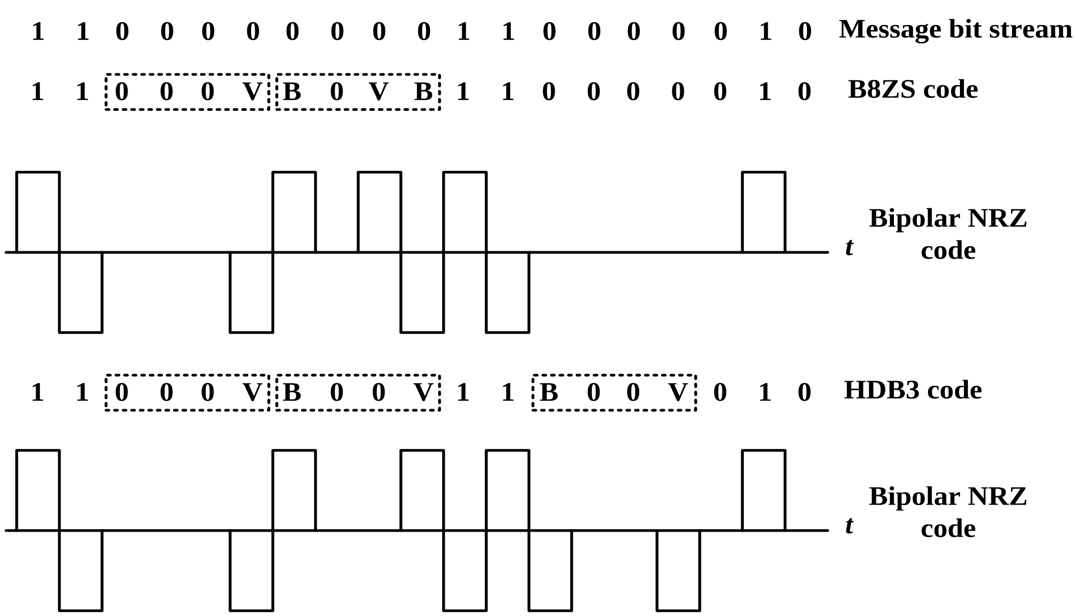


Fig. 7.5 (a) B8ZS (b) HDB3 coding for bit pattern ‘1100000000110000010’

7.3 Power Spectral Density (PSD) of Various Line Codes

The PSD of a random signal shows the distribution of the signal's power content at various frequencies in the frequency domain. The power spectral density is the Fourier transform of the autocorrelation of the signal.

The line coding consists of impulses, therefore, the autocorrelation function of $x(t)$ is

$$R_x(\tau)=\frac{1}{T_b}\sum_{n=-\infty}^{\infty}R_n\delta(\tau-nT_b) \tag{7.6}$$

where, $R_n=\lim_{N\to\infty}\frac{1}{N}\sum_k a_k a_{k+n}$ (7.7)

with $R_0=\lim_{N\to\infty}\frac{1}{N}\sum_k a_k^2$

Hence, the PSD $S_x(\omega)$ is given by

$$S_x(\omega)=\frac{1}{T_b}\sum_{n=-\infty}^{\infty}R_n e^{-jn\omega T_b}$$

Since, autocorrelation function is even function, so

$$S_x(\omega)=\frac{1}{T_b}\left(R_0+2\sum_{n=1}^{\infty}R_n\cos n\omega T_b\right) \tag{7.8}$$

The filter has transfer function as $P(\omega)$, So, output PSD is

$$S_y(\omega)=|P(\omega)|^2 S_x(\omega)=\frac{|P(\omega)|^2}{T_b}\sum_{n=-\infty}^{\infty}R_n e^{-jn\omega T_b} \tag{7.9}$$

$$=\frac{|P(\omega)|^2}{T_b}\left(R_0+2\sum_{n=1}^{\infty}R_n\cos n\omega T_b\right) \tag{7.10}$$

7.3.1 PSD of Polar Signaling

In polar line coding, bit '1' is encoded by pulse $p(t)$ and '0' is encoded by $-p(t)$. So, a_k is equally distributed as

$$a_k=1 \text{ or } -1 \Rightarrow a_k^2=1$$

Hence, $R_0=\lim_{N\to\infty}\frac{1}{N}\sum_k a_k^2$

Since, $a_k^2=1$, for all N pulses, Hence,

$$\sum_k a_k^2=1+1+.....+\text{upto } N \text{ terms} = N$$

So, $R_0=\lim_{N\to\infty}\frac{1}{N}\times N=1$ (7.11)

Moreover, both a_k and a_{k+1} are either 1 or -1. So, the possible combinations of $a_k a_{k+1}$ are

Possible combinations of $a_k a_{k+1}$

a_k \ a_{k+1}	-1	+1
-1	+1	-1
+1	-1	+1

$\Rightarrow a_k a_{k+1}$ = either +1 or -1

As shown above, out of 4 terms, there are two terms as +1 and two terms as -1. Therefore, in general, on average, out of N terms, N/2 terms are 1 and N/2 terms are -1. Hence,

$$R_1 = \lim_{N\to\infty} \frac{1}{N} \sum_k a_k a_{k+1} = \lim_{N\to\infty} \frac{1}{N} \left[\frac{N}{2}(1) + \frac{N}{2}(-1) \right] = 0$$

Similarly,

$$R_n = 0 \qquad for \quad n \geq 1 \tag{7.12}$$

Let the polar signaling is a half-width rectangular pulse scheme, i.e. RZ line coding. Then $p(t)$ is a rectangular pulse with $T_b/2$ time duration and given as

$$p(t) = \text{rect}\left(\frac{t}{T_b/2} \right) = \text{rect}\left(\frac{2t}{T_b} \right) \tag{7.13}$$

The Fourier transform of the Eq. (7.13) is given by

$$P(\omega) = \frac{T_b}{2} \text{sinc}\left(\frac{\omega T_b}{4} \right) \tag{7.14}$$

Therefore, output PSD is obtained as

$$S_y(\omega) = \frac{|P(\omega)|^2}{T_b} R_o \tag{7.15}$$

Substituting the value of R_o

$$S_y(\omega) = \left[\frac{T_b}{2} \text{sinc}\left(\frac{\omega T_b}{4} \right) \right]^2 \Big/ T_b$$

$$= \frac{T_b}{4} \text{sinc}^2\left(\frac{\omega T_b}{4} \right) \tag{7.16}$$

The spectrum of $S_y(\omega)$ is shown in Fig. 7.6.

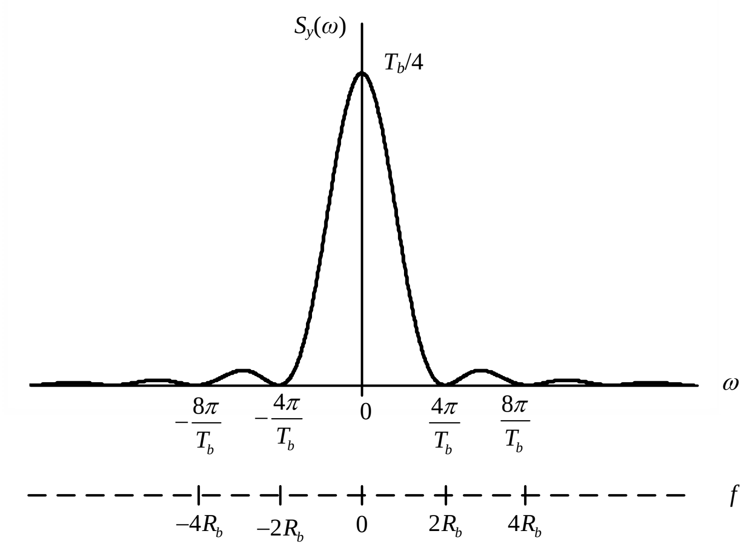


Fig. 7.6 The PSD of polar RZ signaling

Fig, 7.6 shows that the essential bandwidth (first non-dc null frequency) of polar signaling for half-width rectangular pulse scheme (RZ) is $2R_b$ Hz (Four times of theoretical bandwidth). However, the essential bandwidth can be reduced to R_b Hz. for a full-width rectangular pulse scheme, i.e. NRZ line coding, but still, it is twice of theoretical bandwidth.

Polar signaling has the following advantages:

1. A most efficient scheme for power requirement perspective.
2. Transparent, i.e. there is always a pulse, either positive or negative.

Polar signaling has the following limitations also

1. No error detection or error correction capability.
2. Excessive bandwidth is required.
3. Non zero PSD at $\omega = 0$, therefore, it rules out the use of AC coupling

7.3.2 PSD of Bipolar Signaling

There are three levels for pulses representation $0, p(t)$ and $-p(t)$ in bipolar signaling. On average, out of *N* terms, half (i.e. *N*/2) terms of the a_k's are 0s and the remaining half (i.e. *N*/2) are either 1 or -1, with $a_k^2 = 1$. So,

$$R_o = \lim_{N\to\infty} \frac{1}{N}\left[\frac{N}{2}(\pm 1)^2 + \frac{N}{2}(0)^2\right] = \frac{1}{2} \tag{7.17}$$

Further, the possible combinations of $a_k a_{k+1}$'s are 00, 01, 10 and 11. If all combinations are equally likely, then each has *N*/4 combinations. Since bit 0 is represented as no pulse, i.e. $a_k = 0$. So,

$a_k a_{k+1} = 0$; *for* 00, 01 and 10 (3*N*/4 combinations);

$a_k a_{k+1} = -1$ *for* 11 (consecutive pulses have opposite polarities with *N*/4 combinations);

Hence,

$$R_1 = \lim_{N\to\infty} \frac{1}{N} \sum_k a_k a_{k+1} = \lim_{N\to\infty} \frac{1}{N} \left[\frac{N}{4}(-1) + \frac{3N}{4}(0) \right] = -\frac{1}{4} \qquad (7.18)$$

Similarly, the possible combinations of $a_k a_{k+2}$ are 000, 001, 010, 011, 100, 110, 101 and 111 with $N/8$ combinations each.

The first six combinations have either first bit or last bit or both bits as 0. Therefore, $6N/8$ combinations have

$$a_k a_{k+2} = 0$$

In 101, the first and last bits are 1. Since consecutive pulses have opposite polarities, therefore, $N/4$ combinations have

$$a_k a_{k+2} = -1$$

In 111, the first and last bits will have the same polarities; therefore, $N/4$ combinations have

$$a_k a_{k+2} = 1$$

Hence,

$$R_2 = \lim_{N\to\infty} \frac{1}{N} \sum_k a_k a_{k+2} = \lim_{N\to\infty} \frac{1}{N} \left[\frac{6N}{8}(0) + \frac{N}{8}(-1) + \frac{N}{8}(1) \right] = 0 \qquad (7.19)$$

Similarly,

$$R_n = 0 \qquad \text{for} \quad n > 1 \qquad (7.20)$$

Therefore, output PSD is

$$S_y(\omega) = \frac{|P(\omega)|^2}{T_b} \left(R_0 + 2\sum_{n=1}^{\infty} R_n \cos n\omega T_b \right)$$

$$= \frac{|P(\omega)|^2}{T_b} \left(R_0 + 2R_1 \cos \omega T_b \right)$$

$$= \frac{|P(\omega)|^2}{T_b} \left(\frac{1}{2} - \frac{2}{4} \cos \omega T_b \right) = \frac{|P(\omega)|^2}{2T_b} \left(1 - \cos \omega T_b \right)$$

$$\Rightarrow S_y(\omega) = \frac{|P(\omega)|^2}{T_b} \sin^2\left(\frac{\omega T_b}{2}\right)$$

$$S_y(\omega) = \frac{T_b}{4} \operatorname{sinc}^2\left(\frac{\omega T_b}{4}\right) \sin^2\left(\frac{\omega T_b}{2}\right) \tag{7.21}$$

The spectrum of $S_y(\omega)$ is shown in Fig. 7.7.

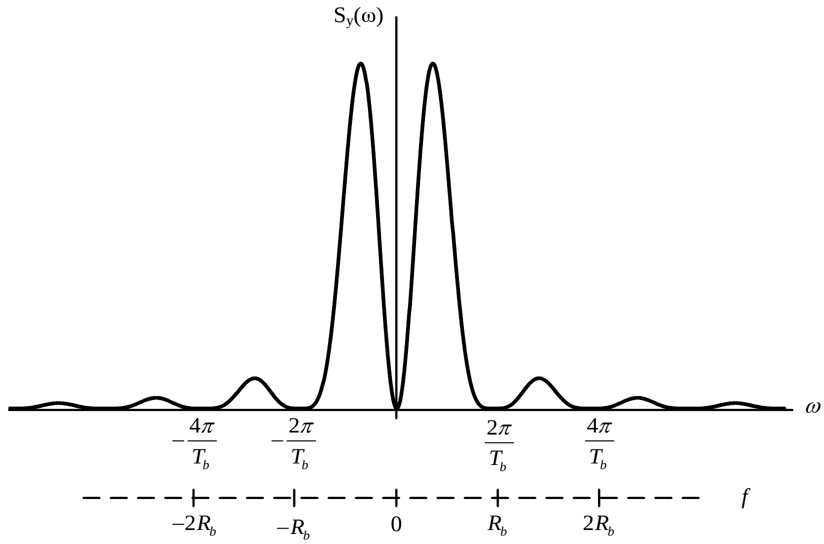


Fig. 7.7 The PSD of bipolar RZ signaling

Fig. 7.7 shows that the essential bandwidth is R_b Hz for bipolar signaling for both half and full pulse width.

Bipolar signaling shows the following advantages:

1. Bandwidth efficient code.
2. It has a spectrum with DC null.
3. It has a single error detection capability.

Bipolar signaling has the following limitations also

1. It requires twice as much power as polar signal needs.
2. Not transparent.

7.3.3 PSD of On-Off Signaling

In on-off signaling, bit '1' is encoded by pulse $p(t)$ and '0' is encoded by 0. So, a_k is equally distributed as

$$a_k = 1 \text{ or } 0$$

Therefore, on average, out of *N* terms, half (i.e. *N*/2) terms are 1 and the remaining *N*/2 terms are 0. Hence

Hence,

$$R_0 = \lim_{N\to\infty} \frac{1}{N}\left[\frac{N}{2}(1)^2 + \frac{N}{2}(0)^2\right] = \frac{1}{2} \tag{7.22}$$

The possible combinations of $a_k a_{k+1}$ are 00, 01, 10 and 11. If all combinations are equally likely, then each has $N/4$ combinations. Since bit 0 is represented as no pulse, i.e. $a_k = 0$, So

$a_k a_{k+1} = 0$; *for* 00, 01 and 10 ($3N/4$ combinations);

$a_k a_{k+1} = 1$ *for* 11 ($N/4$ combinations);

Hence,

$$R_1 = \lim_{N\to\infty} \frac{1}{N}\sum_k a_k a_{k+1} = \lim_{N\to\infty} \frac{1}{N}\left[\frac{3N}{4}(0) + \frac{N}{4}(1)\right] = \frac{1}{4} \tag{7.23}$$

Similarly, the possible combinations of $a_k a_{k+2}$ are 000, 001, 010, 011, 100, 101, 110 and 111 with $N/8$ combinations each.

The first six combinations have either first bit or last bit or both bits as 0. Therefore, $6N/8$ combinations have

$a_k a_{k+2} = 0$

In 101, the first and last bits are 1. Since consecutive pulses have opposite polarities, So, $N/4$ combinations have

$a_k a_{k+2} = 1$

In 111, the first and last bits will have the same polarities, So, $N/4$ combinations have

$a_k a_{k+2} = 1$

Hence,

$$R_2 = \lim_{N\to\infty} \frac{1}{N}\sum_k a_k a_{k+2} = \lim_{N\to\infty} \frac{1}{N}\left[\frac{6N}{8}(0) + \frac{N}{8}(1) + \frac{N}{8}(1)\right] = \frac{1}{4} \tag{7.24}$$

Similarly,

$$R_n = \frac{1}{4} \qquad \textit{for} \quad n \geq 1 \tag{7.25}$$

$$\begin{aligned} S_x(\omega) &= \frac{1}{2T_b} + \frac{1}{4T_b}\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} e^{-jn2\pi f T_b} \\ &= \frac{1}{4T_b} + \frac{1}{4T_b}\sum_{n=-\infty}^{\infty} e^{-jn2\pi f T_b} \end{aligned} \tag{7.26}$$

Since, the exponential summation term is represented as impulse train, i.e.

$$\sum_{n=-\infty}^{\infty} e^{-jn2\pi f T_b} = \frac{1}{T_b}\sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_b}\right) \tag{7.27}$$

So,

$$S_x(\omega) = \frac{1}{4T_b} + \frac{1}{4T_b^2} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_b}\right) \quad (7.28)$$

Substituting the value from Eq. (7.28) into Eq. (7.9) to get the output PSD as

$$S_y(\omega) = \frac{|P(\omega)|^2}{4T_b}\left[1 + \frac{1}{T} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_b}\right)\right] \quad (7.29)$$

Therefore, the PSD for half width rectangular pulse is

$$S_y(\omega) = \frac{T_b}{16} \text{sinc}^2\left(\frac{\omega T_b}{4}\right)\left[1 + \frac{1}{T_b} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_b}\right)\right] \quad (7.30)$$

The spectrum of $S_y(\omega)$ consists of continuous term as well as discrete term both, as shown in Fig. 7.8.

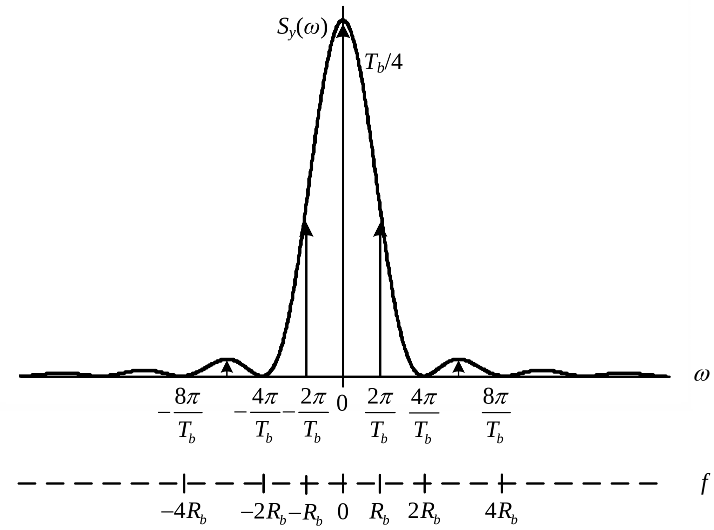


Fig. 7.8 The PSD of bipolar RZ signaling

For a full-width rectangular pulse scheme, i.e. NRZ line coding, the essential bandwidth is half but still two times the theoretical bandwidth.

On-off signaling has the following limitations:

1. It requires twice as much power as polar signal needs.
2. Less immune to noise interference
3. Not transparent.
4. No error detection or correction capability.
5. Excessive bandwidth is required.
6. Non zero PSD at $\omega = 0$, therefore, it rules out the use of AC coupling.

ADDITIONAL SOLVED EXAMPLES

SE7.1 Draw the waveform of unipolar RZ, unipolar NRZ and polar RZ for the bitstream of 10100010.

Sol: The line coding of '10100010' in different format is given in Fig. 7.9.

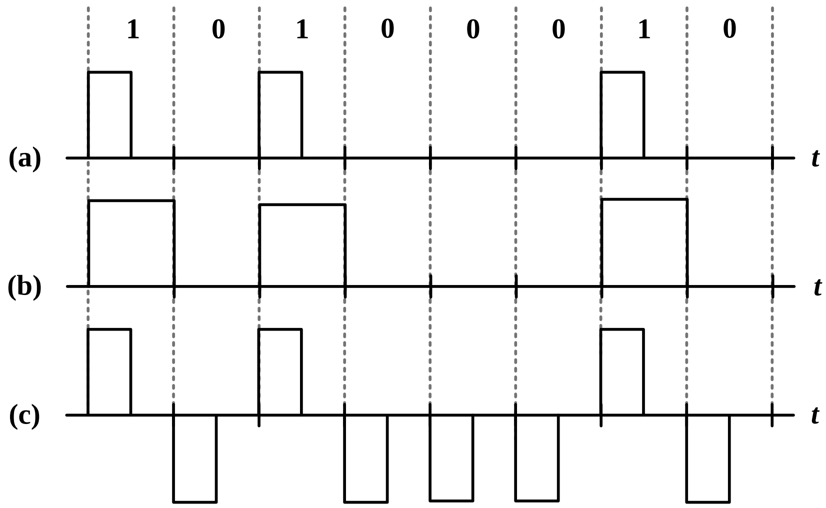


Fig. 7.9 Line coding of '10100010' in (a) unipolar RZ (b) unipolar NRZ (c) Polar RZ

SE7.2 Repeat the above problem for Bipolar NRZ and spilt phase Manchester format.

Sol: The line coding of '10100010' in different format is given in Fig. 7.10.

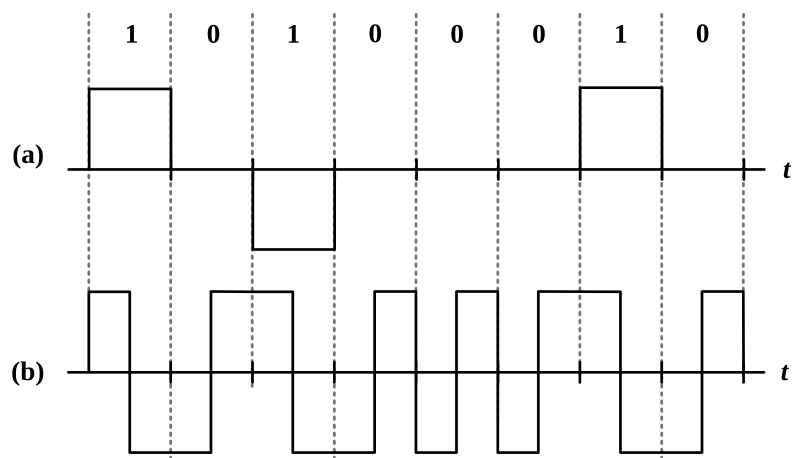


Fig. 7.10 Line coding of '10100010' in (a) Bipolar NRZ (b) Spilt phase Manchester

SE7.3 The binary data 1101010110 is transmitted over a baseband channel. Draw the waveform of transmitted data using the following format:

1. Polar RZ
2. Unipolar NRZ
3. Split phase Manchester format
4. Polar quaternary NRZ signaling.

Sol: The line coding of '1101010110' in different format is given in Fig. 7.11.

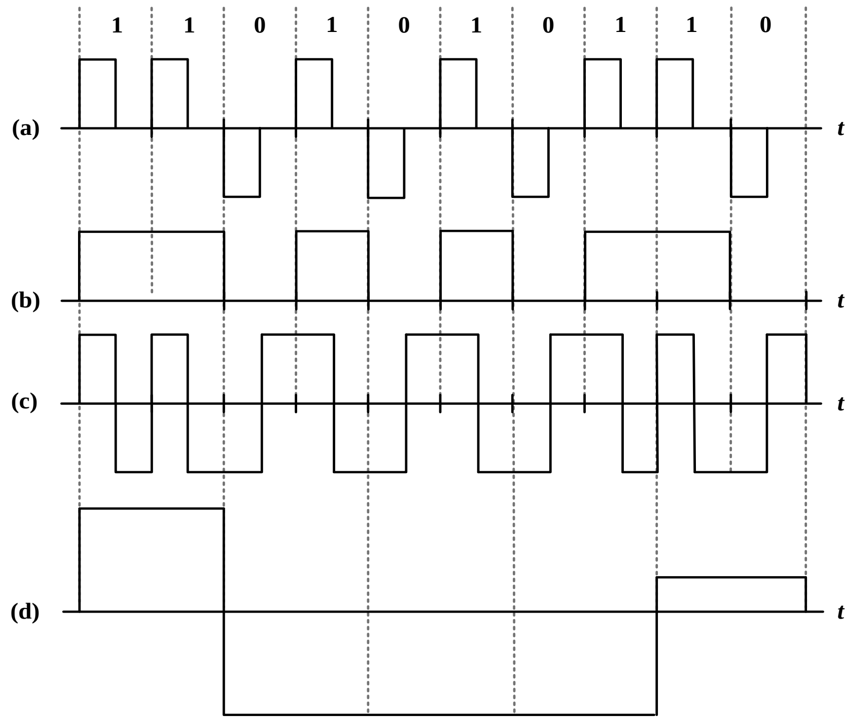


Fig. 7.11 Line coding of '1101010110' in (a) Polar RZ (b) Unipolar NRZ (c) Spilt phase Manchester (d) Polar quaternary NRZ

SE7.4 Draw the waveforms of the bitstream 0011010110 transmitted in the following format:

1. Polar RZ
2. Unipolar RZ
3. Split phase Manchester format

Sol: The line coding of '0011010110' in different format is given in Fig. 7.12.

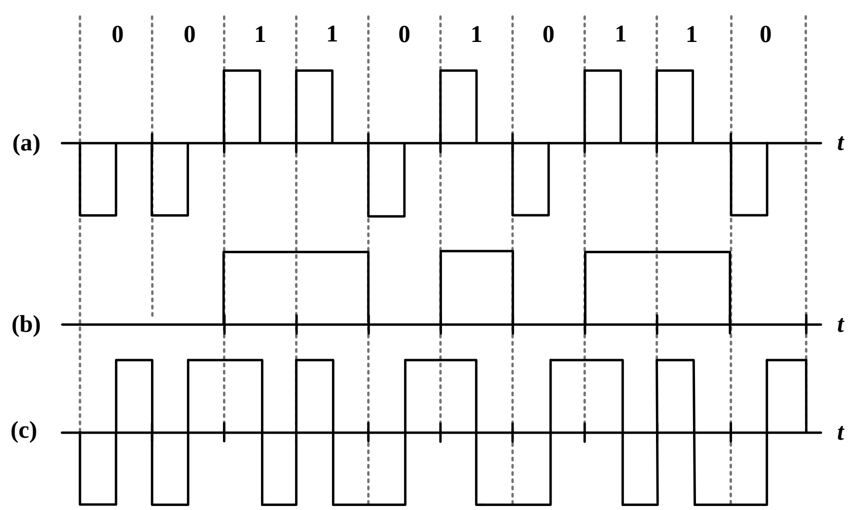


Fig. 7.12 Line coding of '0011010110' in (a) Polar RZ (b) Unipolar NRZ (c) Spilt phase Manchester

PROBLEMS

P7.1 Represent the data 1011010100 using the following digital data formats with a diagram:

(i) Unipolar RZ

(ii) Polar RZ

(iii) Unipolar NRZ

(iv) Split phase Manchester

P7.2 Write advantages and limitations of the following line codes:

(i) Unipolar

(ii) Polar

(iii) Bipolar

P7.3 Explain the desired characteristics of an ideal line code

P7.4 Derive the expression for PSD of bipolar line code for a full-width rectangular pulse.

P7.5 The binary data 1001010111 is transmitted over a baseband channel. Draw the waveform of transmitted data using the following format:

5. Unipolar NRZ
6. Polar RZ
7. Split phase Manchester format
8. Polar quaternary NRZ signaling.

MULTIPLE-CHOICE QUESTIONS

MCQ7.1 Symbol '0' in polar RZ format is represented as

(a) Pulse is transmitted for half the duration
(b) Negative voltage
(c) Zero voltage
(d) Both (a) and (b)

MCQ7.2 AMI is also known as

(a) Manchester coding
(b) Pseudo ternary coding
(c) Polar NRZ coding
(d) None of the above

MCQ7.3 In……… format, the positive half interval pulse is followed by a negative half interval pulse for bit '1'.

(a) Polar RZ
(b) Bipolar NRZ
(c) Polar NRZ
(d) Manchester

MCQ7.4 In polar coding technique

(a) '1' is transmitted by a positive pulse and '0' is transmitted by zero volt
(b) '1' is transmitted by a positive pulse and '0' is transmitted by a negative pulse
(c) Both (a) and (b)
(d) None of the above

MCQ7.5 In unipolar RZ coding technique

(a) '0' is transmitted by zero volt
(b) '1' is transmitted by a positive pulse
(c) Both (a) and (b)
(d) None of the above

MCQ7.6 Which waveforms are also called line codes?

(a) PAM (c) FM
(b) PCM (d) AM

MCQ7.7 In …….. format, logic '1' is represented by half bit wide pulse and logic 0 is represented by the absence of pulse?

(a) Unipolar RZ (c) RZ-AMI
(b) Bipolar RZ (d) Manchester coding

MCQ7.8 In………..format, three-level, positive, negative and zero are used to transmit the bitstream.

(a) Unipolar (c) Polar
(b) Bipolar (d) None of the above

MCQ7.9 ………….substitutes four consecutive zero with 000V and B00V.

(a) HDB3 (c) B4B8
(b) B8ZSf (d) None of the above

MCQ7.10 Which encoding type always has a nonzero average amplitude?

(a) Unipolar (c) Bipolar
(b) Polar (d) All the above

MCQ ANSWERS

MCQ7.1	(d)	MCQ7.6	(c)
MCQ7.2	(b)	MCQ7.7	(a)
MCQ7.3	(d)	MCQ7.8	(b)
MCQ7.4	(b)	MCQ7.9	(a)
MCQ7.5	(c)	MCQ7.10	(a)

CHAPTER 8

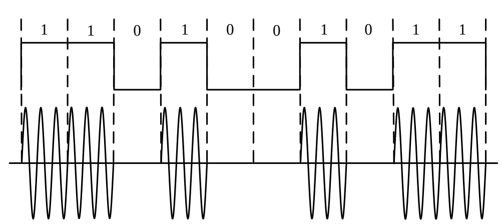
DIGITAL MODULATION

Types

1. *Amplitude shift keying (ASK)*
2. *Frequency shift keying (FSK)*
3. *Phase shift keying (PSK)*

Highlights

Representation



8.1 Introduction

Digital modulation is highly secure with more information capacity type modulation technique. As the signal is transmitted by only two levels, either +*V* or 0 (or –*V*), the modulation techniques are termed keying. These approaches come under the category of continuous-wave modulation since the carrier signal is continuous in nature. According to the variation in parameters, the digital modulation is divided into the following three categories:

1. Amplitude shift keying (ASK)
2. Frequency shift keying (FSK)
3. Phase shift keying (PSK)

All the above-mentioned techniques with their generation and detection methods are explained in the following subsections.

8.2 Amplitude Shift Keying (ASK)

ASK is simply a type of amplitude modulation in which the amplitude of the signal is varied according to the transmitted binary data. Let the transmitted message $m(t)$ has bitstream of 1101001011 where $m(t) = +V$ for binary data '1' and $m(t) = 0$ for binary data '0' as shown in Fig. 8.1. The transmitted ASK signal is expressed as:

$$\phi_{\text{ASK}}(t) = m(t) A_c \cos \omega_c t \tag{8.1}$$

For $m(t) = +V\,(\text{HIGH})$, $m(t) = 1$ and $\phi_{\text{ASK}}(t) = A_c \cos \omega_c t$

For $m(t) = 0\,(\text{LOW})$, $m(t) = 0$ and $\phi_{\text{ASK}}(t) = 0$

It is seen from Fig. 8.1 that the carrier signal is ON for $m(t) = 1$ and OFF for $m(t) = 0$. Therefore, ASK is also known as on-off keying (OOK). The disadvantage of this technique is that it does not give the satisfactory value of the probability of error which is explained in this chapter later.

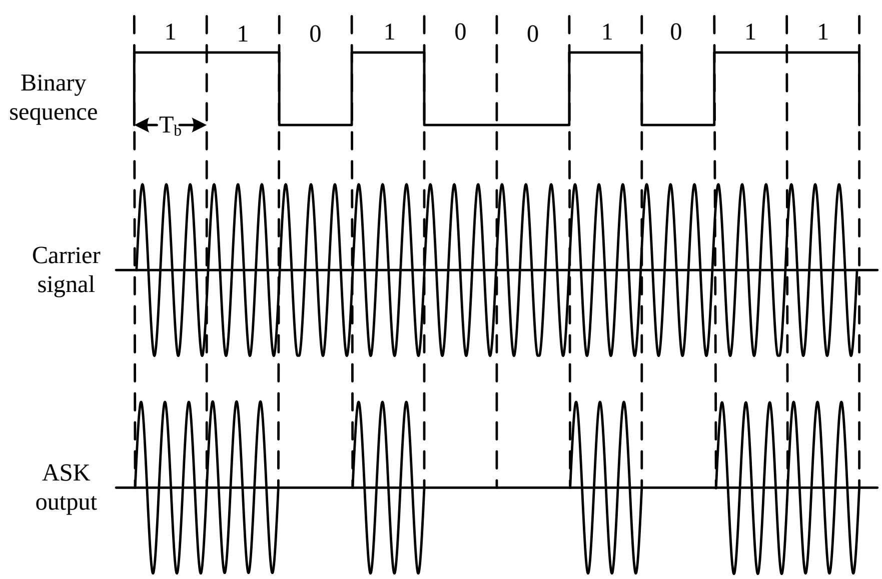


Fig. 8.1 ASK modulated waveform

8.2.1 ASK Modulator

The block diagram of the ASK modulator is shown in Fig. 8.2. It consists of an oscillator for carrier signal generation, a bandpass filter to shape the modulated signal and a binary encoded message signal operated switch.

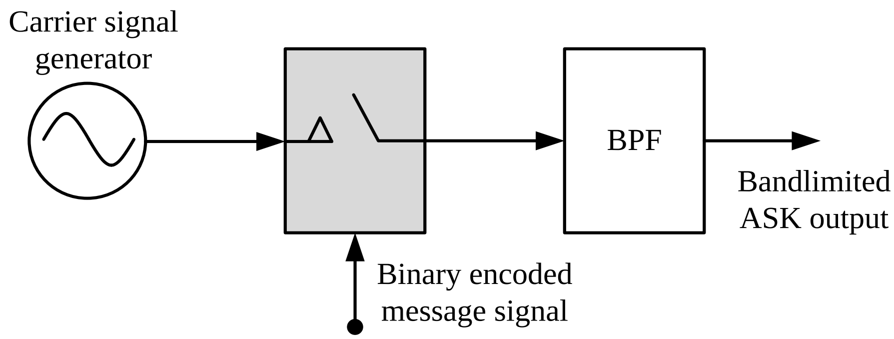


Fig. 8.2 ASK modulator

The working operation of the ASK modulator is as follows:

The oscillator generates the high-frequency continuous carrier signal. The binary encoded message signals enable the switch in either the ON or OFF position. If $m(t)=1$, the switch is closed (ON) and allows the carrier signal to appear at the input of BPF and if $m(t)=0$, the switch is opened (OFF) and has no input at the BPF. The BPF shapes the pulse according to the characteristics of the filter.

8.2.2 ASK Detector/Demodulator

A demodulator or detector circuit is used to recover the desired message signal from the modulated waveform. There are two types of ASK detector/demodulator:

1. Synchronous ASK detector/demodulator
2. Asynchronous ASK detector/demodulator

Similar to coherent demodulation of the continuous modulation, if the frequency of carrier pulse is equal to the pulse signal frequency at the receiver end, then this detection method is synchronous detection and the circuit arrangement is called synchronous ASK demodulator. Otherwise, it is known as an asynchronous ASK demodulator.

8.2.2.1 Synchronous ASK Demodulator

Like AM-SC synchronous demodulator, if the carrier signal generated at the receiver end in the ASK demodulator is in the same phase with the carrier signal at the transmitter side in the ASK modulator, then such type of detection/demodulation is called synchronous (coherent) ASK detection/demodulation. The synchronous ASK detector consists of a multiplier, an LPF and a comparator circuit. The block diagram of the synchronous ASK detector is shown in Fig. 8.3. The multiplier output $m_d(t)$ is

$$m_d(t) = \phi_{\text{ASK}}(t) \times \cos\omega_c t$$

$$= \left[m(t) A_c \cos\omega_c t\right] \times \cos\omega_c t$$

$$= m(t) A_c \cos^2 \omega_c t$$

$$= \frac{m(t) A_c}{2}\left[1 + \cos 2\omega_c t\right] \tag{8.2}$$

The high-frequency term of $\pm 2\omega_c$ is filtered out by the LPF. The output of LPF is demodulated ASK signal is given as

$$m_o(t) = \frac{m(t) A_c}{2} \tag{8.3}$$

The demodulated output $m_o(t)$ is a transmitted message signal with amplification/attenuation of $A_c/2$.

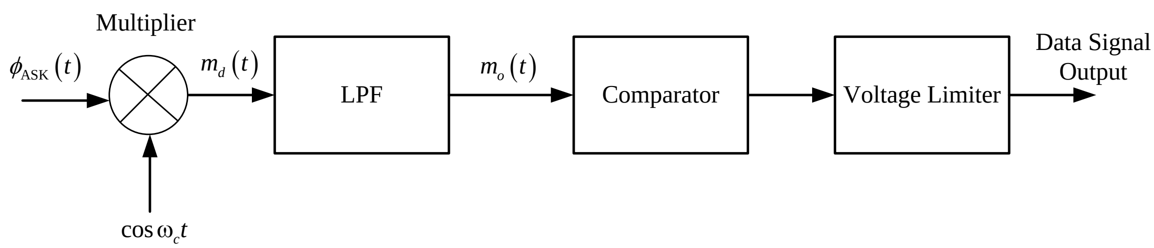


Fig. 8.3 Synchronous ASK detector

If the transmitted message bit is ‘1’, i.e., $m(t) = 1$,

$$m_o(t) = \frac{A_c}{2} \tag{8.4}$$

If the transmitted message is ‘0’, i.e., $m(t) = 0$,

$$m_o(t) = 0 \tag{8.5}$$

Further, a voltage limiter is used to limit the amplitude of the voltage at a defined value, i.e. reshape the pulse.

8.2.2.2 Asynchronous ASK Demodulator

The only difference between the coherent (synchronous) detector and the non-coherent (asynchronous) detector is that in the latter one, carrier signals at the transmitter side and receiver end are not in the same phase with each other. The asynchronous detector comprises a square law device (like a diode), an LPF and a comparator. The LPF is used to reconstruct the original message signal by eliminating the high-frequency signal components from the output of the square law device. The block diagram of the asynchronous ASK demodulator is shown in Fig. 8.4.

The working operation of asynchronous ASK demodulator is as follows:

The modulated ASK signal is provided at the input of the half-wave rectifier. The rectifier clips the negative half of the input and provides the positive half at the input of the LPF. The LPF eliminates the higher frequency component and provides an output similar to the envelope detector output of AM signal. Further, this signal is applied at the input of the comparator circuit for equivalent digital representation. The voltage limiter circuit is used to keep the signal amplitude at a pre-determined level by suppressing the fluctuation due to noise

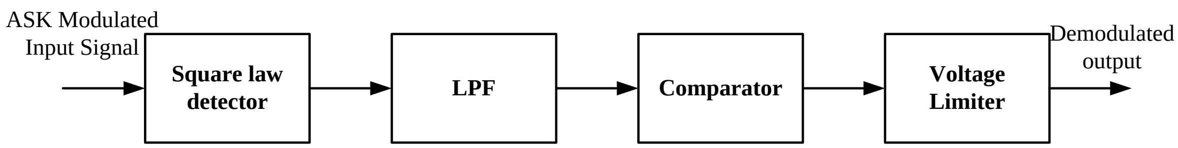


Fig. 8.4 Asynchronous ASK demodulator

ASK Applications

The applications of ASK are given below:

1. Home automation devices
2. Wireless base stations
3. Industrial networks devices
4. Low-frequency RF applications

8.3 Frequency Shift Keying (FSK)

Like FM, the frequency of carrier signal changes in accordance with digital input in FSK. The simplest form of FSK is binary frequency shift keying (BFSK) which uses two different frequencies for binary (‘1’ and ‘0’) information transmission. If the input message is ‘1’ (HIGH), the frequency is high and if the input message is ‘0’ (LOW), the frequency is low in FSK. The frequencies for ‘1’ and ‘0’ are named as mark and space frequencies, respectively. The FSK output waveform for binary message sequence 11011001011 is shown in Fig. 8.5.

Mathematically, the FSK waveform is represented as:

$$\phi_{\text{FSK}}(t) = A_c \cos(\omega_c \pm \varphi)t \tag{8.6}$$

Here, if the input is ‘1’, the term φ is additive in nature and the plus (+) sign is used to increase the frequency. Further, if ‘0’ is applied as input, the term φ is subtracted from ω_c and the minus (-) sign is used to decrease the frequency.

8.3.1 FSK Modulator

A schematic representation of the FSK modulator is shown in Fig. 8.6. It consists of two oscillators and one binary input sequence equivalent to the message.

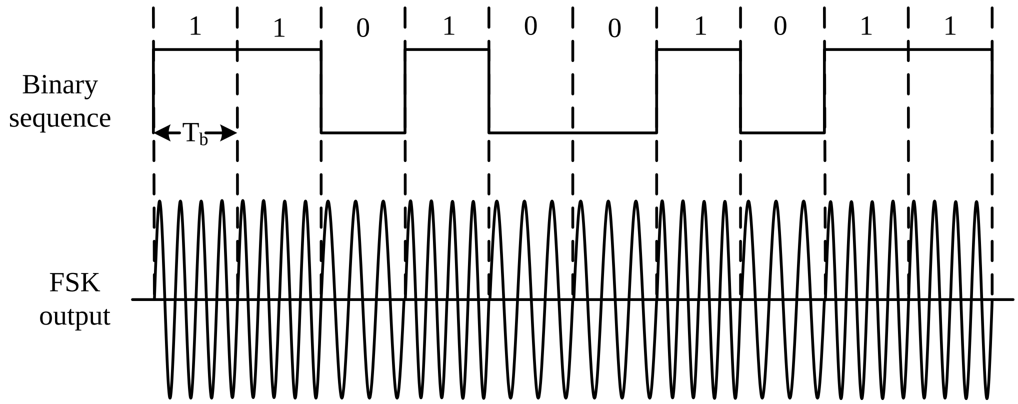


Fig. 8.5 FSK modulated waveform

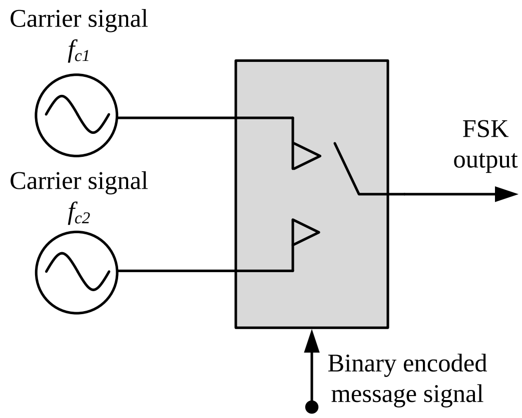


Fig. 8.6 FSK modulator

Two carrier signals of lower and higher frequencies are produced by the two different oscillators, as shown in Fig. 8.6. Both of these two oscillators are connected to a switch and an internal clock to avoid the abrupt phase discontinuities during the transmission of the modulating signal. The carrier frequencies are chosen according to the binary input of message signal.

8.3.2 FSK Demodulator

Following two types of FSK demodulator are used

1. Synchronous (coherent) demodulator/detector
2. Asynchronous (non-coherent) demodulator/detector

8.3.2.1 Synchronous FSK Detector

Like other synchronous detectors, the synchronous FSK detector needs carrier signals at the receiver stage in the same phase as the carrier signal at the transmitter stage. The only difference is that the synchronous FSK detector needs two different carrier signals since there are two different carrier signals at the transmitter end. The schematic diagram of the synchronous FSK demodulator (detector) is presented in Fig. 8.7. It comprises two mixers with local oscillator circuits for carrier signals generation, one difference amplifier and one LPF.

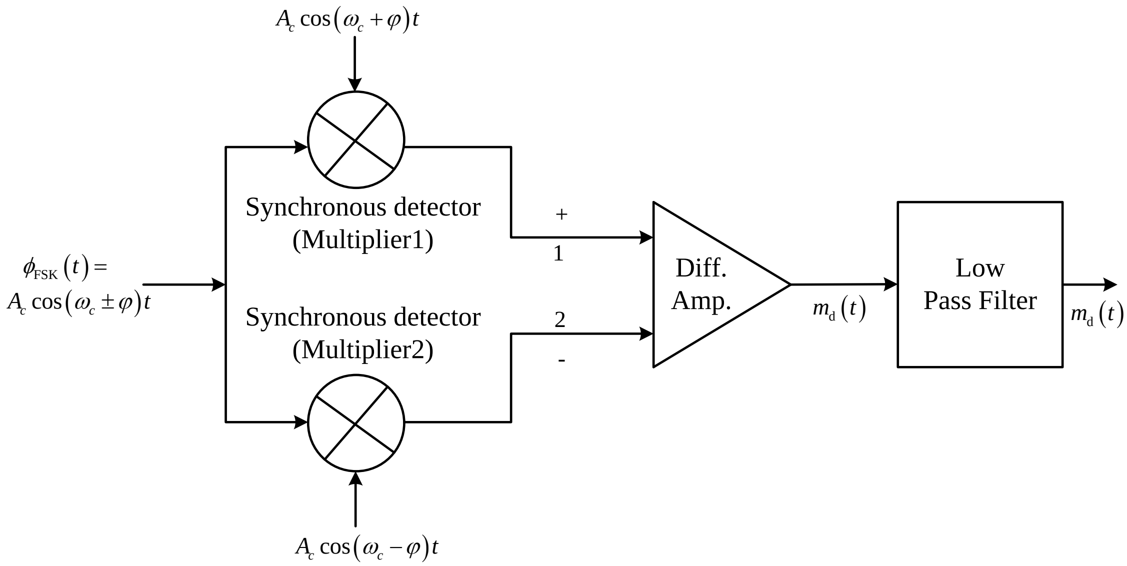


Fig. 8.7 Synchronous FSK detector

The working operation of the synchronous FSK demodulator is as follows:

Let, the FSK modulated signal is $\phi_{\text{FSK}}(t) = A_c \cos(\omega_c \pm \varphi)t$,

If $\phi_{\text{FSK}}(t) = A_c \cos(\omega_c + \varphi)t$, the non-inverting input ('+' terminal) of the difference amplifier is given as

$$A_c \cos(\omega_c + \varphi)t \times \cos(\omega_c + \varphi)t = \frac{A_c}{2}\left[1 + \cos 2(\omega_c + \varphi)t\right] \tag{8.7}$$

Similarly, the input at inverting terminal ('-' terminal) of the difference amplifier is given as

$$A_c \cos(\omega_c + \varphi)t \times \cos(\omega_c - \varphi)t = \frac{A_c}{2}\left[\cos 2\omega_c t + \cos 2\varphi t\right] \tag{8.8}$$

So, the output of the difference amplifier is given by

$$\begin{aligned} m_d(t) &= \frac{A_c}{2}\left[1 + \cos 2(\omega_c + \varphi)t\right] - \frac{A_c}{2}\left[\cos 2\omega_c t + \cos 2\varphi t\right] \\ &= \frac{A_c}{2} + \frac{A_c}{2}\left[\cos 2(\omega_c + \varphi)t - \cos 2\omega_c t - \cos 2\varphi t\right] \end{aligned} \tag{8.9}$$

Similarly, if $\phi_{\text{FSK}}(t) = A_c \cos(\omega_c - \varphi)t$

$$m_d(t) = -\frac{A_c}{2} + \frac{A_c}{2}\left[\cos 2\omega_c t + \cos 2\varphi t - \cos 2(\omega_c - \varphi)t\right] \tag{8.10}$$

The high-frequency terms are filtered out by LFP. Therefore, the output of the LPF is given by $m_o(t) = \pm\frac{A_c}{2}$.

If $m_o(t) = +\frac{A_c}{2}$, the decision is that '1' is received.

If $m_o(t) = -\frac{A_c}{2}$, the decision is that '0' is received.

For easy separation of two signals, $\varphi T >> \pi$.

8.3.2.2 Asynchronous FSK Detector

The block diagram of the asynchronous FSK demodulator is shown in Fig. 8.8. It consists of two BPFs (BPF-1 and BPF-2), two envelop detectors and one decision circuit.

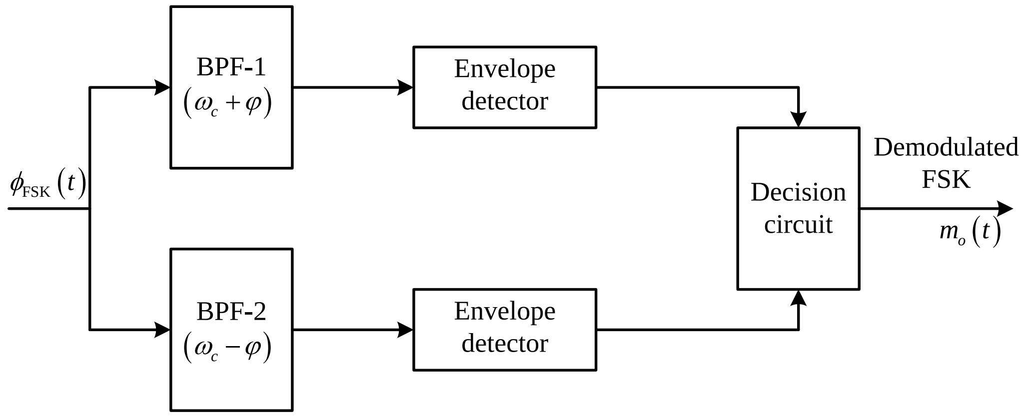


Fig. 8.8 Asynchronous FSK detector

The BPF-1 is tuned to the Mark frequency $(\omega_c+\varphi)$ and the BPF-2 is tuned to Space frequency $(\omega_c-\varphi)$. Now, the output obtained from these two filters look like an ASK signal which are forwarded to envelop detectors. According to the outputs of the envelope detectors, the decision circuit reshaped the waveform into a rectangular pulse to detect the binary bit either '1' or '0'.

8.4 Phase Shift Keying (PSK)

In this method, the phase of the carrier signal is shifted according to the input message. There are different types of PSK according to the combination of input bits patterns. The simplest form of the PSK is binary PSK (BPSK) which transmits the information in a two-phase shift ('π' or '0') in the carrier signal. Other forms of PSK are quadrature phase-shift keying (QPSK), differential phase-shift keying (DPSK) etc. In this section, PSK is considered for BPSK and other PSKs are explained in this chapter later.

8.4.1 Binary Phase Shift Keying (BPSK)

Since BPSK takes only two phases of reversal, it is also called 2-phase PSK or phase reversal keying. BPSK is a digital version of a double sideband suppressed carrier modulation scheme. Like DSB-SC, the BPSK is generated by the multiplication of the carrier signal with the binary sequence of the message signal.

The PSK waveform for the binary message $m(t)$ is represented as

$$\phi_{PSK}(t)=A_c\cos\left[\omega_c t+\varphi(t)\right] \qquad (8.11)$$

For $m(t)='1'$, $\rightarrow \varphi(t)=0$, So,

$$\phi_{PSK}(t)=A_c\cos\omega_c t$$

For $m(t) = '0'$, $\rightarrow \varphi(t) = \pi$, So

$$\phi_{\text{PSK}}(t) = -A_c \cos \omega_c t$$

Therefore, $\phi_{PSK}(t) = \pm A_c \cos(\omega_c t)$ (8.12)

Thus,

$$\begin{aligned} \phi_{PSK}(t) &= +A_c \cos(\omega_c t) \quad \text{for} \quad m(t) = 1 \\ \phi_{PSK}(t) &= -A_c \cos(\omega_c t) \quad \text{for} \quad m(t) = 0 \end{aligned} \tag{8.13}$$

The PSK modulated output for input bit pattern 1101001011 is shown in Fig. 8.9. The output is π radian phase-shifted from the carrier if the input bit is '0'. No phase shift occurs if the input is '1'.

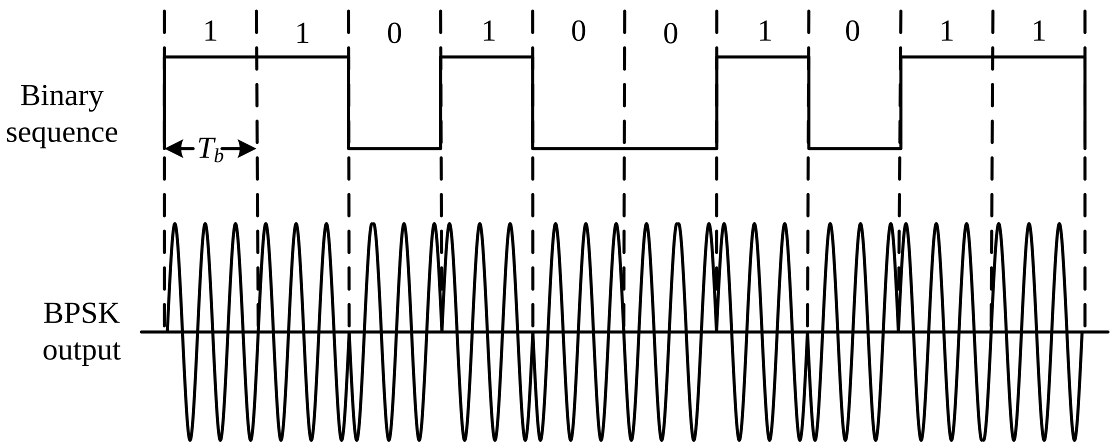


Fig. 8.9 PSK modulated waveform for bitstream of '1101001011'

8.4.1.1 BPSK Modulator

The block diagram of the BPSK modulator is shown in Fig. 8.10. It consists of a balanced modulator with two inputs 1) carrier signal and 2) binary sequence of the message signal. The local oscillator is used to generate the carrier signal.

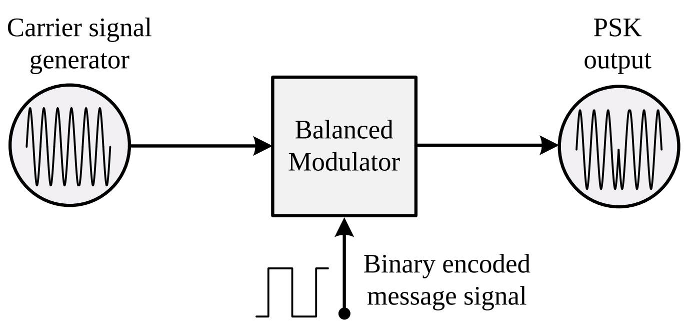


Fig. 8.10 BPSK modulator

The BPSK signal is generated by the multiplication of both inputs in a balanced modulator. The 0° phase shift is provided for '1' binary input and the 180° phase reversal is provided for '0' input. The BPSK modulated waveform is shown in Fig. 8.11.

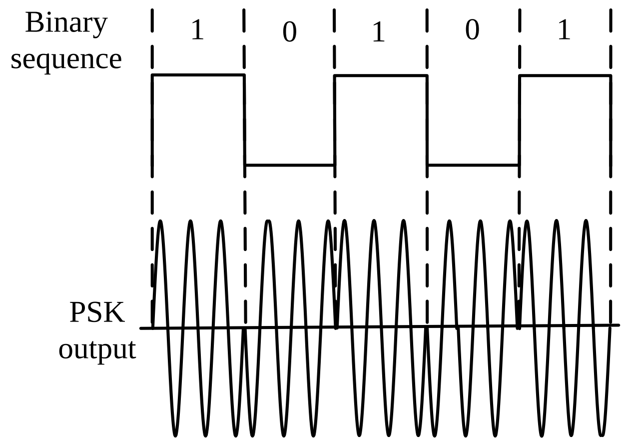


Fig. 8.11 BPSK modulated waveform

8.4.1.2 BPSK Demodulator

The synchronous detection method is used for the demodulation of the BPSK signal. The block diagram of a BPSK demodulator is shown in Fig. 8.12. The synchronising circuit, which consists of a square law detector, BPF and frequency divider, is used to generate the locally synchronising carrier.

Let the BPSK input is given as

$$\phi_{\text{PSK}}(t) = A_c \cos\left[\omega_c t + \varphi(t)\right] \tag{8.14}$$

The output of the square law device is

$$\phi_{sld}(t) = A_c^2 \cos^2\left[\omega_c t + \varphi(t)\right]$$

$$= \frac{A_c^2}{2}\left[1 + \cos 2\left(\omega_c t + \varphi(t)\right)\right] \tag{8.15}$$

$\phi_{\text{sld}}(t)$ is applied at the input of BPF $(\pm 2\omega_c)$, which passes only the term of $\cos 2\left(\omega_c t + \varphi(t)\right)$ and further the frequency divider arrangement gives the desired term of $\cos\left(\omega_c t + \varphi(t)\right)$.

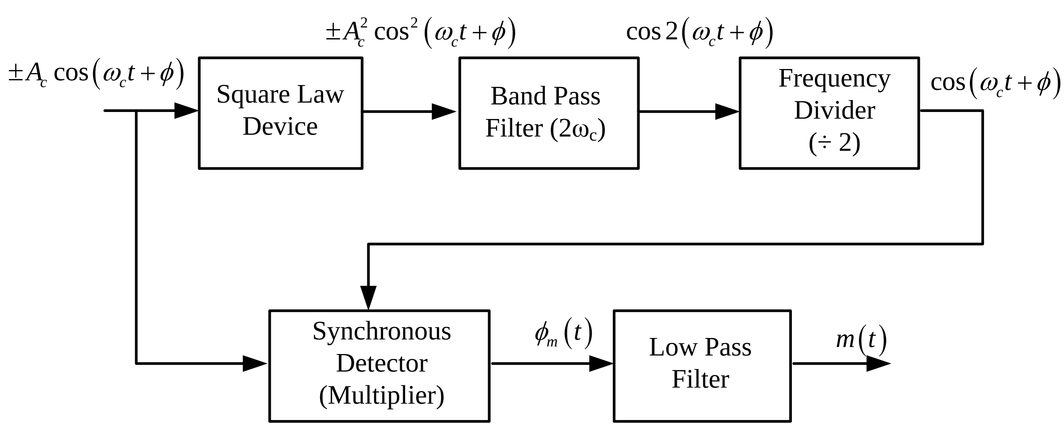


Fig. 8.12 BPSK demodulator

This synchronised carrier signal is multiplied with the BPSK modulated signal in a multiplier. The output of the multiplier is

$$\phi_m(t) = A_c\cos(\omega_c t + \varphi(t)) \times \cos(\omega_c t + \varphi(t))$$
$$= \frac{A_c}{2}\left[1 + \cos 2(\omega_c t + \varphi(t))\right] \quad (8.16)$$

The LPF separates the DC term of $\phi_m(t)$, i.e. $m(t) = \frac{A_c}{2}$. For easy separation of DC term, $\omega_c T >> \pi$.

Applications

1. Bio-metric or contactless operations
2. Wireless LANs,
3. RFID and Bluetooth communications

8.5 *M*-ary Encoding

For the reduction of bandwidth, a combination of more than or equal to two bits is used to transmit the information. This technique is called *M*-ary technique which is further categorised into the following types:

1. *M*-ary ASK
2. *M*-ary FSK
3. *M*-ary PSK

In this chapter, only *M*-ary PSK has been discussed.

8.5.1 *M*-ary PSK Modulation Techniques

In digital representation, if *N* bits are used to represent total *M* different numbers, then the relation between *N* and *M* is given by:

$$N = \log_2 M$$
$$\Rightarrow M = 2^N$$

where, *M* is the number of combinations possible with *N* bits.

Let *N* =3, then $M = 2^3 = 8$, i.e. total of 8 numbers (0 to 7) are represented by 3-bits.

Similarly, in *M*-ary PSK, *M* different combinations are used to represent *M* different symbols. The expression for *M*-ary PSK modulated signal is given as

$$\phi_{M-ary}(t) = A_c\cos(\omega_c t + \varphi) \quad (8.17)$$

where, φ is given as

$$\varphi_i = \frac{2\pi}{M}(i-1) + \text{constant} \quad (8.18)$$

where $i = 1, 2, ..., M$

If $M = 4$ and constant = 0, then four phases are

$$\varphi_{(i=1)} = \frac{2\pi}{4}(1-1)+0=0 \qquad \varphi_{(i=3)} = \frac{2\pi}{4}(3-1)+0=\pi$$

$$\varphi_{(i=2)} = \frac{2\pi}{4}(2-1)+0=\frac{\pi}{2} \qquad \varphi_{(i=3)} = \frac{2\pi}{4}(4-1)+0=\frac{3\pi}{2}$$

The above 4-ary PSK is called QPSK.

If $M=4$ and constant = $\pi/4$, then four phases are

$$\varphi_{(i=1)} = \frac{2\pi}{4}(1-1)+\frac{\pi}{4}=\frac{\pi}{4} \qquad \varphi_{(i=3)} = \frac{2\pi}{4}(3-1)+\frac{\pi}{4}=\frac{5\pi}{4}$$

$$\varphi_{(i=2)} = \frac{2\pi}{4}(2-1)+\frac{\pi}{4}=\frac{3\pi}{4} \qquad \varphi_{(i=3)} = \frac{2\pi}{4}(4-1)+\frac{\pi}{4}=\frac{7\pi}{4}$$

The above 4-ary PSK with π/4 phase is called π/4-QPSK.

Since a single signal uses multiple bits transmission (more than one bit), the required channel bandwidth is reduced.

8.5.2 Quadrature Phase Shift Keying (QPSK)

As already mentioned, the phase reversal for QPSK is 0, π/2, π, 3π/2. Similarly, the phase reversal for π/4-QPSK is - π/4, 3π/4, -3π/4, π/4. The bit combinations and the phase reversal for QPSK and π/4-QPSK are presented in Table 8.1 and Table 8.2, respectively.

Table 8.1 QPSK

S. No.	S_1	S_0	φ
1.	0	0	0
2.	0	1	π/2
3.	1	0	π
4.	1	1	3π/2

Table 8.2 π/4-QPSK

S. No.	S_0	S_1	φ
1.	0	0	- π/4
2.	0	1	3π/4
3.	1	0	-3π/4
4.	1	1	π/4

This technique is further extended for 8-ary PSK $(M=8 \Rightarrow N=3 \text{ bits})$ and 16-ary PSK $(M=16 \Rightarrow N=4 \text{ bits})$. So, QPSK is 4-ary PSK which sends the two bits at a time for transmission of a message. The advantage of 4-ary PSK or QPSK is that it reduces the data bit rate to half. The carrier signal and the outputs of QPSK and π/4-QPSK for message bit pattern '00111001' are shown in Fig. 8.13 and Fig. 8.14, respectively.

8.5.2.1 QPSK Modulator

The schematic diagram of the QPSK modulator is presented in Fig. 8.15. It consists of a bit splitter, a 2-bit serial to parallel converter, a local oscillator for carrier generation, two balanced modulators and a summer circuit.

The working principle of the QPSK modulator is as follows:

The baseband signal's odd bits (i.e., 1^{st} bit, 3^{rd} bit, etc.) and even bits (i.e., 2^{nd} bit, 4^{th} bit, etc.) are separated by the bits splitter at the modulator input. After that, the separated bits are multiplied with carrier signals to generate the odd BPSK and even BPSK. Finally, the odd BPSK bits and even BPSK bits are added in the summer to generate the QPSK output signal.

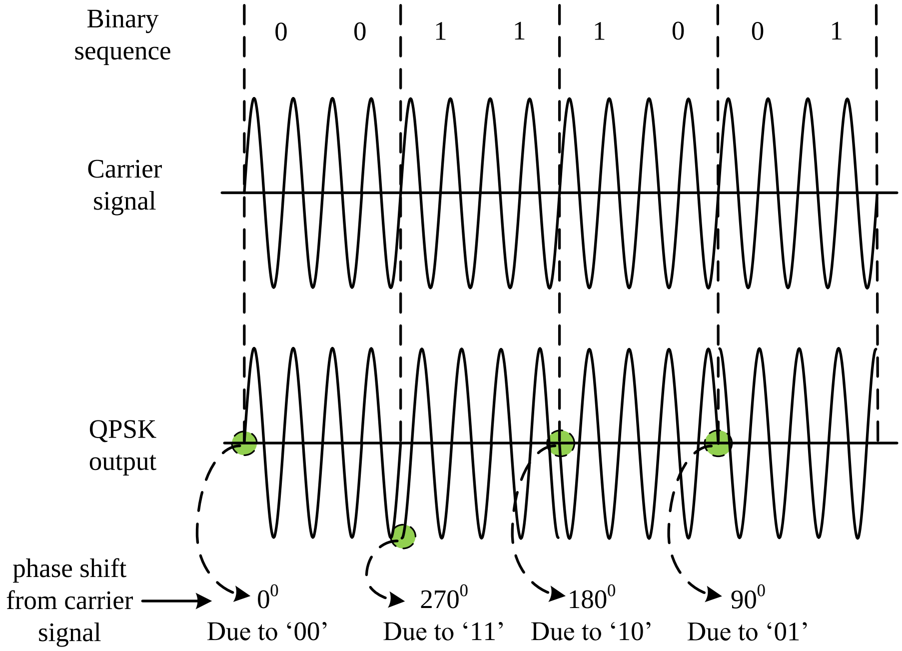


Fig. 8.13 Output waveform of QPSK (4-ary PSK)

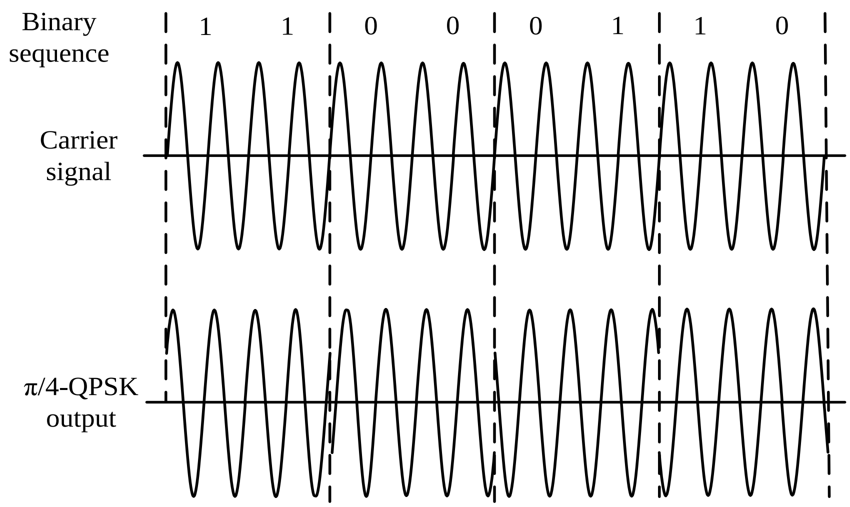


Fig. 8.14 Output waveform of π/4-QPSK

8.5.2.2 QPSK Demodulator

The synchronous detection method is used for the QPSK detector. The block diagram of the QPSK demodulator is shown in Fig. 8.16. It consists of two product modulators, one local oscillator, two BPF, two integrators or decision circuits and one 2-bit parallel to serial converter. The working operation of the QPSK demodulator is as follows:

The QPSK modulated signal is multiplied with the two carrier signals of in-phase and π/2-phase shift in balanced modulators. The multiplied signal is passed through the LPF to recover the original bits of the message. These bits are reshaped in pulses by the integrator or decision circuits. Finally, a 2-bit parallel to serial converter is used to rearrange the data to recover the original bitstream message.

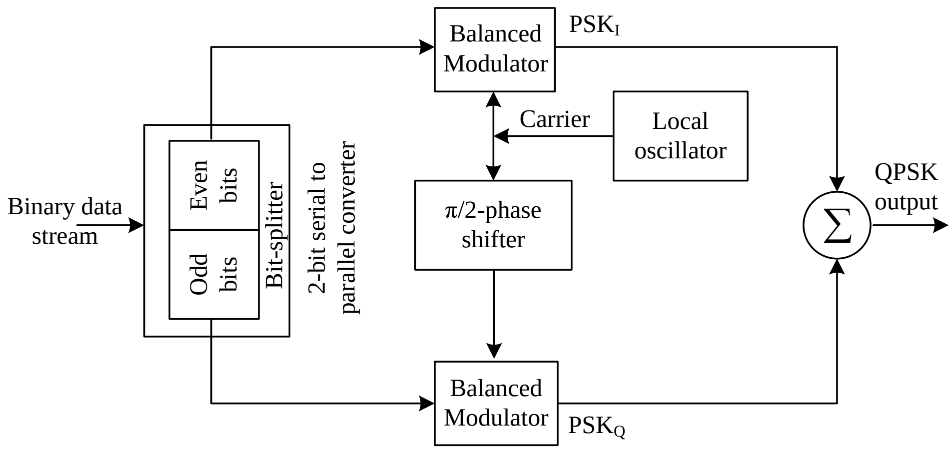


Fig. 8.15 QPSK modulator

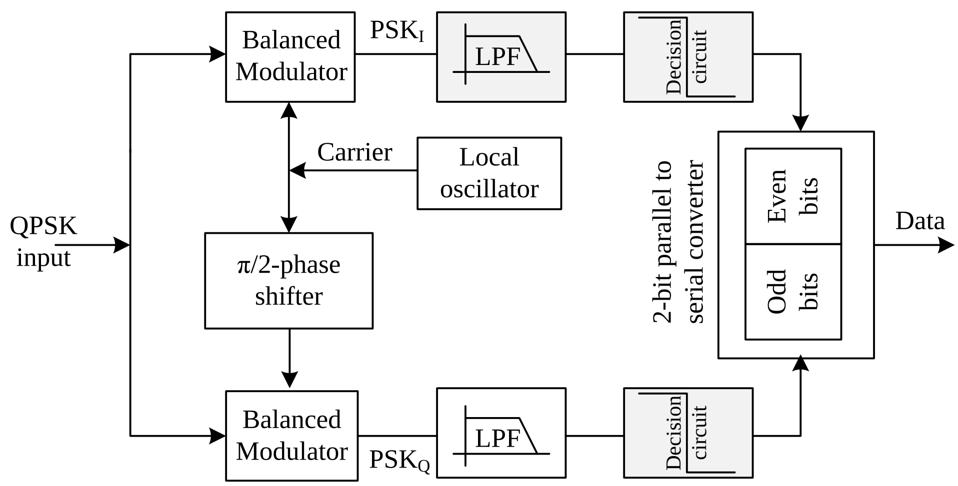


Fig. 8.16 QPSK demodulator

8.6 Differential Phase Shift Keying (DPSK)

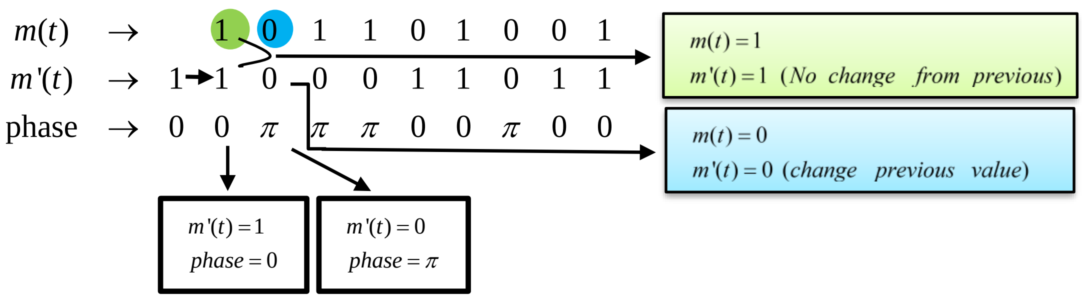
PSK uses synchronising circuit at the receiver end, which increases the complexity of the demodulator construction. Therefore, another approach that shifts the phase of the modulated signal relative to the previous informative bit to get rid of the complexity of the synchronising circuit is developed. This method is called differential phase-shift keying (DPSK). DPSK method doesn't need a reference oscillator.

Let $m(t)$ is message bitstream to be transmitted. One auxiliary message bit $m'(t)$ is generated from $m(t)$ by a logic circuit. It is assumed that the first bit is arbitrary (either '1' or '0'). The subsequent bit of $m'(t)$ is generated by a rule given below:

If $m(t) = '1'$, No change in $m'(t)$

If $m(t) = '0'$, $m'(t)$ changes from ‘1’ to ‘0’ or from ‘0’ to ‘1’.

If the arbitrary bit of $m'(t)$ is chosen as ‘1’, the DPSK operation for the bitstream $m(t) = '101101001'$ is given below:



Similarly, if the arbitrary bit of $m'(t)$ is chosen as ‘0’, then

$$\begin{array}{lccccccccccc} m(t) & \rightarrow & & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ m'(t) & \rightarrow & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ \text{phase} & \rightarrow & \pi & \pi & 0 & 0 & 0 & \pi & \pi & 0 & \pi & \pi \end{array} \tag{8.19}$$

The output diagram of the DPSK for $m(t) = '101101001'$ is given in Fig. 8.17.

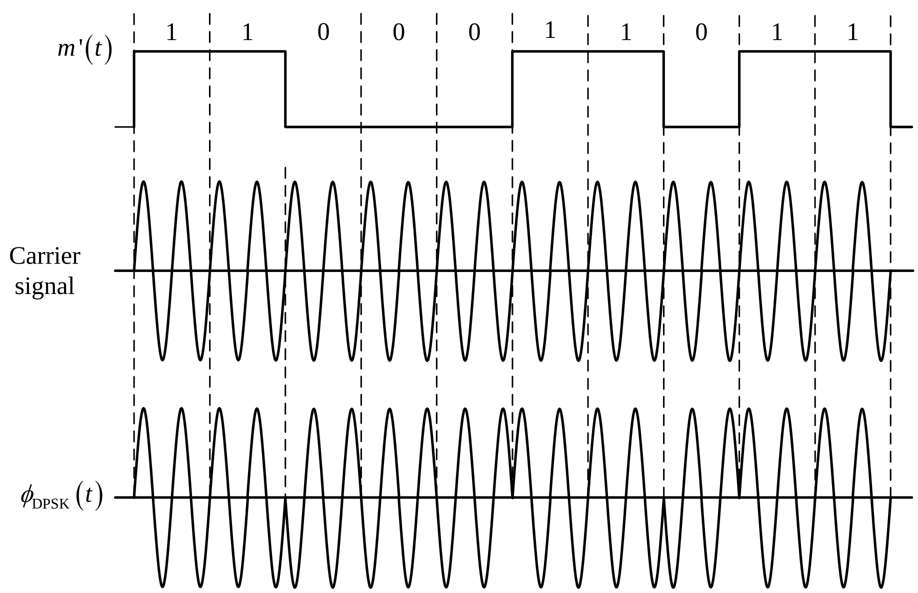


Fig. 8.17 Waveform of DPSK modulation

From the DPSK output diagram, it is clear that the output is reversed, i.e. π phase shift if the $m'(t) = '0'$ and no phase change occurs if $m'(t) = '1'$.

8.6.1 DPSK Modulator

The block diagram of the DPSK modulator is shown in Fig. 8.18(a). Since no reference signal is used in DPSK, therefore, the transmitted signal itself is used as a reference signal. The operation of the DPSK modulator is previously explained for both the arbitrary bits, either ‘1’ or ‘0’.

8.6.2 DPSK Demodulator

In the DPSK demodulator, the balanced modulator receives two inputs 1) DPSK signal and 2) 1-bit delay DPSK signal, as shown in Fig. 8.18(b). The message signal recovery waveforms by the DPSK demodulator is shown in Fig. 8.19. The bitstream obtained by the DPSK demodulator for arbitrary digit '1' is

$$m'(t) = '1100011011' \tag{8.20}$$

For similar input, the detected output is '1'; otherwise, it is '0'. Thus, the detected output bitstream is $'101101001'$ which is same as the binary sequence of the message signal. The LPF is used to reshape the signal in pulse shape. This signal is passed through the LPF and further applied to the comparator circuit to obtain the digital equivalent of the original message signal.

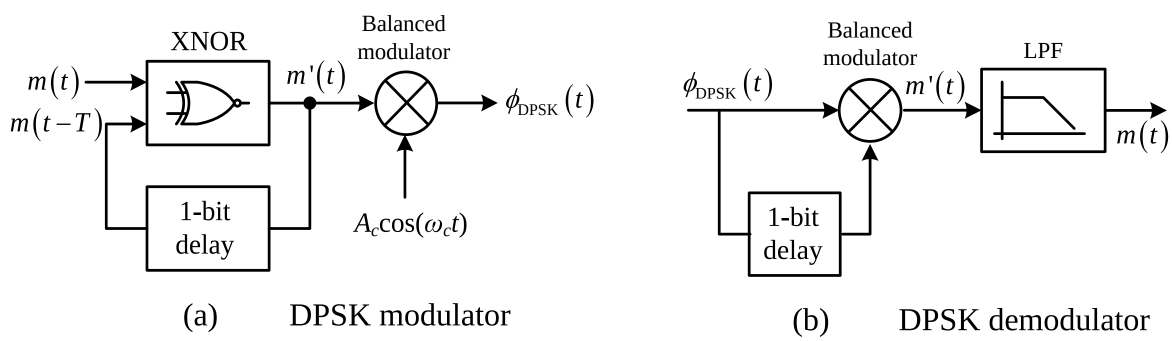


Fig. 8.18 (a) DPSK modulator (b) DPSK demodulator

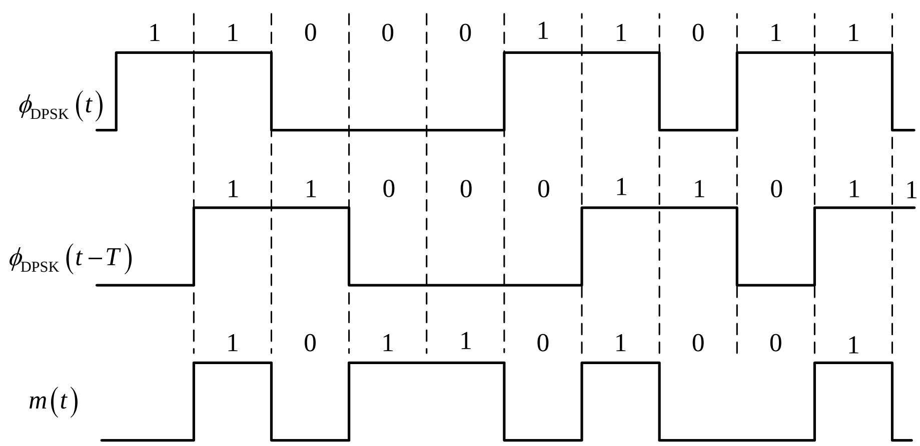


Fig. 8.19 DPSK demodulator output for modulated bitstream 1100011011

Advantages

1. Bandwidth requirement is low in comparison with BPSK modulation.
2. Construction of the receiver is simple as there is no need for synchronising circuit at the receiver end in DPSK.

Disadvantages

1. The bit error rate is higher than that of BPSK.
2. The noise interference is more in DPSK.

8.7 Baud Rate and Minimum Bandwidth

Baud rate is defined as the rate of signal transmission over the communication channel. To send the signal over the communication channel, the required minimum theoretical bandwidth is called Nyquist bandwidth.

Let R_b is the bit rate in bps and the bandwidth of the channel is B Hz, then the relationship between both parameters is given as

$$R_b = 2B \tag{8.21}$$

For example, if a communication medium has a bandwidth of 3 kHz, then it can transmit a maximum of 6 kbps through it. Another way to increase the transmission rate over the same bandwidth channel is using multilevel signaling, i.e. more than one bit is transmitted simultaneously over the channel.

Let *M*-ary encoding (*M*-combinations) is used for the communication; then the bit rate is given as

$$R_b = B\log_2 M \tag{8.22}$$

Since *M* combinations are represented by *N*-bits as

$$N = \log_2 M \tag{8.23}$$

So, $$R_b = B \times N \tag{8.24}$$

Therefore, the necessary bandwidth to pass the *M*-ary modulated carrier is

$$B = \frac{R_b}{N} \tag{8.25}$$

The baud rate is the bit rate divided by the number of bits required to encode one single element.

So, $$\text{Baud rate} = \frac{R_b}{N} \tag{8.26}$$

Further, bandwidth efficiency is defined as

$$\eta_{BW} = \frac{\text{Transmission bit rate (bps)}}{\text{Minimum bandwidth (Hz)}} = \frac{Bits/\sec}{Hertz} \tag{8.27}$$

8.7.1 Baud Rate and Bandwidth ASK

In ASK, there are only two levels either $A_c \cos(\omega_c t)$ or 0 (i.e. $M = 2$), So the required number of bits is $N = \log_2 2 = 1$

Therefore, the Baud rate is given as

$$\text{Baud rate} = \frac{R_b}{N} = \frac{R_b}{1} = R_b$$

Bandwidth is also given as

$$B = \frac{R_b}{1} = R_b$$

8.7.2 Baud Rate and Bandwidth for FSK

The Baud rate of FSK is obtained by putting $N=1$, i.e.

$$\text{Baud rate} = \frac{R_b}{N} = \frac{R_b}{1} = R_b$$

If the mark and the space frequencies of FSK are f_m and f_s, then the minimum bandwidth is

$$B = \left|\left(f_m + R_b\right) - \left(f_s - R_b\right)\right| = \left|\left(f_m - f_s\right)\right| + 2R_b$$

The baud rate and the bandwidth for few other popular modulation techniques are shown in Table 8.3.

Table 8.3 Baud rate and the bandwidth for a few popular modulations

S. N.	**Methods**	**Number of Symbols** (M)	**Bits/ symbol** $(N = \log_2 M)$	**Baud rate** (R_b/N)
1.	**BPSK**	2	1	R_b
2.	**QPSK**	4	2	$R_b/2$
3.	**8-PSK**	8	3	$R_b/3$
4.	**16-PSK**	16	4	$R_b/4$
5.	**8-QAM**	8	3	$R_b/3$
6.	**16-QAM**	16	4	$R_b/4$

8.8 Constellation Diagram

Constellation diagram is a scheme representation of digitally modulated signal such as PSK or QAM. In other words, the constellation diagram is a graphical representation way to relate the output symbols and bits of each symbol. The signal is represented by a two-dimensional *xy*-plane scatter diagram in the polar plane at the symbol sampling instant in the constellation diagram. The distance of the symbol sampling point from the origin is a measurement of the amplitude of the signal and the phase shift of the signal is measured by the angle from the horizontal axis in the counter clockwise direction. The constellation diagram of the 8-PSK is shown in Fig. 8.20.

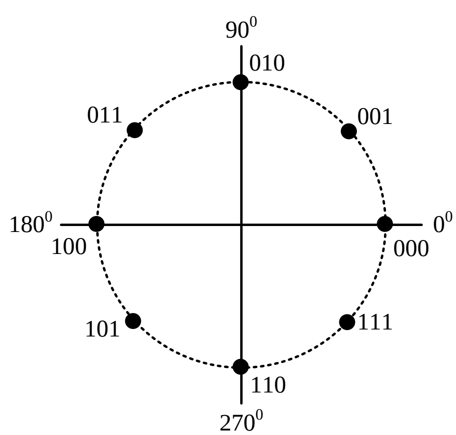


Fig. 8.20 Constellation diagram of the 8-PSK

The 8-PSK shows eight symbols to be transmitted and therefore, each symbol is represented by the combination of 3 bits.

The angle of each symbol instant is measured by the formula

$$\varphi_i = \frac{2\pi}{M}(i-1) + \text{const.} \tag{8.28}$$

For 8-PSK, $M = 8,\ i = (1, 2, ...8)$, constant = 0

QPSK (4-PSK) is four symbols with 2-bits each symbol PSK modulation technique, i.e., in QPSK, 4-message symbols are represented by the combination of 2-bits. Two possible constellation diagrams of the QPSK are shown in Fig. 8.21(a)-(b).

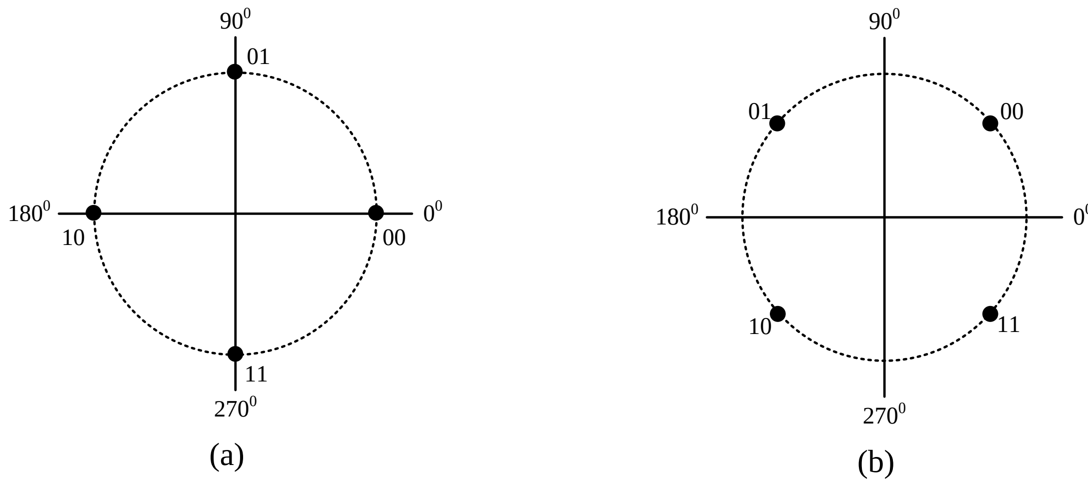


Fig. 8.21 Constellation diagrams of the (a) QPSK (b) π/4-QPSK

Similarly, the 16-PSK constellation diagram is shown in Fig. 8.22

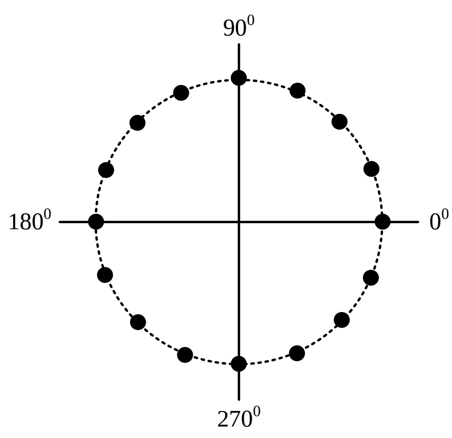


Fig. 8.22 Constellation diagram of 16-PSK

8.9 Baseband Signal Receiver

Any binary encoded signal is transmitted by a sequence of voltage level +*V* for '1' and 0 or –*V* for '0'. Any bit is represented by a signal for a certain time duration T_b, as shown in Fig. 8.23.

At the receiver end, the sample value is taken at a particular sample time $t = T_s$ in a bit interval. If the value is negative, the received bit is '0' and if it is positive, the received bit is '1'. Let bit '1' is transmitted, but at the sampling instant, a noise voltage of magnitude larger than *V* with opposite polarity is added, then the sampled value will be negative and an error will occur in the decision. A baseband receiver shown in Fig. 8.24 received the bit with duration of T_b.

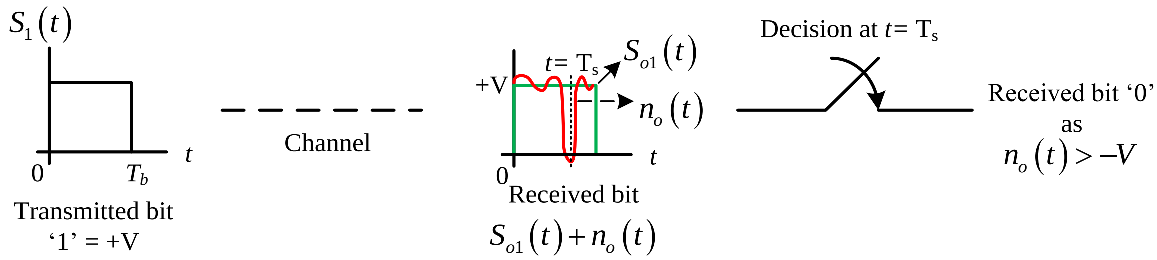


Fig. 8.23 Bit representation and effect of noise

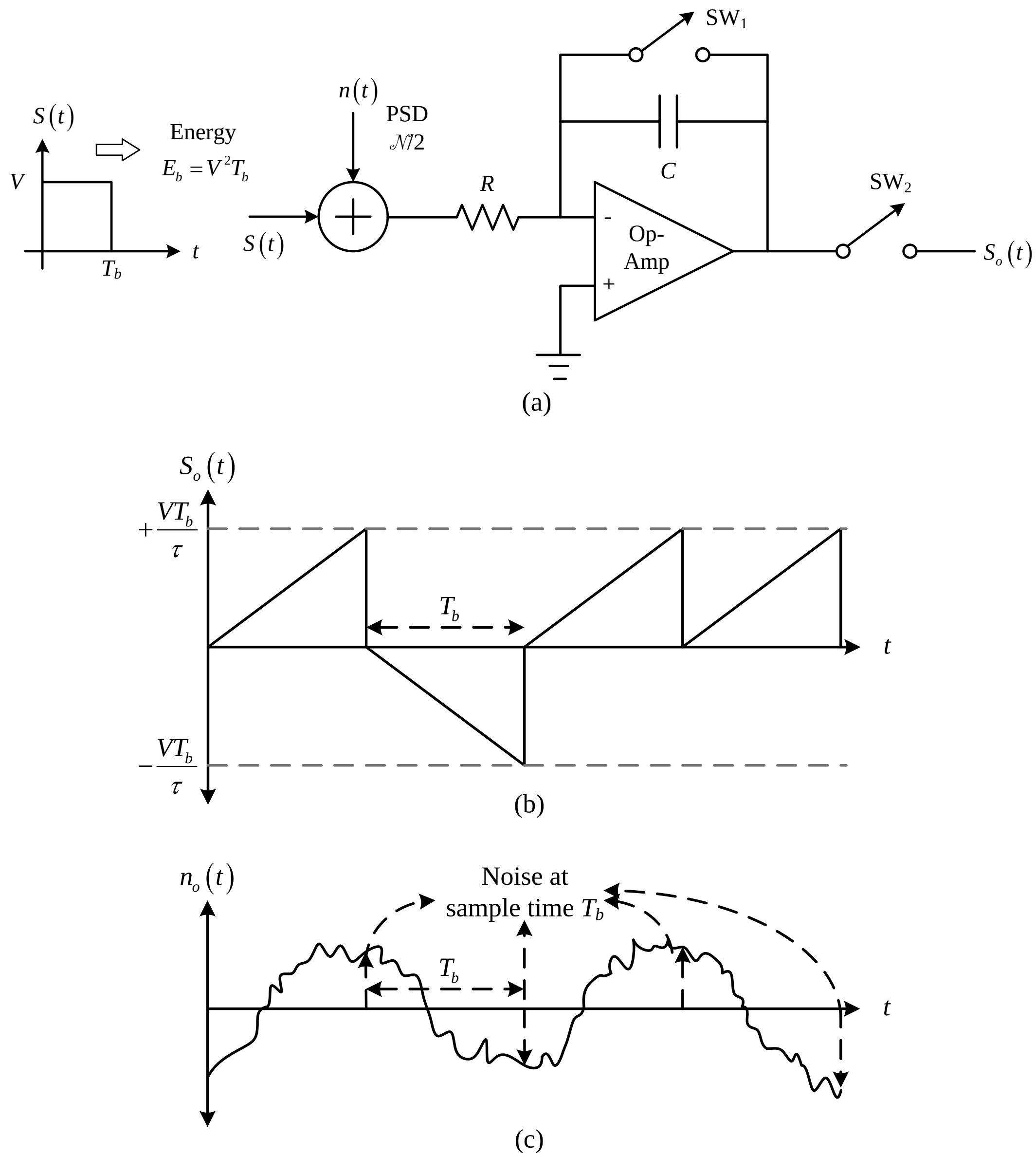


Fig. 8.24 (a) Baseband signal receiver (b) Output sampled signal voltage (c) Output sampled noise voltage

Let the time constant is $\tau = RC$

The output voltage level of the receiver is

$$V_o(t) = \frac{1}{\tau}\int_0^{T_b}\left[S(t)+n(t)\right]dt$$
$$= \underbrace{\frac{1}{\tau}\int_0^{T_b} S(t)\,dt}_{\text{Output signal component } S_o(t)} + \underbrace{\frac{1}{\tau}\int_0^{T_b} n(t)\,dt}_{\text{Output noise component } n_o(t)} \tag{8.29}$$

If the input voltage level is V volt, then output sampled signal component is

$$S_o(T_b) = \frac{1}{\tau}\int_0^{T_b} V dt = \frac{VT_b}{\tau} \tag{8.30}$$

The output signal voltage $S_o(t)$ is a ramp signal as shown in Fig. 8.24(b). The magnitude of the signal voltage is maximum at the time $t = T_b$, therefore, the sampling instant is selected at $t = T_b$ to maximize the SNR.

The output noise voltage sample is

$$n_o(T_b) = \frac{1}{\tau}\int_0^{T_b} n(t)\,dt \tag{8.31}$$

The variance of $n_o(T_b)$ is

$$\sigma_o^2 = \overline{n_o^2(T_b)} = \frac{\mathcal{N}T_b}{2\tau^2} \tag{8.32}$$

The signal to noise ratio at the output is

$$\left(\frac{S}{N}\right)_{output} = \frac{S_o}{N_o} = \frac{P_s}{P_n} = \frac{(VT_b/\tau)^2}{\mathcal{N}T_b/2\tau^2}$$

$$\Rightarrow \frac{S_o}{N_o} = \frac{2V^2T_b}{\mathcal{N}} \tag{8.33}$$

8.10 Probability of Error/ Bit Error Rate (BER)

Consider the two cases of an error

1. If the transmitted bit is ‘1’, then the output voltage level is $S_o(T_b) = \frac{VT_b}{\tau}$. The error will occur if $n_o(T_b) < -\frac{VT_b}{\tau}$ (Region 1 of Fig. 8.25).
2. If the transmitted bit is ‘0’, then the output voltage level is $S_o(T_b) = -\frac{VT}{\tau}$. The error will occur if $n_o(T_b) > \frac{VT_b}{\tau}$ (Region 2 of Fig. 8.25).

Both the cases are shown by the shaded area of Fig. 8.25.

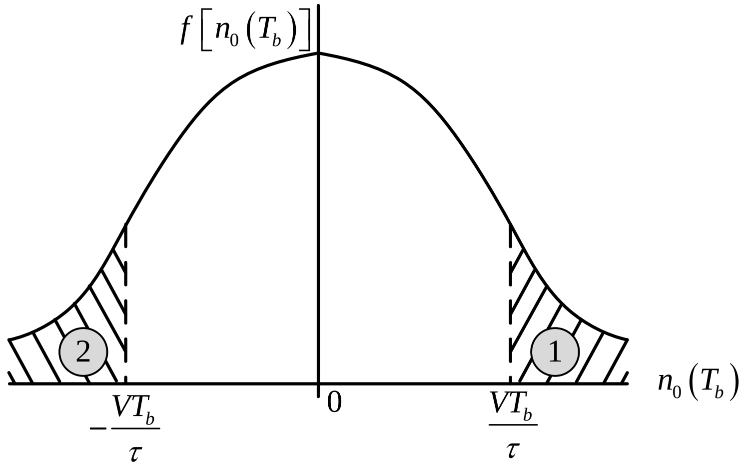


Fig. 8.25 Probabilty density of noise

Let the transmitted bit is '1', i.e. output signal voltage is $S_o\left(T_b\right)=\frac{VT_b}{\tau}$. So, the probability of error is obtained by the integration of the shaded region 1 as

$$P_e=\int_{\frac{VT_b}{\tau}}^{\infty} f\left[n_o\left(T_b\right)\right]dn_o\left(T_b\right) \tag{8.34}$$

As we know, the PSD of Gaussian noise is given as,

$$f\left[n_o\left(T_b\right)\right]=\frac{1}{\sqrt{2\pi\sigma_o^2}}e^{-n_o^2\left(T_b\right)/2\sigma_o^2} \tag{8.35}$$

So, $$P_e=\int_{\frac{VT_b}{\tau}}^{\infty}\frac{1}{\sqrt{2\pi\sigma_o^2}}e^{-\frac{n_o^2\left(T_b\right)}{2\sigma_o^2}}dn_o\left(T_b\right) \tag{8.36}$$

Let $z=\frac{n_o\left(T_b\right)}{\sqrt{2}\sigma_o}\Rightarrow dz=\frac{1}{\sqrt{2}\sigma_o}dn_o\left(T_b\right)$

Limit $n_o\left(T_b\right)\rightarrow\infty\Rightarrow z\rightarrow\infty;$ $n_o\left(T_b\right)\rightarrow\frac{VT_b}{\tau}\Rightarrow z\rightarrow V\sqrt{\frac{T_b}{\mathcal{N}}}$ (From Eq. (8.32))

Hence,

$$P_e=\int_{z=\left(V\sqrt{\frac{T_b}{\mathcal{N}}}\right)}^{\infty}\frac{1}{\sqrt{\pi}}e^{-z^2}dz\Rightarrow\frac{1}{2}\underbrace{\left(\frac{2}{\sqrt{\pi}}\int_{z=\left(V\sqrt{\frac{T_b}{\mathcal{N}}}\right)}^{\infty}e^{-z^2}dz\right)}_{erfc(z)}$$

$$\Rightarrow P_e=\frac{1}{2}erfc\left(V\sqrt{\frac{T_b}{\mathcal{N}}}\right)=\frac{1}{2}erfc\left(\frac{V^2T_b}{\mathcal{N}}\right)^{1/2}$$
$$\Rightarrow P_e=\frac{1}{2}erfc\left(\frac{E_b}{\mathcal{N}}\right)^{1/2} \tag{8.37}$$

where, E_b is the energy of transmitted bit pulse (Fig. 8.24). The maximum value of P_e is 0.5 and it decreases rapidly with increment in $E_b/\mathcal{N}$ as shown in Fig. 8.26. Moreover, if the entire signal is entirely lost in the noise, the receiver cannot be wrong more than half the time on average.

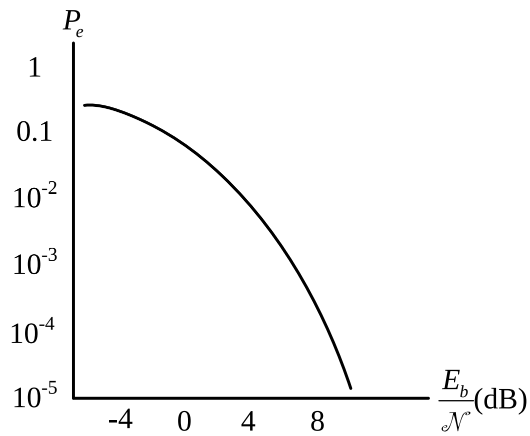


Fig. 8.26 Plot between P_e and $E_b/\mathcal{N}$

The performance of digital communication can be improved by reducing the bit error rate and hence by increasing the signal to noise ratio. The signal to noise ratio can be increased by the use of the matched filter.

8.11 The Optimum Filter

As previously explained, the performance of the baseband receiver could be improved by reducing the probability of error. Therefore, a filter is cast off, which gives the minimum probability of error. This filter is called the optimum filter.

Let the high and the low voltage levels of the transmitted signal has values as $S_1(t)$ and $S_2(t)$ respectively. Further, the corresponding sampled outputs are $S_{o1}(T_b)$ and $S_{o2}(T_b)$ respectively. In the absence of noise, transmitted and received information is equal (Fig. 8.27(a)) i.e.

$$S_{o1}(T_b) = S_1(T_b) \qquad \text{and} \qquad S_{o2}(T_b) = S_2(T_b)$$

In the presence of noise, the corresponding output voltage levels are

$$\begin{aligned} v_{o1}(T_b) &= S_{o1}(T_b) + n_o(T_b) \quad \text{for input } S_1(T_b) \quad \text{and} \\ v_{o2}(T) &= S_{o2}(T_b) + n_o(T_b) \quad \text{for input } S_2(T_b) \end{aligned} \tag{8.38}$$

The decision on transmitted signal depends upon whether the received signal is close to are $S_{o1}(T_b)$ or $S_{o2}(T_b)$ (Fig. 8.27(b)). Therefore, a threshold voltage level or decision boundary is decided and given as

$$v_{th}(t) = \frac{S_{o1}(T_b) + S_{o2}(T_b)}{2} \tag{8.39}$$

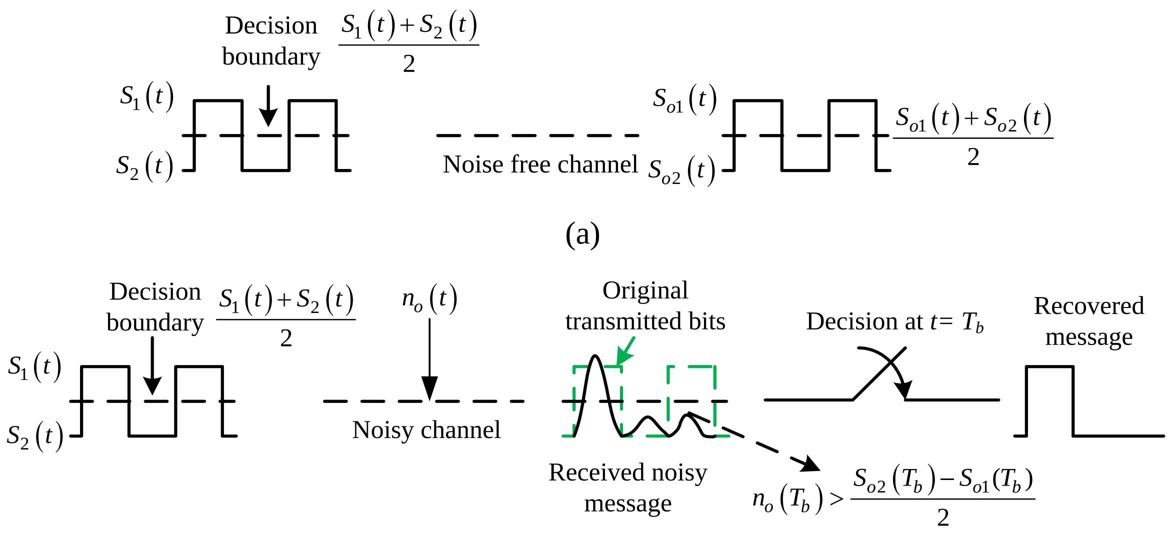


Fig. 8.27 (a) Transmitted and received messages for noise-free channel (b) Transmitted and received messages for noisy channel

Let $S_1(T_b)$ is transmitted, then there will be an error in received signal if noise is negative and such that

$$S_{o1}(T_b)+n_o(T_b)>\frac{S_{o1}(T_b)+S_{o2}(T_b)}{2}$$

$$n_o(T_b)>\frac{S_{o1}(T_b)+S_{o2}(T_b)}{2}-S_{o1}(T_b) \qquad (8.40)$$

$$n_o(T_b)>\frac{S_{o2}(T_b)-S_{o1}(T_b)}{2}$$

Similarly, if $S_2(T_b)$ is transmitted, then an error occurs if noise is negative and greater than

$$n_o(T_b)>\frac{S_{o1}(T_b)-S_{o2}(T_b)}{2} \qquad (8.41)$$

Hence, the probability of error is

$$P_e=\int_{\frac{S_{o1}(T_b)-S_{o2}(T_b)}{2}}^{\infty} f\left[n_o(T_b)\right]dn_o(T_b)$$

$$P_e=\int_{\frac{S_{o1}(T_b)-S_{o2}(T_b)}{2}}^{\infty}\frac{e^{-n_o^2(T_b)/2\sigma_o^2}}{\sqrt{2\pi}\sigma_o}dn_o(T_b)$$

Let, $z=\frac{n_o(T_b)}{\sqrt{2}\sigma_o}\Rightarrow dz=\frac{1}{\sqrt{2}\sigma_o}dn_o(T_b)$

$$P_e = \int_{\frac{S_{o1}(T_b)-S_{o2}(T_b)}{2\sqrt{2}\sigma_o}}^{\infty} \frac{1}{\sqrt{\pi}} e^{-z^2} dz = \frac{1}{2} \underbrace{\left(\frac{2}{\sqrt{\pi}} \int_{z=\frac{S_{o1}(T_b)-S_{o2}(T_b)}{2\sqrt{2}\sigma_o}}^{\infty} e^{-z^2} dz \right)}_{erfc(z)}$$

$$= \frac{1}{2} erfc\left[\frac{S_{o1}(T_b) - S_{o2}(T_b)}{2\sqrt{2}\sigma_o} \right] \tag{8.42}$$

So, P_e decreases with increment in $\frac{S_{o1}(T_b) - S_{o2}(T_b)}{2\sqrt{2}\sigma_o}$. Hence, an optimum filter is used to maximize the ratio

$$\gamma = \frac{S_{o1}(T_b) - S_{o2}(T_b)}{\sigma_o} = \frac{p(T_b)}{\sigma_o} \textbf{ or } \gamma^2 = \frac{p^2(T_b)}{\sigma_0^2} \tag{8.43}$$

Therefore, the error probability is given as

$$P_e = \frac{1}{2} erfc\left[\frac{S_{o1}(T_b) - S_{o2}(T_b)}{2\sqrt{2}\sigma_o} \right] = \frac{1}{2} erfc\left[\frac{\left(S_{o1}(T_b) - S_{o2}(T_b)\right)^2}{8\sigma_o^2} \right]^{1/2}$$

$$\Rightarrow P_e = \frac{1}{2} erfc\left[\frac{1}{8}\gamma^2 \right]^{1/2} \tag{8.44}$$

The minimum value of error probability is obtained when γ has its maximum value. So,

$$(P_e)_{\min} = \frac{1}{2} erfc\left[\frac{1}{8}\gamma_{\max}^2 \right]^{1/2} \tag{8.45}$$

The Eq. (8.45) is very important to find the error probabilities of ASK, FSK and PSK.

If the input noise is white noise, i.e., PSD of noise is $\mathcal{N}/2$, the optimum filter is called a matched filter which is explained in the next section

8.12 Matched Filter

In digital communication, the transmitted bits are either '0' or '1'. Any pulse obtained for a duration of T_b is assumed as a bit, as shown in Fig 8.28.

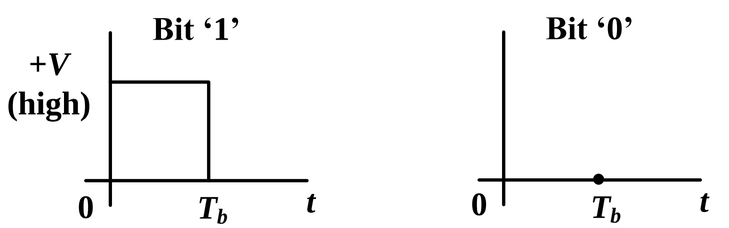


Fig. 8.28 Representation of bits '1' and '0'

If this signal is transmitted over a noise-free channel, the received bit will be the same as the transmitted bit. In another case, if the channel is noisy, then the transmitted bit is distorted by the noise and the correctness of the received bit depends upon the value of the noise signal.

Let the transmitted bit is ‘1’; this bit is a $+V$ magnitude with T_b duration pulse signal. The channel is noisy; therefore, additive noise $n_o(t)$ distorts the received bit. This received bit is sampled at the time instant $t = T_s$. Usually, $T_s = T_b$ to increase the SNR and further, this signal is compared to make the decision for the received signal through the comparator. The other input of the comparator is a signal with a decision boundary $\left(\frac{V+0}{2} = \frac{V}{2}\right)$. As shown in Fig 8.29(a), the noise has a magnitude less than $V/2$ at sampling instant; therefore, the received bit is the same as a transmitted bit ‘1’.

In another example, if the noise has a magnitude greater than $V/2$, the received signal is ‘0’ with an error in detection.

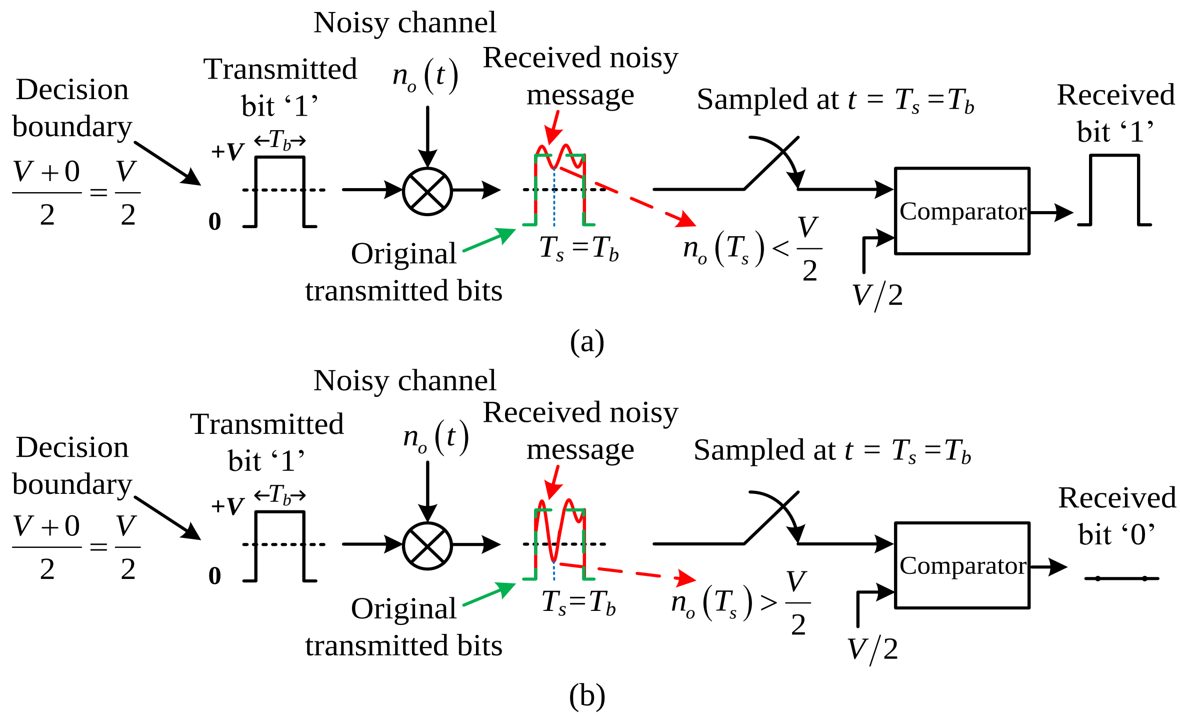


Fig. 8.29 Received bit for the magnitude of noise (a) less than the threshold (b) greater than the threshold

Therefore, a filter is used to improve the signal to noise ratio (SNR) in the presence of additive noise. This filter is called a matched filter or optimal linear filter. The block diagram of the matched filter with its application in the receiver is shown in Fig. 8.30.

The input of the matched filter is message signal with additive noise, i.e. noisy message as

$$m_n(t) = m(t) + n_o(t) \tag{8.46}$$

Let the impulse response of the matched filter is $h(t)$. Therefore, the output of the matched filter is

$$m_m(t) = m_n(t) * h(t) = \left(m(t) + n_o(t)\right) * h(t)$$
$$= \underbrace{m(t) * h(t)}_{\text{Message component}} + \underbrace{n_o(t) * h(t)}_{\text{Noise component}} \qquad (8.47)$$

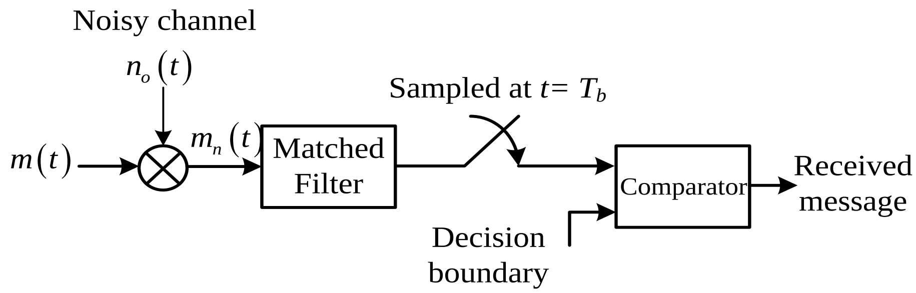


Fig. 8.30 The block diagram of the matched filter with its application in the receiver

Let the Fourier transforms of all signals are given as

$$m(t) \xrightarrow{\text{FT}} M(\omega)$$
$$n_o(t) \xrightarrow{\text{FT}} N_o(\omega) \qquad (8.48)$$
$$h(t) \xrightarrow{\text{FT}} H(\omega)$$

Since the matched filter is used to improve the SNR; therefore we calculate the signal power P_s and noise power P_n individually.

Noise is additive in nature with PSD of $\mathcal{N}/2$. So the output noise power is

$$P_n = \frac{\mathcal{N}}{2} \int_{-\infty}^{\infty} \left|H(\omega)\right|^2 df \qquad (8.49)$$

The signal component is the

$$m_s(t) = m(t) * h(t) \xrightarrow{\text{FT}} M_s(\omega) = M(\omega).H(\omega) \qquad (8.50)$$

The signal component is obtained by inverse Fourier transform as

$$m_s(t) = \int_{-\infty}^{\infty} M(\omega).H(\omega) e^{j2\pi ft} df \qquad (8.51)$$

The value of the signal at sampling instant $t = T_b$ is

$$m_s(T_b) = \int_{-\infty}^{\infty} M(\omega).H(\omega) e^{j2\pi fT_b} df \qquad (8.52)$$

This is a constant value. So, the signal power is given as

$$P_s = \left|m_s(T_s)\right|^2 = \left|\int_{-\infty}^{\infty} M(\omega).H(\omega) e^{j2\pi fT_b} df\right|^2 \qquad (8.53)$$

Hence, SNR is given as

$$SNR = \frac{P_s}{P_n} = \frac{\left|\int_{-\infty}^{\infty} M(\omega).H(\omega) e^{j2\pi fT_b} df\right|^2}{\frac{\mathcal{N}}{2} \int_{-\infty}^{\infty} \left|H(\omega)\right|^2 df} \qquad (8.54)$$

According to Schwartz inequality

$$\left|\int_{-\infty}^{\infty} x(t).y(t)dt\right|^2 \le \int_{-\infty}^{\infty} |x(t)|^2 dt.\int_{-\infty}^{\infty} |y(t)|^2 dt \tag{8.55}$$

if $\quad y(t) = x^*(t) \quad$ (Conjugate to one another)

So,

$$SNR = \frac{P_s}{P_n} = \frac{\left|\int_{-\infty}^{\infty} M(\omega).H(\omega)e^{j2\pi fT_b}df\right|^2}{\frac{\mathcal{N}}{2}\int_{-\infty}^{\infty}|H(\omega)|^2 df} \le \frac{\int_{-\infty}^{\infty}\left|M(\omega)e^{j2\pi fT_b}\right|^2 df.\int_{-\infty}^{\infty}|H(\omega)|^2 df}{\frac{\mathcal{N}}{2}\int_{-\infty}^{\infty}|H(\omega)|^2 df}$$

$$\Rightarrow SNR \le \frac{2}{\mathcal{N}}\underbrace{\int_{-\infty}^{\infty}|M(\omega)|^2 df}_{\text{Energy of the signal}} \tag{8.56}$$

Therefore, maximum SNR is

$$(SNR)_{\max} = \frac{2E_b}{\mathcal{N}} \qquad \left(E_b = \text{Bit energy} = \int_{-\infty}^{\infty}|M(\omega)|^2 df\right) \tag{8.57}$$

So, the impulse response of the matched filter is

$$H(\omega) = \text{conj}\left(M(\omega)e^{j2\pi fT_b}\right) = M^*(\omega)e^{-j2\pi fT_b} \tag{8.58}$$

The impulse response in the time domain is obtained by inverse Fourier transform of Eq. (8.58)

$$h(t) = \int_{-\infty}^{\infty} H(\omega)e^{j2\pi ft}df \tag{8.59}$$

Substituting the value of $H(f)$ from Eq. (8.58) into Eq. (8.59)

$$h(t) = \int_{-\infty}^{\infty} M^*(\omega)e^{j2\pi ft}e^{-j2\pi fT_b}df \tag{8.60}$$

$$\Rightarrow h(t) = \int_{-\infty}^{\infty} M^*(\omega)e^{-j2\pi f(T_b-t)}df = \left[\underbrace{\int_{-\infty}^{\infty} M(\omega)e^{j2\pi f(T_b-t)}df}_{\text{Fourier representation of } m(T_b-t)}\right]^* \tag{8.61}$$

$$\Rightarrow h(t) = \left[m(T_b-t)\right]^*$$

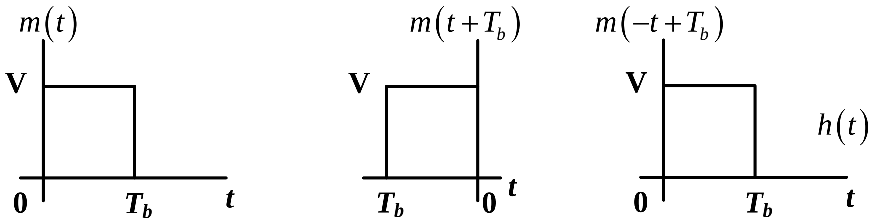
Therefore, the impulse response of the matched filter is shifted version of the message signal but similar in shape. Therefore, the filter is named as a matched filter.

It is concluded that

(i) $(SNR)_{\max} = \frac{2E_b}{\mathcal{N}}$

(ii) $h(t) = \left[M(T_b-t)\right]^*$

Let $m(t)$ is a pulse with time duration of T_b, then the transfer function $h(t)$ of matched filter is shown in figure below



If the conjugate term is considered, $m^*(-t+T_b)$ is equal to $h(t)$ which is similar in shape to input bit. Therefore, the filter is named as matched filter.

8.13 Probability of Error in ASK

As we know, the transmitted signal in ASK modulation is given by

$\phi_{\text{ASK},1}(t) = A_c \cos(\omega_c t)$ (if '1' is transmitted) and

$\phi_{\text{ASK},0}(t) = 0$ (if '0' is transmitted) (8.62)

The difference of both bits is

$$p(t) = \phi_{\text{ASK},1}(t) - \phi_{\text{ASK},0}(t) = A_c \cos(\omega_c t) \tag{8.63}$$

The ratio is

$$\begin{aligned}
\gamma_{\max}^2 &= \frac{2}{\mathcal{N}} \int_0^{T_b} p^2(t)\,dt = \frac{2}{\mathcal{N}} \int_0^{T_b} A_c^2 \cos^2(\omega_c t)\,dt \\
&= \frac{2A_c^2}{\mathcal{N}} \int_0^{T_b} \frac{1+\cos(2\omega_c t)}{2}\,dt \\
&= \frac{2A_c^2}{\mathcal{N}} \left[\frac{t}{2} + \frac{\sin(2\omega_c t)}{4\omega_c} \right]_0^{T_b} \\
&= \frac{2A_c^2}{\mathcal{N}} \left[\frac{T}{2} + \frac{\sin(2\omega_c T_b)}{4\omega_c} \right]
\end{aligned}$$

Since, ω_c is very high, therefore, $\frac{\sin(2\omega_c T_b)}{4\omega_c} << 1$, Hence

$$\gamma_{\max}^2 = \frac{A_c^2 T_b}{\mathcal{N}} \tag{8.64}$$

Hence, the probability of error in ASK is given by

$$\begin{aligned}
P_e &= \frac{1}{2} erfc\left[\frac{1}{8} \gamma_{\max}^2 \right]^{1/2} \\
&= \frac{1}{2} erfc\left[\frac{A_c^2 T_b}{8\mathcal{N}} \right]^{1/2}
\end{aligned} \tag{8.65}$$

The energy of the signal is given by

$$E_b = \frac{A_c^2}{2} T_b$$

So, the error probability in terms of energy is given as

$$\Rightarrow P_e = \frac{1}{2} erfc\left[\frac{E_b}{4\mathcal{N}}\right]^{1/2} \qquad (8.66)$$

8.14 Probability of Error in FSK

The modulated signal in FSK is represented as

$$\phi_{\text{FSK},1}(t) = A_c \cos(\omega_c + \varphi)t \qquad \text{(if '1' is transmitted)} \qquad \text{and}$$

$$\phi_{\text{FSK},0}(t) = A_c \cos(\omega_c - \varphi)t \qquad \text{(if '0' is transmitted)} \qquad (8.67)$$

So, $p(t) = \phi_{\text{FSK},1}(t) - \phi_{\text{FSK},0}(t)$

$$= A_c\left[\cos(\omega_c + \varphi)t - \cos(\omega_c - \varphi)t\right] \qquad (8.68)$$

The ratio is given by

$$\gamma_{\max}^2 = \frac{2}{\mathcal{N}} \int_0^{T_b} \left[p(t)\right]^2 dt$$

$$= \frac{2}{\mathcal{N}} \int_0^{T_b} A_c^2 \left[\cos(\omega_c + \varphi)t - \cos(\omega_c - \varphi)t\right]^2 dt$$

$$= \frac{2A_c^2 T_b}{\mathcal{N}}\left[1 - \frac{\sin 2\varphi T_b}{2\varphi T_b} + \frac{1}{2}\frac{\sin\left[2(\omega_c + \varphi)T_b\right]}{2(\omega_c + \varphi)T_b} - \frac{1}{2}\frac{\sin\left[2(\omega_c - \varphi)T_b\right]}{2(\omega_c - \varphi)T_b} - \frac{\sin 2\omega_c T_b}{2\omega_c T_b}\right] \qquad (8.69)$$

Consider the following assumption for bit separation, i.e., ω_c is very high

$$\omega_c T_b >> 1 \qquad \text{and} \qquad \omega_c >> \varphi$$

Therefore, the terms with ω_c as a denominator term is much less and can be neglected. Hence,

$$\gamma_{\max}^2 = \frac{2A_c^2 T_b}{\mathcal{N}}\left[1 - \frac{\sin 2\varphi T_b}{2\varphi T_b}\right] \qquad (8.70)$$

The maximum value of γ, i.e. $\gamma_{\max}$ is achieved when $\frac{\sin 2\varphi T_b}{2\varphi T_b}$ is minimum. Therefore,

$$\left(\sin 2\varphi T_b\right)_{\min} = -1 = \sin\left((2n+1)\frac{\pi}{2}\right) \quad n = 1, 3, ...$$

$$2\varphi T_b = \frac{3\pi}{2} \qquad (8.71)$$

Substituting the value of $2\varphi T_b$

$$\gamma_{\max}^2 = \frac{2A_c^2 T_b}{\mathcal{N}}\left[1 - \frac{-1}{3\pi/2}\right]$$

$$= \frac{2A_c^2 T_b}{\mathcal{N}}\left[1 + \frac{2}{3\pi}\right] = \frac{2.42 A_c^2 T_b}{\mathcal{N}} \qquad (8.72)$$

Hence, the probability of error is given by

$$P_e = \frac{1}{2} erfc\left[\frac{1}{8}\gamma_{\max}^2\right]^{1/2}$$

$$= \frac{1}{2} erfc\left[\frac{1}{8}\times\frac{2.42A_c^2T_b}{\mathcal{N}}\right]^{1/2} = \frac{1}{2} erfc\left[0.3\frac{A_c^2T_b}{\mathcal{N}}\right]^{1/2} \quad (8.73)$$

$$\Rightarrow P_e = \frac{1}{2} erfc\left[0.6\frac{E_b}{\mathcal{N}}\right]^{1/2} \quad (8.74)$$

8.15 Probability of Error in PSK

The modulated signal in PSK is represented as

$$\phi_{\text{PSK},1}(t) = A_c\cos(\omega_c t) \quad \text{(if '1' is transmitted)} \qquad \text{and}$$

$$\phi_{\text{PSK},0}(t) = -A_c\cos(\omega_c t) \quad \text{(if '0' is transmitted)} \quad (8.75)$$

The difference of both bits is

$$p(t) = \phi_{\text{PSK},1}(t) - \phi_{\text{PSK},0}(t) = 2A_c\cos(\omega_c t) \quad (8.76)$$

The ratio is

$$\gamma_{\max}^2 = \frac{2}{\mathcal{N}}\int_0^{T_b} p^2(t)\,dt = \frac{2}{\mathcal{N}}\int_0^{T_b} 4A_c^2\cos^2(\omega_c t)\,dt$$

$$= \frac{8A_c^2}{\mathcal{N}}\int_0^{T_b}\frac{1+\cos(2\omega_c t)}{2}\,dt$$

$$= \frac{8A_c^2}{\mathcal{N}}\left[\frac{T_b}{2}+\frac{\sin(2\omega_c T_b)}{4\omega_c}\right] \quad (8.77)$$

Since, ω_c is very high, therefore, $\frac{\sin(2\omega_c T_b)}{4\omega_c} << 1$, Hence

$$\gamma_{\max}^2 = \frac{4A_c^2T_b}{\mathcal{N}} \quad (8.78)$$

Hence, the probability of error is given by

$$P_e = \frac{1}{2} erfc\left[\frac{1}{8}\gamma_{\max}^2\right]^{1/2} = \frac{1}{2} erfc\left[\frac{A_c^2T_b}{2\mathcal{N}}\right]^{1/2} \quad (8.79)$$

So, the error probability in terms of energy is given as

$$\Rightarrow P_e = \frac{1}{2} erfc\left[\frac{E_b}{\mathcal{N}}\right]^{1/2} \quad (8.80)$$

8.16 Probability of Error in DPSK

The probability of error is given by

$$P_e = \frac{1}{2} e^{-E_b/\mathcal{N}} \quad (8.81)$$

A plot of error probabilities is shown in Fig. 8.31.

P_e curves for ASK,FSK, PSK and DPSK

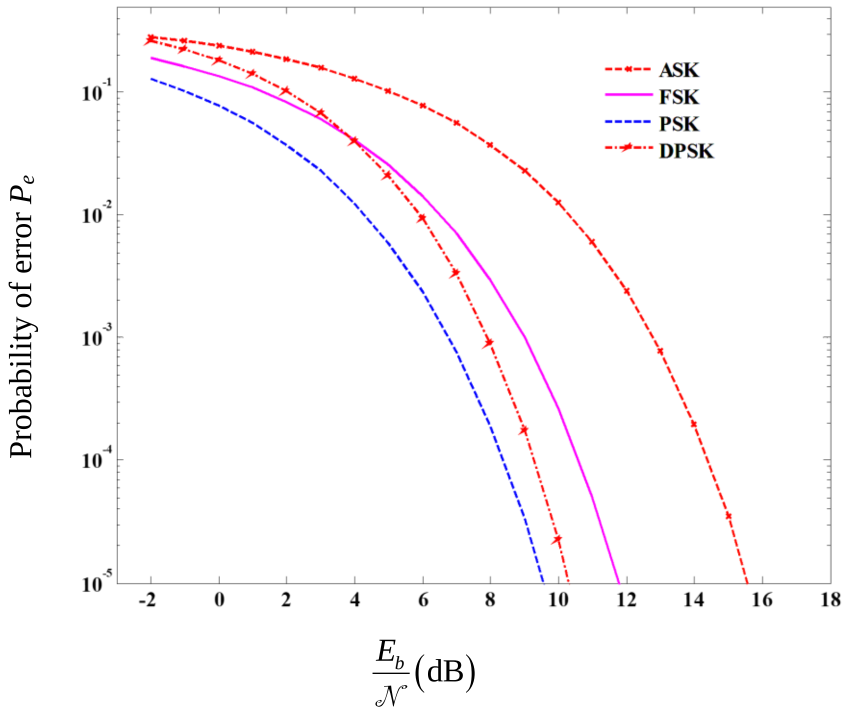
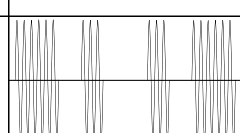
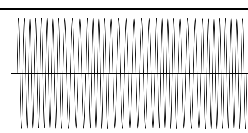
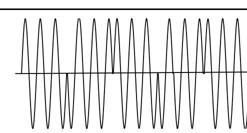


Fig. 8.31 The probabilities of errors

8.17 ASK vs. FSK vs. PSK

A comparison between ASK, FSK and PSK based on different constraints are shown in Table 8.4.

Table 8.4 Comparison between ASK, FSK and PSK

S. N.	Parameters	ASK	FSK	PSK
1.	**Variable characteristics**	Amplitude	Frequency	Phase
2.	**Complexity**	Simple	Moderate	Very complex
3.	**BER**	High	Low	Very low
4.	**Noise immunity**	Poor	Better than ASK	Better than FSK
5.	**Bit rate**	<100 bits/sec	< 1200 bits/sec	high bit rates
6.	**Applications**	Optical fibre	Radio transmission	Wireless LANs and Bluetooth
7.	**Waveform**			

ADDITIONAL SOLVED EXAMPLES

SE8.1 What type of modulation is shown in Fig. 8.32?

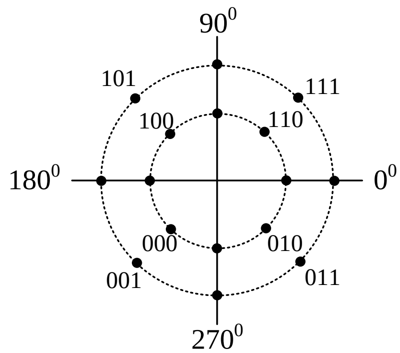


Fig. 8.32 Figure of problem SE8.1

Sol: In the given constellation diagram, there are eight symbols with four possible phases and two possible amplitudes. Such type of modulation scheme is called 8-quadrature amplitude modulation (8-QAM). In this scheme, both the amplitude and phase of the carrier signal is modulated. The eight possible output symbols might be $2\cos\left(\omega_c t \pm 45^o\right)$, $4\cos\left(\omega_c t \pm 45^o\right)$, $2\cos\left(\omega_c t \pm 135^o\right)$ $4\cos\left(\omega_c t \pm 135^o\right)$. The eight symbols are represented by the combination of the 3-bit pattern.

SE8.2 For an 8-PSK system operating with an information bit rate of 30 kbps, determine

a. Baud rate,

b. minimum bandwidth,

c. bandwidth efficiency

Sol: Given data:

For 8-PSK, the number of bits/symbol $2^N = 8 \Rightarrow N = 3$

Bit rate, $R_b = 30$ kbps = 30000

a. Baud rate $R_{baud} = \dfrac{R_b}{N} = \dfrac{30000}{3} = 10000$ symbols/sec

b. Minimum bandwidth $R_{baud} = \dfrac{R_b}{N} = \dfrac{30000}{3} = 10000$

c. Bandwidth efficiency $= \dfrac{\text{Transmission bit rate}}{\text{minimum bandwidth}}$

$$= \frac{30000}{10000} = 3 \text{ bits per second per cycle of bandwidth}$$

SE8.3 Using the signal constellation shown in Fig. 8.33, answer the following questions.

(a) What type of modulation does this represent?

(b) How many symbols are represented (*M*)?

(c) How many bits per symbol are used (*N*)?

(d) If the Baud is 10,000 symbols/second, what is the bit rate (R_b)?

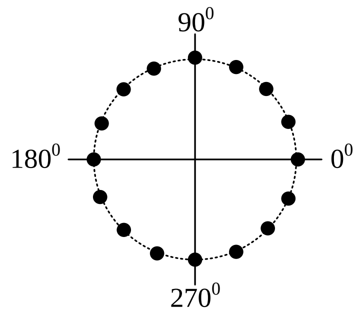


Fig. 8.33 Figure of problem SE8.3

Sol: In the given constellation diagram, there are 16 symbols with different phases. The amplitude of each symbol is the same. Therefore,

(a) This is 16-PSK modulation

(b) There are 16 symbols, i.e. $M = 16$.

(c) Bits/ symbol is N which is given as

$$2^N = 16$$
$$\Rightarrow N = 4$$

(d) Bit rate $R_b = N$ x Baud rate = 4 x 10000 = 40000 bps = 40 kbps

SE8.4 Determine the baud and minimum bandwidth necessary to pass a 20 kbps binary signal using amplitude shift keying.

Sol: The baud rate is the transmission of symbols per second. It is related to the bit rate as

$$R_{baud} = \frac{R_b}{N}$$

In binary ASK, number of bits $N = 1$;

So, $R_{baud} = \dfrac{20000}{1} = 20000$ symbols/sec

The minimum bandwidth is $(\text{BW})_{\min} = 10000\,\text{Hz}$.

SE8.5 What types of modulation are shown in Figs. 8.34(a)-(b)? Draw the modulated waveform for the message signal with pattern 11000110 using these modulation techniques

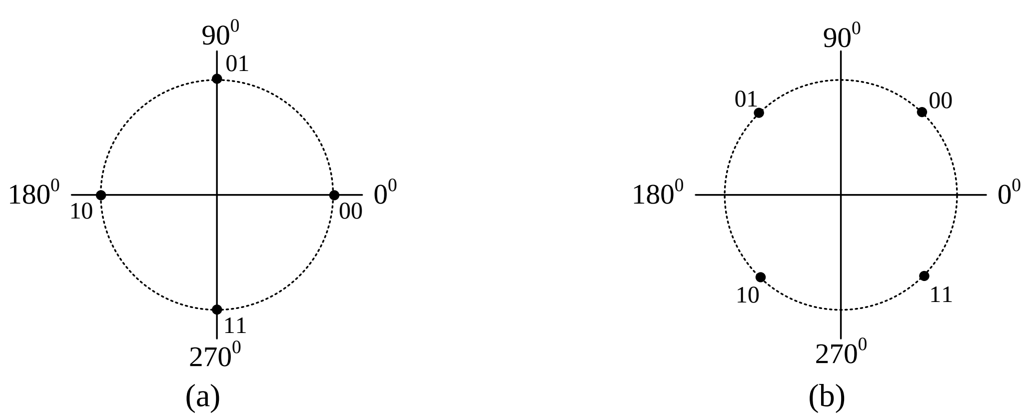
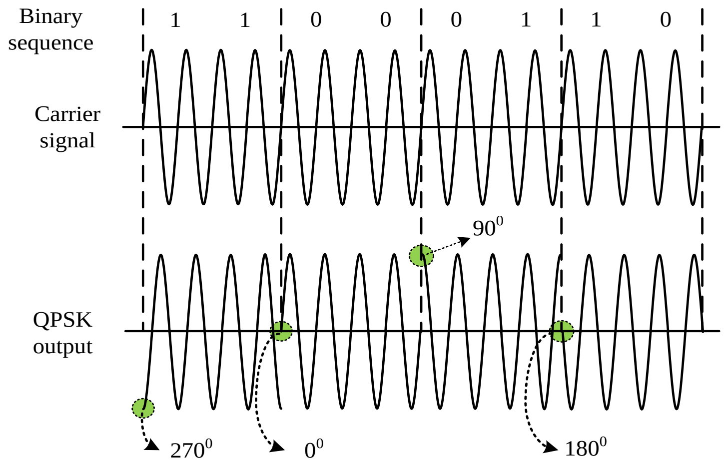


Fig. 8.34 Figure of problem SE8.3

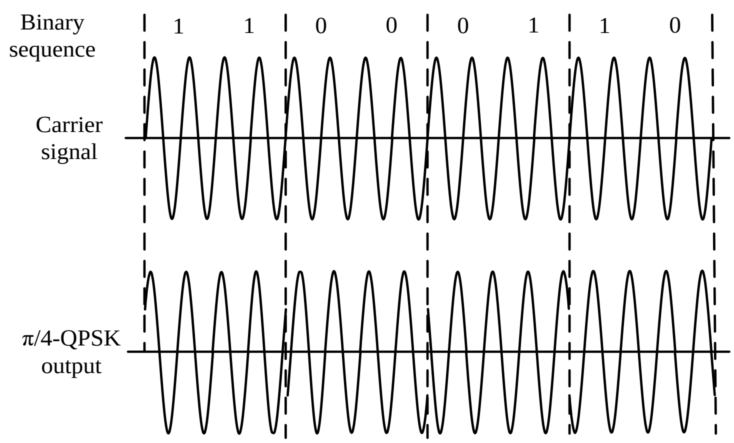
Sol: (a) In the given constellation diagram, there are four symbols with different phases and fixed amplitude. Therefore, this is the 4-PSK or QPSK modulation. The phases for each symbol is given in the table below.

S. No.	S_1	S_0	ϕ
1.	0	0	0
2.	0	1	$\pi/2$
3.	1	0	π
4.	1	1	$3\pi/2$



(b). In the given constellation diagram, there are four symbols with different phases with $\pi/4$ phase shift and fixed amplitude. Therefore, this is the $\pi/4$-QPSK modulation. The phases for each symbol is given in the table below.

S. No.	S_1	S_0	ϕ
1.	0	0	$-\pi/4$
2.	0	1	$3\pi/4$
3.	1	0	$-3\pi/4$
4.	1	1	$\pi/4$



SE8.6 For 16-PSK and a transmission system with a 15 kHz bandwidth, determine the maximum bit rate.

Sol: For 16-PSK, the number of bits $N = 4$

So, Four bits are transmitted in 1 Hz bandwidth. Therefore, the maximum bit rate

$$R_b = 4 \times 15000 = 60000 \text{ bps}$$

SE8.7 For a given data rate, the bandwidth of B_P a BPSK signal and the bandwidth B_o of the OOK signal are related as

(a) $B_P = \dfrac{B_o}{4}$

(b) $B_P = \dfrac{B_o}{2}$

(c) $B_P = B_o$

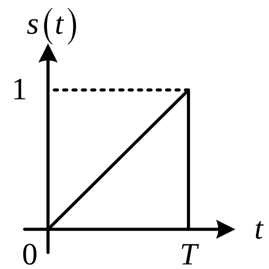
(d) $B_P = 2B_o$ **(GATE: 1998)**

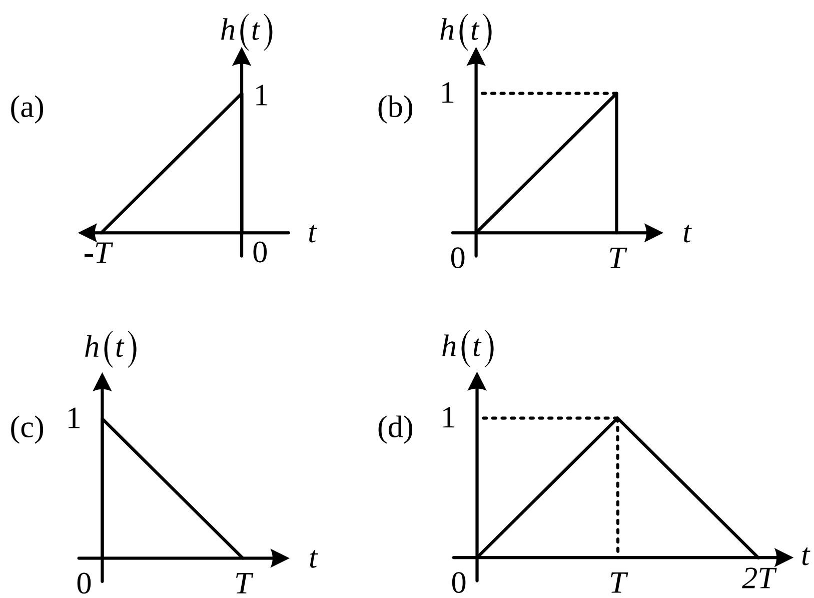
Sol: The number of bits for the representation of BPSK and OOK both is 2. Therefore, the bandwidth of BPSK and OOK is the same i.e.

$B_p = B_o$

Hence, option (c) is correct.

SE8.8 Consider the pulse shape $s(t)$ as shown below. The impulse response $h(t)$ of the filter matched to this pulse is **(GATE: 2010)**





Sol: Impulse response of the matched filter is given by

$$h(t) = s(t - T)$$

Hence, option (c) is correct.

PROBLEMS

P8.1 What are the different types of analog and digital modulation techniques?

P8.2 Distinguish between ASK, FSK and PSK techniques.

P8.3 Draw the block diagram of the amplitude shift keying system and explain the working operation.

P8.4 What is coherent and non-coherent modulation?

P8.5 Describe the coherent and non-coherent modulation and detection of ASK/On-Off Keying.

P8.6 Explain the FSK modulation scheme with its modulator and demodulator.

P8.7 Describe the coherent and non-coherent modulation and detection of PSK.

P8.8 Draw the bandwidth spectrum for ASK, FSK and PSK.

P8.9 How is the drawback of PSK removed? Explain that technique.

P8.10 Explain DPSK signal generation with a neat diagram.

P8.11 Explain the working of Quadrature Phase Shift Keying (QPSK) transmitter and receiver. Draw the constellation diagram and derive the probability of the error for the same.

P8.12 Briefly explain the generation of the FSK signal. Also, discuss its probability of error.

P8.13 Explain working of Quadrature Phase Shift Keying (QPSK) transmitter and receiver. Draw the constellation diagram and phase diagram. Derive the probability of the error for same.

P8.14 Derive the expression of BER for FSK modulation.

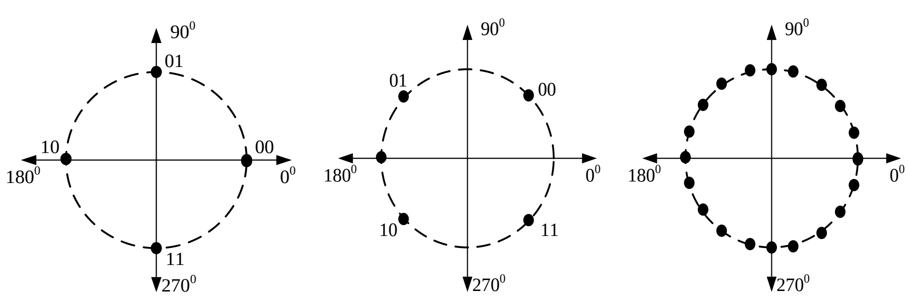
P8.15 Explain the importance of matched filter.

NUMERICAL PROBLEMS

P8.16 Draw ASK, FSK and PSK waveforms for the bitstream 1011001.

P8.17 Determine the minimum bandwidth for a BPSK modulator with a carrier frequency of 40 MHz and an input bit rate of 500 kbps?

P8.18 What type of modulation is this and draw the modulated waveform for the message signal with pattern 11000110 using the modulation technique shown in the Fig. 8.35(a)-(c).



(a) (b) (c)

Fig. 8.35 Figures of problems 8.18

P8.19 Determine (a) the peak frequency deviation, (b) minimum bandwidth, and (c) baud rate for a binary FSK signal with a mark frequency of 49 kHz, a space-frequency of 51 kHz, and an input bit rate of 2 kbps.

P8.20 For a BPSK modulator with a carrier frequency of 70 MHz and an input bit rate of 10 Mbps, Determine the maximum and minimum upper and lower side frequencies, draw the output spectrum, determine the minimum Nyquist bandwidth, and calculate the baud rate.

P8.21 Determine the baud rate and minimum bandwidth necessary to pass a 10 kbps binary signal using ASK.

P8.22 For a QPSK modulator with an input data rate (Rb) equal to 10 Mbps and a carrier frequency of 70 MHz, determine the minimum double-sided Nyquist bandwidth $\left(f_{Nyquist}\right)$ and the baud rate.

P8.23 For an 8-PSK modulator with an input data rate (R_b) equal to 10 Mbps and a carrier frequency of 70 MHz, determine the minimum double-sided Nyquist bandwidth $\left(f_{Nyquist}\right)$ and the baud rate.

P8.24 Binary data has to be transmitted over a telephone link that has a usable bandwidth of 3000 Hz and a maximum achievable signal-to-noise power ratio of 6 dB at its output.

(a) Determine the maximum signaling rate and probability of error if a coherent ASK scheme is used for transmitting binary data through this channel.

(b) If the data is maintained at 300 bits/sec, calculate the error probability.

MULTIPLE-CHOICE QUESTIONS

MCQ8.1 Which of the following has the least noise immunity?

(a) ASK (c) PSK

(b) FSK (d) QAM

MCQ8.2 ……….rate is always less than or equal to …….. rate.

(a) Baud, bit (c) Baud, base

(b) Bit, baud (d) Base, Baud

MCQ8.3 If the bit rate is 1200 bps and there are 4 bits for each signal element, then the baud rate is

(a) 4800 (c) 400

(b) 1200 (d) 300

MCQ8.4 An ASK modulated signal has a bit rate of 2000 bps, the baud rate is

(a) 2000 (c) 4000

(b) 1000 (d) None of the above

MCQ8.5 The baud rates for 2-PSK and 4-PSK modulated signals with the bit rate of 2000 bps are

(a) 2000, 4000 (c) 2000, 1000

(b) 1000, 2000 (d) 4000, 2000

MCQ8.6 The bit rates for 8-QAM and 32-QAM modulated signals with the baud rate of 2000 bps are

(a) 16000, 64000

(b) 64000, 16000

(c) 10000, 6000

(d) 6000, 10000

MCQ8.7 OOK is a type of…….

(a) QAM modulation

(b) FSK modulation

(c) ASK modulation

(d) PSK modulation

MCQ8.8 If the baud rate of the ASK modulated signal is 5000 Hz, the minimum bandwidth is

(a) 1000 (c) 5000

(b) 2500 (d) 10000

MCQ8.9 The number of points in the constellation of an 8-PSK modulation is ………the number of points for an 8-QAM.

(a) Equal to (c) Less than

(b) Greater than (d) None of the above

MCQ8.10 In ……….. modulation, the bit rate is eight times the baud rate

(a) 8-QAM (c) 256-QAM

(b) 64-QAM (d) None of the above

MCQ8.11 Which has the same probability of error?

(a) BPSK and QPSK

(b) BPSK and ASK

(c) BPSK and PAM

(d) BPSK and QAM

MCQ8.12 Which FSK has no phase discontinuity?

(a) Uniform FSK

(b) Discrete FSK

(c) Continuous FSK

(d) None of the above

MCQ8.13 For the generation of FSK, the data pattern will be

(a) NRZ pattern

(b) RZ pattern

(c) Split-phase Manchester

(d) None of the above

MCQ8.14 The bit rate of a digital communication system is 64 Mbits/sec. The baud rate will be in QPSK modulation techniques

(a) 64 Mbps (c) 16 Mbps

(b) 32 Mbps (d) 8 Mbps

MCQ8.15 QAM stands for

(a) Quadrature Amplitude Modulation

(b) Quadrature Amplitude Masking

(c) Quadrature Amplitude Marking

(d) All of the above

MCQ8.16 QPSK symbol consists of

(a) 1bit (c) 2 bits

(b) 1byte (d) 4 bits

MCQ8.17 Which of the following has the maximum probability of error

(a) DPSK (c) FSK

(b) PSK (d) ASK

MCQ8.18 Bit stream is partitioned into even and odd streams in ……..system

(a) FSK (c) QPSK

(b) ASK (d) BPSK

MCQ8.19 If the bit rate for an FSK signal is 1200 bps, the baud rate is

(a) 1200 (c) 400

(b) 800 (d) 200

MCQ8.20 How many carrier frequencies are used in BFSK?

(a) 1 (c) 3

(b) 2 (d) 4

MCQ ANSWERS

MCQ8.1	(a)	MCQ8.11	(c)
MCQ8.2	(a)	MCQ8.12	(c)
MCQ8.3	(d)	MCQ8.13	(a)
MCQ8.4	(a)	MCQ8.14	(b)
MCQ8.5	(c)	MCQ8.15	(a)
MCQ8.6	(d)	MCQ8.16	(c)
MCQ8.7	(c)	MCQ8.17	(d)
MCQ8.8	(c)	MCQ8.18	(c)
MCQ8.9	(a)	MCQ8.19	(a)
MCQ8.10	(c)	MCQ8.20	(b)

CHAPTER 9

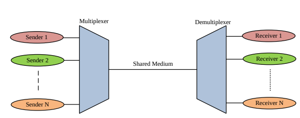
MULTIPLEXING AND INFORMATION THEORY

Definition

Multiplexing is a simultaneous transmission technique that sends multiple message signals from different sources over a common channel.

Highlights

Representation



9.1 Introduction

Multiplexing is a simultaneous transmission technique that sends multiple message signals from different sources over a common or shared communication channel at the same time in the form of a complex signal, whereas demultiplexing is the process to recover the original signals from this complex signal at the receiver end. The block diagram of the multiplexing-demultiplexing concept is shown in Fig. 9.1.

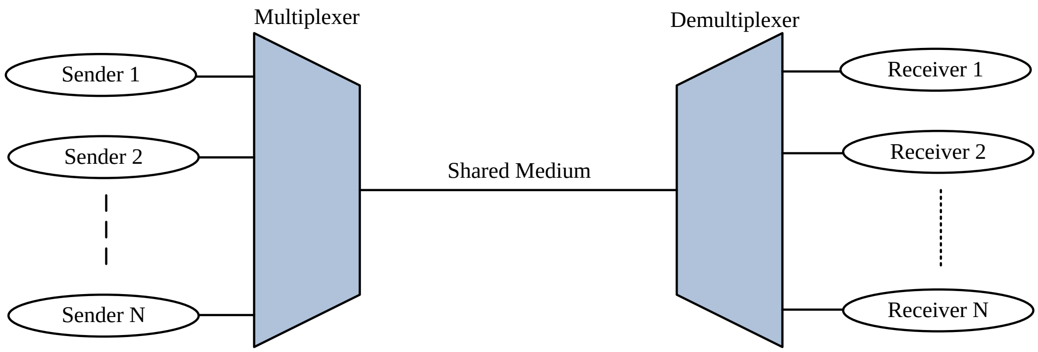


Fig. 9.1 The concept of multiplexing/ demultiplexing

9.2 Need for Multiplexing

The task of many signal's transmission simultaneously can be achieved by setting one transmitter-receiver pair for each pair separately. But, this is an expensive method; therefore, there is a need for multiplexing. A daily life example of multiplexing is the sending of many channels over a cable by the T.V. distributor. Multiplexing permits several signals to be combined and transmitted over a single communication link.

The classification of different types of multiplexing techniques is shown in Fig. 9.2.

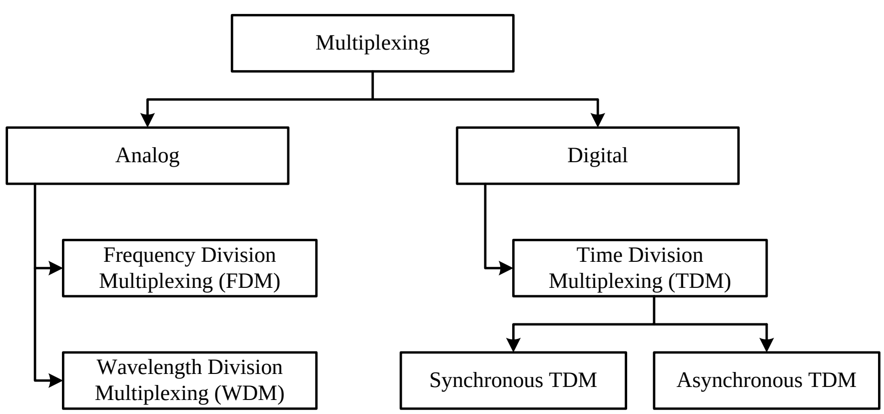


Fig. 9.2 Classification of multiplexing

9.3 Type of Multiplexing

There are two types of multiplexing techniques:

1. Analog multiplexing
2. Digital multiplexing

9.3.1 Analog Multiplexing

If the analog signals are combined as a data stream to transmit on a communication medium, analog multiplexing is used. The analog multiplexing is divided into two categories according to their frequency (FDM) or wavelength (WDM).

(i) Frequency Division Multiplexing (FDM)

FDM is the most widely used analog multiplexing. In this technique, the different frequencies are combined as a data stream for sending them over a communication channel as a single signal.

Example: A traditional T.V. transmitter uses FDM to send many channels over a single cable.

(ii) Wavelength Division Multiplexing (WDM)

In WDM, different wavelength's data streams are transmitted in the form of the light spectrum. Since the wavelength is inversely proportional to the frequency; therefore, frequency of the signal decreases with increment in the wavelength.

Example: Optical fibre Communications to merge different wavelength signals into a single light.

9.3.2 Digital Multiplexing

In digital multiplexing, data streams are sent over the communication medium in the form of discrete frames or packets. Time-division multiplexing is a form of digital multiplexing.

Time Division Multiplexing (TDM)

In TDM, the time frame is divided into slots and further, one slot is allotted for each channel to send the message over a single communication channel. There are following two types of TDM techniques:

(i) Synchronous TDM

In Synchronous TDM, the time frame is divided into the same number of slots as the number of devices are, i.e. if there are *n* devices, there are *n* slots. Further, the multiplexer (MUX) is used to assign the same slot to each message at every time. Therefore, the sampling rate for each message is equal in synchronous TDM.

(ii) Asynchronous TDM

Unlike synchronous TDM, if the allotted device sends nothing for its allotted slot, then that slot is allotted to another device. Therefore, the sampling rate is not the same for each message.

9.4 Frequency Division Multiplexing (FDM) vs. Time Division Multiplexing (TDM)

FDM and TDM both are very popular and widely used multiplexing techniques. In FDM, a guard band is required, whereas a synchronization pulse is needed in TDM. A comparison between FDM and TDM for different parameters is shown in Table 9.1.

Table 9.1 Comparison between FDM and TDM

S. No.	Parameters	FDM	TDM
1.	**Definition**	Frequency division multiplexing.	Time division multiplexing.
2.	**Type**	Analog multiplexing	Digital multiplexing
3.	**Signal**	Works only with analog signal.	Works well with analog as well as digital signals both.
4.	**Mandatory input**	Guard band	Synchronization pulse
5.	**Efficiency**	FDM is less efficient in comparison with TDM.	TDM is efficient.
6.	**Conflict**	FDM has high conflict.	TDM has low conflict.
7.	**Sharing**	Frequency is shared in FDM	Time is shared in TDM

9.5 Fundamentals of Time Division Multiplexing

Time division multiplexing is a method of serial transmission or reception of more than one independent analog signal or digital signal over a single common channel. A synchronized switch is used at each end of the transmission line to transmit or receive the signal for a fraction of time. A simple TDM system is shown in Fig. 9.3. Normally, the TDM method is used when the bit rate of the transmitted signal is less than that of the transmission medium.

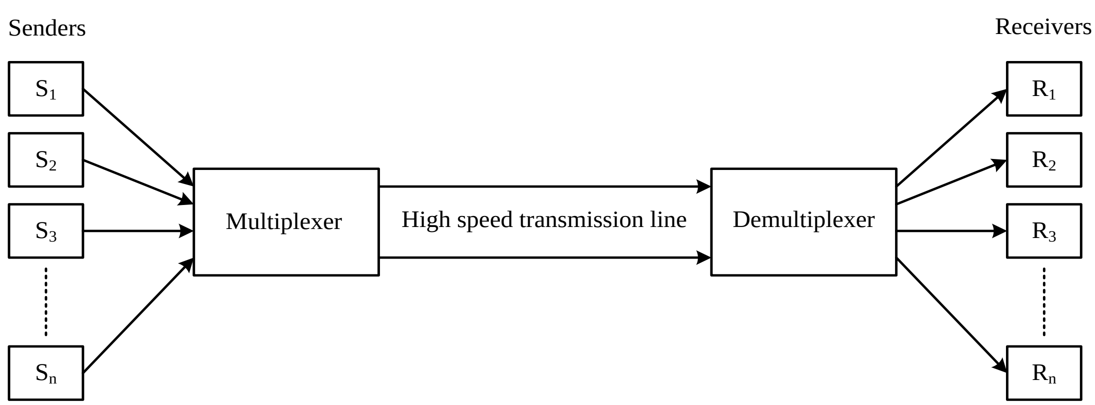


Fig. 9.3 The concept of TDM

The different functions performed at the transmitter and the receiver ends in TDM multiplexing are given below:

At Transmitter End

Following functions are performed at the transmitter end

1. The information or message streams are simultaneously transmitted on time-sharing basis by a synchronized switch.
2. During the transmission, each signal takes its own individual time slot.
3. The slots may be permanently assigned.

At Receiver End

Following functions are performed at the receiver end

1. A synchronized decommutator or sampler is used to assign the message stream to the individual receivers.
2. Noise distortion is eliminated by the low pass filter.

T1 carrier system is an example of a time-division multiplexing system.

9.6 Digital Hierarchy

Normally two classes of multiplexers are used:

1. **Multiplexer to combine low data rate channels**- such multiplexer combines the channel up to 9600 bits/sec into a signal of data rate up to 64 kbps. This signal is called **"digital signal level 0" (DS0)**. The sampling rate is 8000 samples/sec, i.e. the time duration between each sample is 1/8000 sec (125 μsec). DS0 is the fundamental building block of a digital communication system.
2. **Multiplexer to combine much higher bit rate channels** – There are four levels of multiplexing, as shown in Fig. 9.4.

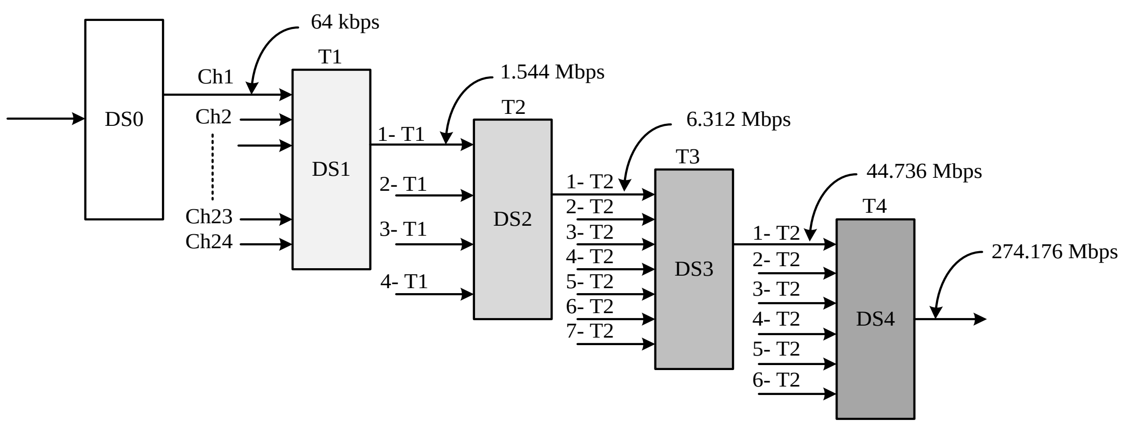


Fig. 9.4 TDM hierarchy rate (T lines)

The above-mentioned hierarchy is called as TDM hierarchy and is explained as:

1. First level is the T1 multiplexer which consists of 24 channels of 64 kbps, i.e. 24 DS0 lines. The output of the T1 multiplexer is the DS1 (digital signal 1) signal at a rate of 1.544 Mbps.
2. Four DS1 signals (shown as 1-T1 to 4-T1) are multiplexed by second-level multiplexer T2 and generates a DS2 (digital signal 2) signal at a rate of 6.312 Mbps as output.
3. Seven DS2 signals (shown as 1-T2 to 7-T2) are multiplexed by third-level multiplexer T3 and generates a DS3 (digital signal 3) signal at a rate of 44.376 Mbps as output.
4. Six DS3 signals (shown as 1-T3 to 6-T3) are multiplexed by a fourth level multiplexer and generates a DS4 (digital signal 4) signal at a rate of 274.176 Mbps as output.

The description of two classes of multiplexers and channels are given in Table 9.2.

Table 9.2 DS services

Class	Name	Multiplexing level	Number of channels	The rate in bits/sec
Class 1: Low data rate	**DS0**	0	1	64 kbps
Class 2: Much higher data rate	**DS1**	1	24	1.544 Mbps
	DS2	2	24 x 4 = 96	6.312 Mbps
	DS3	3	96 x 7 = 672	44.376 Mbps
	DS4	4	672 x 6 = 4032	274.176 Mbps

Most of the countries of the world except North America and Japan are using the PCM hierarchy. PCM hierarchy is employed by the multiplexing of 30 telephone channels (E0 channels) at a rate of 64 kbps to make E1 carrier at the rate of 2.048 Mbps.

Further, four E1 lines are multiplexed into an E2 carrier (rate of 8.448 Mbps), four E2 lines are multiplexed into an E3 carrier (rate of 34.368 Mbps), and four E3 lines are multiplexed into an E4 carrier (rate of 139.264 Mbps). The block diagram of the PCM hierarchy is shown in Fig. 9.5. PCM hierarchy is summarized in Table 9.3.

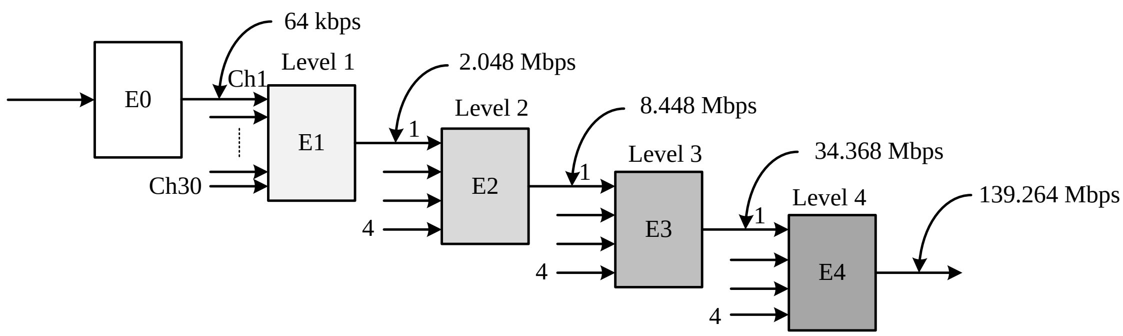


Fig. 9.5 PCM hierarchy rates

Table 9.3 E-lines

Class	Name	Multiplexing level	Number of channels	The rate in bits/sec
Class 1: Low data rate	**E0**	0	1	64 kbps
Class 2: Much higher data rate	**E1**	1	30	2.048 Mbps
	E2	2	120	8.448 Mbps
	E3	3	480	34.368 Mbps
	E4	4	1920	139.264 Mbps

9.7 T1 Carrier System: PCM/TDM System

The T-carriers, developed by AT&T Bell Laboratories, are high-speed time division multiplexed digital services. There are four types of T-carrier lines:

Table 9.4 Different types of T-carrier lines

S. No.	Services	T-carrier line	Transmission speed
1.	DS1	T1-carrier	1.544 Mbps
2.	DS2	T2-carrier	6.312 Mbps
3.	DS3	T3-carrier	44.736 Mbps
4.	DS4	T4-carrier	274.176Mbps

T1-carrier stands for transmission one carrier which uses pulse code modulation and time-division multiplexing (PCM/TDM). T1 line generally uses 24 voice channels with a bandwidth around 300 Hz to 3000 Hz and each channel support 64 kbps transmission speed. The sampling speed is selected as 8 kHz. The block diagram representation of PCM encoded samples from 24 voice band channels for transmission is shown in Fig. 9.6. T1-carrier technology is used in the United States and Canada and it has been a North American digital multiplexing standard since 1963. T1 used Alternate Mark Inversion (AMI) to eliminate the DC component and to reduce frequency bandwidth.

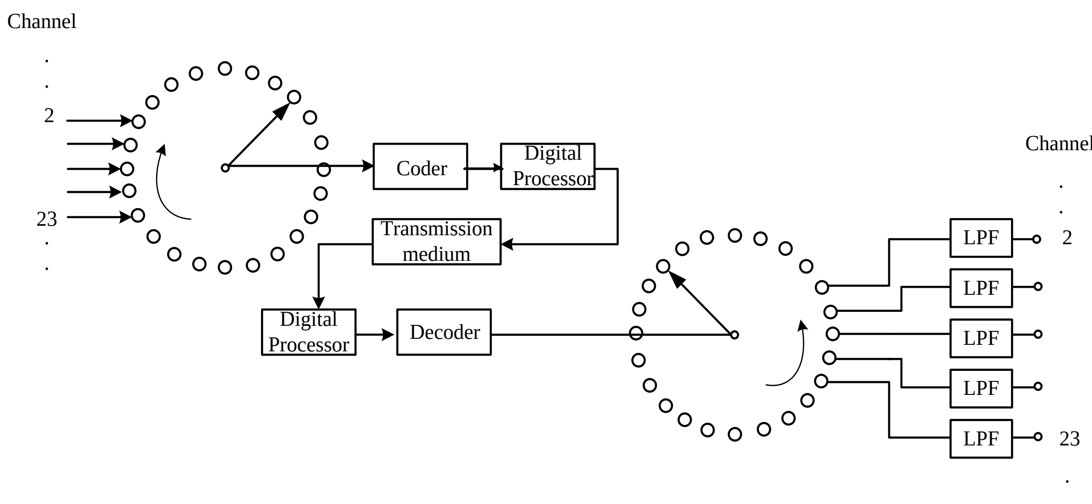


Fig. 9.6 T1 carrier system

Following few points to be noted:

1. Message streams of all 24 channels are simultaneously transmitted on a time-sharing basis by a synchronized switch or sampler and represented as a TDM-PAM signal.
2. This PAM signal is quantized and encoded into 8-binary pulses by the encoder.
3. These binary codes are converted into a digital signal (PCM) for transmission over the channel.

4. A 24 x 1 digital switch called multiplexer is used to get the one TDM output from 24 independent input channels.
5. This PCM output is sequentially transmitted over a common transmission line through the multiplexer.
6. Regenerative repeaters are used at approximately 6000 *ft* to detect the pulses, to remove the noise and to retransmit new pulses.
7. At the receiver end, the signals are demultiplexed, decoded, and further reconstructed to obtain the desire audio signal.
8. The T1-carrier system performs the operation of sampling, encoding and multiplexing of 24 voiceband channels.
9. Instead of a mechanical switch, a high-speed electronic switching circuit is used T1 carrier system. This switch is called electronic Commutator.

9.7.1 Synchronization and Signaling of TI Carrier System

In the T1 carrier system, each channel contains an 8-bit PCM codeword (one sample). There are 24 channels in the T1 carrier system. A segment containing one codeword (8 bits) or one sample from each of the 24 channels is called a **frame.** The sampling rate is 8000 samples/sec; therefore, the time slot of each sample is

$$\frac{1}{8000} = 125\ \mu\text{sec}$$

Each frame has 24 x 8 = 192 information bits as

$$\frac{24\ \text{channels}}{\text{frame}} \times \frac{8\ \text{bits}}{\text{channel}} = 192 \quad \text{bits/frame}$$

One 64 kbps PCM encoded sample is transmitted for each voice band channel with 8000 samples/sec (frame). Therefore, the line speed is

$$\frac{192\ \text{bits}}{\text{frame}} \times \frac{8000\ \text{frames}}{\text{sec}} = 1.536\ \text{Mbps}$$

An additional bit (called framing bit) is added at the beginning of each frame to separate the information bits correctly. So, total number of bits is 192 + 1 = 193 bits/frame, shown in Fig. 9.7.

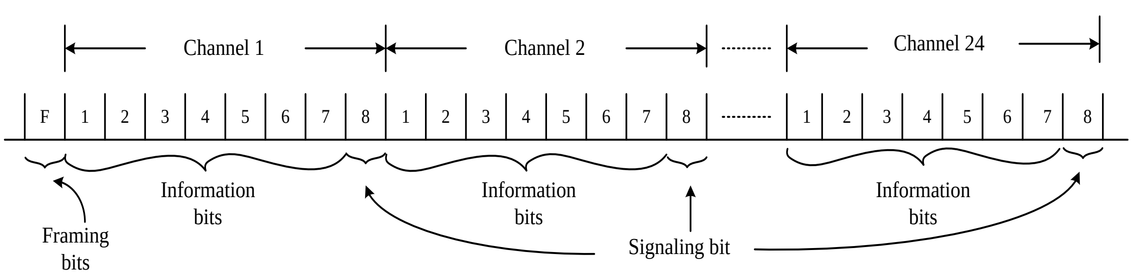


Fig. 9.7 One frame

Now, each frame consists of a total of 193 bits; therefore, line speed for the T1 digital carrier system is

$$\frac{193 \text{ bits}}{\text{frame}} \times \frac{8000 \text{ frames}}{\text{second}} = 1.544 \text{ Mbps}$$

So, the minimum bandwidth of the T1 carrier system is

$$(\text{BW})_{\min} = \frac{1}{2} \times 1.544 \times 10^6 = 772 \text{ kHz}$$

The duration of each bit is

$$\frac{125 \times 10^{-6}}{193} = 0.6477 \ \mu\text{sec}$$

9.7.2 Bit Rate

As previously explained T1 carrier system consists of 193 bits per frame and each frame has $125 \ \mu\text{sec}$ time duration. So, the bit rate of the T1 channel is

$$f_b = \frac{1}{T_b} = \frac{193}{125 \mu \sec} = \frac{193}{125 \times 10^{-6}} = 1.544 \text{ Mbps}$$

The Bandwidth of The TDM-PCM System

- Total number of channels = n (each bandlimited to f_m)
- Length of PCM code = N
- So, quantization levels $L = 2^N$
- Therefore sampling frequency = $2f_m$ (sampling period = $1/2f_m$)
- Since 1 bit is required for the separation of each channel. So total number of separation bits for n channels = n
- One extra bit is added as synchronising bit.

Therefore, the total number of bits/sampling period =

$$\frac{N \ bits}{Channel} \times n \ Channels + n(seperating \ bits) + 1(synchronizing \ bit) = nN + n + 1$$

Therefore, bit duration (T_b) = $\dfrac{\text{Sampling period}}{\text{Total number of bits}}$

$$T_b = \frac{1/2 f_m}{[nN + n + 1]} = \frac{1}{\left[n(N+1)+1\right] 2 f_m} \text{ sec}$$

Hence, bandwidth $\text{BW} = \dfrac{1}{T_b} = \left[n(N+1)+1\right] 2 f_m \text{ Hz}$

Now if $N >> 1$ and $n >> 1$

Approximated bandwidth BW = $2nNf_m$ Hz

9.8 Interleaving

The multiplexing is performed as the bit-by-bit (digit interleaving) basis or word-by-word (word interleaving) basis.

9.8.1 Bit Interleaving

In bit interleaving, bits are taken one at a time from each channel's signal, as shown in Fig. 9.8. There are 8-different channels' signals which are to be time-division multiplexed. Firstly, one bit from each channel's signal is taken at a time and time-division multiplexed. Thereafter, 2nd bits, 3rd bits and so on are taken from each signal. This interleaving is used in North America. The advantages of bit interleaving are as follows:

1. It is simple.
2. There is no buffering, so bit timing is preserved.

9.8.2 Word (byte) Interleaving

Bytes are multiplexed in succession from each channel, as shown in Fig. 9.9. The eight bits or one byte is taken from channel 1. After that, the next byte to be multiplexed is taken from channel 2, channel 3 and so on. Byte interleaving is used in SONET formatted signals and DS1 signals. Byte interleaving takes advantage of technology developed for computers and preserves byte timing.

9.8.3 Frame Interleaving

The schematic of frame interleaving is shown in Fig. 9.10. One frame of each message is transmitted simultaneously. Therefore, it requires larger buffers.

9.9 Synchronization Techniques

There are three types of synchronization techniques:

9.9.1 Carrier Synchronization

The knowledge of both the phase and the frequency of the carrier signal is important for synchronous or coherent detection. The assessment of the carrier frequency and phase is called carrier synchronization.

9.9.2 Frame Synchronization

If the digital system is frame-based, then the assessment of the starting and stopping time of the data frame is necessary. This estimation process is called frame synchronization.

9.9.3 Symbol/bit Synchronization

At the receiver side of the digital system, the output must be sampled at the symbol rate with a specified sampling instant. For this purpose, a clock pulse is required. The process of estimating such clock pulse is called symbol/bit synchronization.

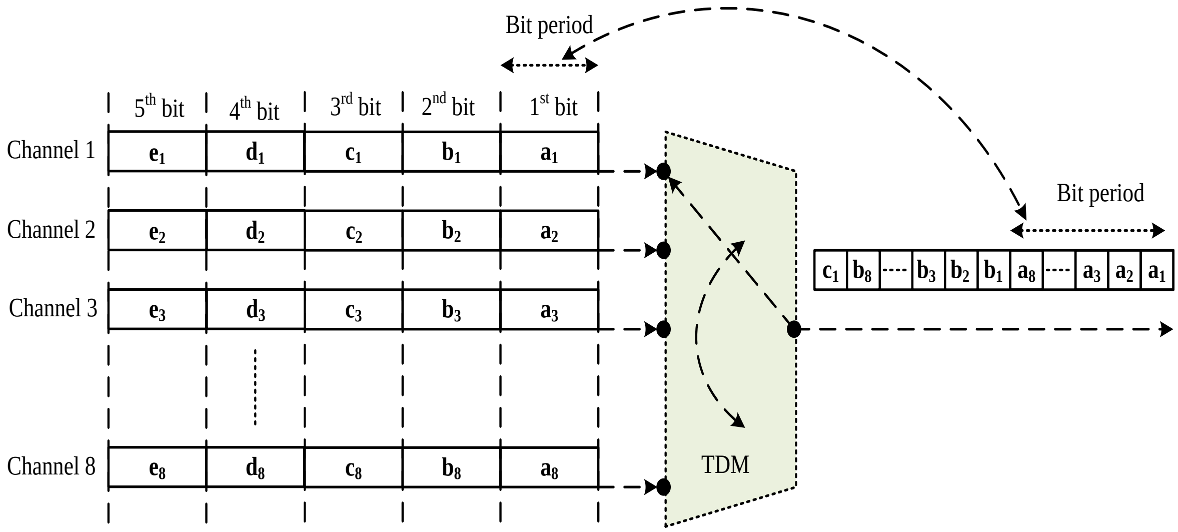


Fig. 9.8 Bit interleaving

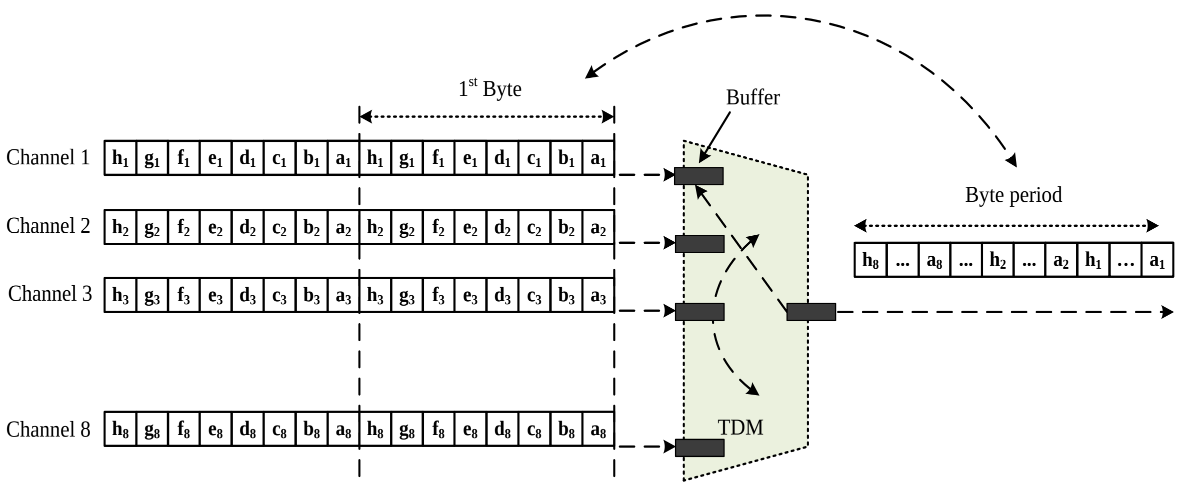


Fig. 9.9 Byte interleaving

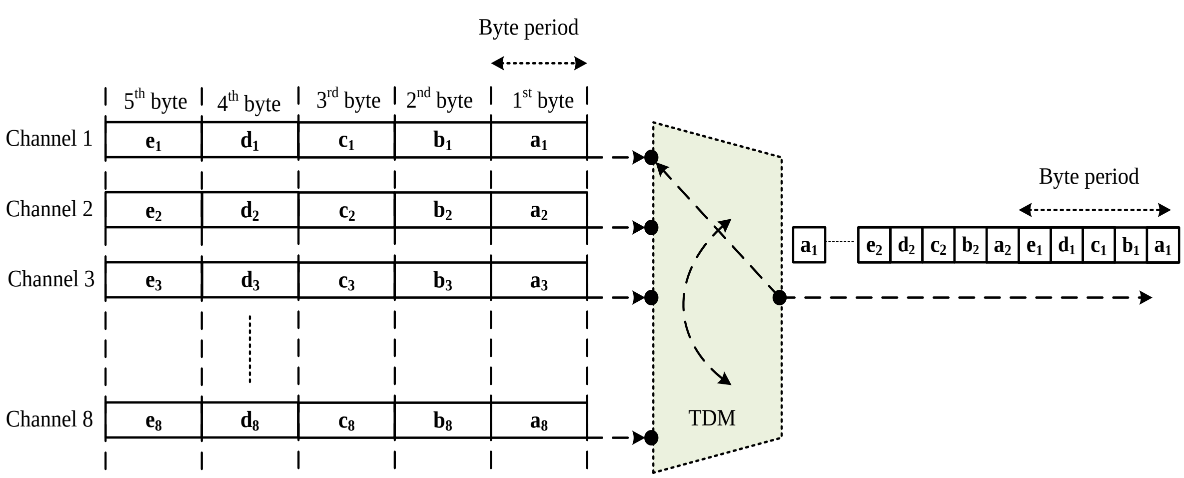


Fig. 9.10 Frame interleaving

9.10 Information

In general terms, information is defined as a measure of surprise or uncertainty (unexpectedness). Therefore, the more unexpected event, the more amount of surprise and convey more information. So, the information contained in a message is directly related to the probability of occurrence, such as

$$\left.\begin{matrix}\begin{pmatrix}\text{High probability event}\\ P\to 1\end{pmatrix}\Rightarrow\begin{pmatrix}\text{Low information}\\ I\to 0\end{pmatrix}\\ \begin{pmatrix}\text{Low probability event}\\ P\to 0\end{pmatrix}\Rightarrow\begin{pmatrix}\text{High information}\\ I\to \infty\end{pmatrix}\end{matrix}\right\}\Rightarrow\ I\propto\log\left(\frac{1}{P}\right)$$

The amount of the information for the binary encoded message is given by

$$I=\log_2\left(\frac{1}{P}\right)\text{ bits} \tag{9.1}$$

Since the information is measured in binary digits (bits); therefore, the logarithm base is selected as 2.

9.11 Entropy

Let a memoryless source x emits n messages $\left(m_1, m_2, ...m_n\right)$ with probabilities $\left(P_1, P_2, ...P_n\right)$ respectively, such that $\left(P_1+P_2+......+P_n=1\right)$. According to Eq. (9.1), the information of the message m_i is

$$I_i=\log_2\left(\frac{1}{P_i}\right)\text{ bits}$$

Therefore, the average information/message is given by

$$\sum_{i=1}^{n}P_iI_i\text{ bits}$$

In information theory, the average amount of the uncertainty or information of a random variable's possible outcomes is measured as entropy. Mathematically, the entropy of message source x is stated as:

$$H(\text{x})=\sum_{i=1}^{n}P_iI_i\ =\sum_{i=1}^{n}P_i\log_2\left(\frac{1}{P_i}\right)\Rightarrow-\sum_{i=1}^{n}P_i\log_2 P_i \tag{9.2}$$

Since the base is 2; therefore, the entropy $H(\text{m})$ is measured in bits and bounded as $0\le H(\text{x})\le\log_2(q)$. If random variable **x** takes the value as constant, the measured entropy is 0. Further, if all transmitted symbols have equal probability $P_i=\frac{1}{q}$, entropy attains its maximum value.

The conditional entropy of *x* for a given *y* is expressed as

$$H(x \mid y) = -\sum_{x,y}^{n} P(x,y) \log_2 P(x \mid y) \tag{9.3}$$

The conditional probability is bounded by

$$0 \le H(x \mid y) \le H(x) \tag{9.4}$$

9.12 Channel Capacity of Discrete Memoryless Channel (DMC)

Let a source emits $\{x_r\}$ set of symbols, whereas the receiver receives $\{y_k\}$ set of symbols that may or may not be identical to the transmitted set of symbols $\{x_r\}$. If the channel is noiseless, then the set of received symbols $\{y_k\}$ uniquely determine the transmitted symbols $\{x_k\}$. There would be a certain amount of uncertainty regarding the transmitted signal when y_j is received for the noisy channel.

Let $P(x_i|y_j)$ is the conditional probability that represents x_i was transmitted when y_j is received. Then the uncertainty in transmitted x_i when y_j received is given by $\log\left[1/P(x_i|y_j)\right]$. Therefore, the average uncertainty $H(\mathrm{x}|\mathrm{y})$ for overall x_i and y_j is expressed as

$$H(x|y) = \sum_i \sum_j P(x_i, y_j) \log\left[1/P(x_i|y_j)\right] \quad \text{bits/symbol} \tag{9.5}$$

For noiseless channels, probabilities are either 0 or 1, i.e. either $P(x_i, y_j) = 0$ or $P(x_i|y_j) = 1$. In both cases

$$H(x|y) = 0 \qquad \text{(for noiseless channel)}$$

Similarly, $P(y_j|x_i) =$ transition probability that represents y_j is received when x_i is transmitted. Above mentioned probability is the characteristics of the channel and the receiver, which is further specified as the channel matrix and given as:

$$\begin{array}{cc} & \text{Outputs} \\ & \begin{array}{cccc} y_1 & y_2 & \cdots & y_k \end{array} \\ \text{Inputs} \begin{array}{c} x_1 \\ x_2 \\ \cdots \\ x_r \end{array} & \begin{pmatrix} P(y_1|x_1) & P(y_2|x_1) & \cdots & P(y_k|x_1) \\ P(y_1|x_2) & P(y_2|x_2) & \cdots & P(y_k|x_2) \\ \cdots & \cdots & \cdots & \cdots \\ P(y_1|x_r) & P(y_2|x_r) & \cdots & P(y_k|x_r) \end{pmatrix} \end{array} \tag{9.6}$$

The reverse probability $P(x_i|y_j)$ is obtained by Bayes' rule:

$$\begin{aligned} P(x_i|y_j) &= \frac{P(y_j|x_i)P(x_i)}{P(y_j)} \\ &= \frac{P(y_j|x_i)P(x_i)}{\sum_i P(x_i,y_j)} \qquad \left(\because P(y_j)=\sum_i P(x_i,y_j)\right) \\ &= \frac{P(y_j|x_i)P(x_i)}{\sum_i P(x_i)P(y_j|x_i)} \qquad \left(\because P(y_j)=\sum_i P(x_i,y_j)=\sum_i P(x_i)P(y_j|x_i)\right) \end{aligned} \tag{9.7}$$

Hence, the probability $P(x_i|y_j)$ is computed with channel matrix $P(y_j|x_i)$ and input symbols probabilities $P(x_i)$.

The relations between the mutual information and entropies are depicted as shown in Fig. 9.11.

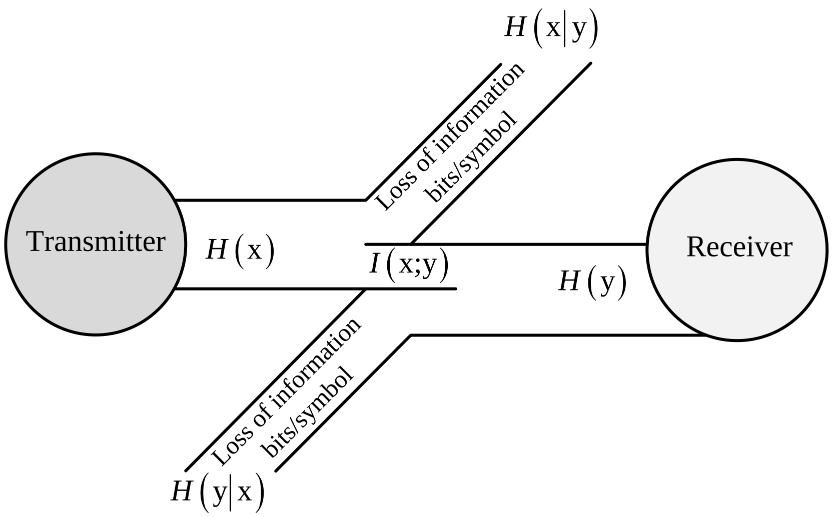


Fig. 9.11 Relations between the mutual information and entropies

Following points to be noted from Fig. 9.11

1. The transmitted information $I(\text{x; y})$ is obtained by the following expression

$$\begin{aligned} &H(\text{x}) = I(\text{x; y}) + H(\text{x}|\text{y}) \\ &\Rightarrow I(\text{x; y}) = H(\text{x}) - H(\text{x}|\text{y}) \end{aligned} \tag{9.8}$$

Similarly, $I(\text{x; y}) = H(\text{y}) - H(\text{y}|\text{x})$, (9.9)

$$I(\text{x; y}) \le \min\{H(\text{x}), H(\text{y})\} \tag{9.10}$$

2. If there is no dependency between input and output of the DMC, i.e. mutual information sharing is zero, then

$$I(\text{x; y}) = 0, \quad H(\text{x}) = H(\text{x}|\text{y}), \quad H(\text{y}) = H(\text{y}|\text{x}) \tag{9.11}$$

3. If the channel is transparent, i.e., there is no uncertainty

$$H(\mathrm{x}|\mathrm{y}) = H(\mathrm{y}|\mathrm{x}) = 0, \qquad I(\mathrm{x;y}) = H(\mathrm{x}) = H(\mathrm{y}) \tag{9.12}$$

Therefore, the information received by the receiver is $H(\mathrm{x})$ for a noiseless channel while there is some loss of information in the received message for a noisy channel. This average loss of information bits/symbol is given by $H(\mathrm{x}|\mathrm{y})$. Therefore, the information received by the receiver for a noisy channel is expressed as

$$I(\mathrm{x;\ y}) = H(\mathrm{x}) - H(\mathrm{x}|\mathrm{y}) \quad \text{bits/symbol} \tag{9.13}$$

where, $I(\mathrm{x;y})$ = mutual information of x and y.

Since,

$$H(\mathrm{x}) = \sum_i P(x_i)\log\frac{1}{P(x_i)} \quad \text{bits} \tag{9.14}$$

So,

$$I(\mathrm{x;\ y}) = \underbrace{\sum_i P(x_i)\log\left[1/P(x_i)\right]}_{=H(\mathrm{x})} - \underbrace{\sum_i\sum_j P(x_i, y_j)\log\left[1/P(x_i|y_j)\right]}_{=H(\mathrm{x|y})}$$

$$\begin{aligned}
&= \sum_i\sum_j P(x_i, y_j)\log\left[1/P(x_i)\right] - \sum_i\sum_j P(x_i, y_j)\log\left[1/P(x_i|y_j)\right] \quad \left(\because \sum_j P(x_i, y_j) = P(x_i)\right)\\
&= \sum_i\sum_j P(x_i, y_j)\log\left[\frac{P(x_i|y_j)}{P(x_i)}\right]\\
&= \sum_i\sum_j P(x_i, y_j)\log\left[\frac{P(x_i, y_j)}{P(x_i)P(y_j)}\right] \qquad \left(\because P(x_i, y_j) = P(y_j)P(x_i|y_j)\right)
\end{aligned} \tag{9.15}$$

Using Bayes' rule

$$I(\mathrm{x;\ y}) = \sum_i\sum_j P(x_i, y_j)\log\left[\frac{P(y_j|x_i)}{P(y_j)}\right] \tag{9.16}$$

Or

$$I(\mathrm{x;\ y}) = \sum_i\sum_j P(x_i)P(y_j|x_i)\log\left[\frac{P(y_j|x_i)}{\sum_i P(x_i)P(y_j|x_i)}\right] \quad \left(\because P(x_i, y_j) = P(x_i)P(y_j|x_i)\right) \tag{9.17}$$

Since, $I(\mathrm{x;\ y})$ is symmetrical w.r.t. x and y, So,

$$I(\mathrm{x;\ y}) = I(\mathrm{y;\ x}) = H(y) - H(y\,|\,x) \tag{9.18}$$

For a given channel, the maximum value of $I(\mathrm{x;\ y})$ for a given set of $P(x_i)$ is channel capacity C_s,

$$C_s = \max_{P(x_i)} I(\mathrm{x;\ y}) \qquad \text{bits/symbol} \tag{9.19}$$

9.13 Binary Symmetric Channel (BSC)

The BSC is a common communication channel model. In this model, the transmitter sends one bit, either '1' or '0' and the receiver receives the one bit either '1' or '0' at a time. A schematic diagram of BSC is shown in Fig. 9.12.

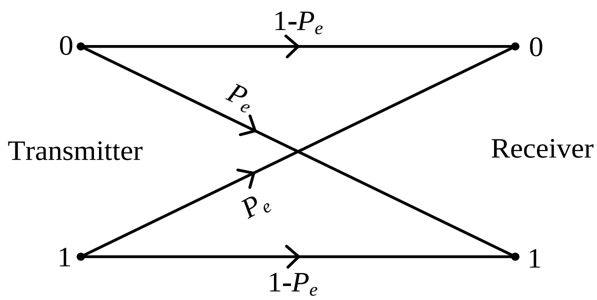


Fig. 9.12 Binary symmetric channel (BSC)

Here P_e is the probability of error, i.e. the received bit is '1' if the transmitted bit is '0' or vice versa. Therefore, the conditional probabilities of BSC are

$$\begin{aligned} P[0|0] &= 1-P_e \qquad & P[1|0] &= P_e \\ P[0|1] &= P_e \qquad & P[1|1] &= 1-P_e \end{aligned} \tag{9.20}$$

The entropy and the channel capacity of the BSC are given as

$$H_{BSC}(x) = \left[P_e \log\frac{1}{P_e} + (1-P_e)\log\left(\frac{1}{1-P_e}\right)\right] \tag{9.21a}$$

$$C_s = 1 - H_{BSC}(x) = 1\text{-}\left[P_e \log\frac{1}{P_e} + (1-P_e)\log\left(\frac{1}{1-P_e}\right)\right] \tag{9.21b}$$

9.14 Concept of Error-Free Communication

As previously explained, any message source (x is replaced with m for message source) with entropy $H(\mathrm{m})$ can be encoded by an average of $H(\mathrm{m})$ digits/message with zero redundancy. If the message is transmitted over a noisy channel, no possibility of error-free communication. Therefore, redundant bits are used to combat the noise.

Let one extra bit is used as a redundant bit with the encoded message of entropy $H(\mathrm{m})$. This extra bit increases the average word length to $H(\mathrm{m})+1$. Further, the efficiency decreases and is given as:

$$\eta = \frac{H(\mathrm{m})}{H(\mathrm{m})+1} \tag{9.22}$$

The redundancy is given by

$$\gamma = 1-\eta = 1-\frac{H(\mathrm{m})}{H(\mathrm{m})+1} = \frac{1}{H(\mathrm{m})+1} \tag{9.23}$$

Shannon proved that error-free communication could be achieved by adding sufficient redundancy. If a message source with entropy $H(\mathrm{m})$ transmits the messages over a binary symmetric channel (BSC) of error probability P_e, then error-free communication could be achieved with an encoded message of word length at least equal to $H(\mathrm{m})/C_s$, where $C_s\,(C_s<1)$ is called channel capacity and is given by:

$$C_s = 1 - \left[P_e \log_2 \frac{1}{P_e} + (1-P_e)\log_2(1-P_e) \right] \quad \text{where, } P_e = \text{Error probability} \tag{9.24}$$

The highest information rate (in units of information per unit time) is represented as the channel capacity of a given channel and can be realized with an arbitrarily small error probability. If $C_s = 0.5$, error-free communication is achieved with an average word length at least equal to

$$\frac{H(\mathrm{m})}{C_s} = \frac{H(\mathrm{m})}{0.5} = 2H(\mathrm{m}) \tag{9.25}$$

Therefore, $H(\mathrm{m})$ redundant digits/message is required for error-free communication, i.e. for every two digits transmitted; there will be one message digit and one redundant digit. Since the maximum efficiency of code is C_s, therefore, the redundancy is

$$\gamma = 1-\eta = 1-C_s = 1-0.5 = 0.5$$

Further, the error probability of the binary signal is given as

$$P_e = e^{-kE_b} \qquad \text{where } E_b = \text{ energy/bit} \tag{9.26}$$

So, the error probability can be reduced by increasing E_b (energy/bit). As we know, the signal power is given by

$$S_e = E_b R_b \qquad \text{where } R_b = \text{bit rate} \tag{9.27}$$

Therefore, increasing E_b means either decreasing bit rate (R_b) for constant signal power (S_i) or increasing signal power (S_i) at the constant bit rate (R_b). Therefore, error-free communication i.e. $P_e \to 0$ is possible if $E_b \to \infty$. This condition is possible if

a) $S_e \to \infty \qquad (R_b = \text{constant})$

b) $R_b \to 0\,(S_e = \text{constant})$

Signal power S_e cannot be increased beyond a certain limit due to physical limitations and the bit rate R_b cannot be zero for the transmission of message bits. Therefore, it seems that error-free communication is not possible over a noisy channel. This thought was in the mind of engineers until Shannon showed that error-free communication is achieved by keeping the bit rate R_b below a certain limit (called channel capacity/sec), i.e.

$P_e \to 0$ if $R_b < C$

The channel capacity/sec is given as

$$C = 2BC_s \qquad \text{where } B = \text{Bandwidth} \tag{9.28}$$

Note: C_s is the maximum possible information transmitted when one symbol or digit is transmitted, whereas C is the maximum rate of information transmitted per second when K symbols or digits are transmitted.

9.15 Shannon–Hartley Theorem

With the assumption of bounded signal power, the **Shannon–Hartley theorem** establishes a channel capacity for a communication medium and tells the maximum amount of information that can be transmitted error-free in the presence of noise over a specified channel bandwidth.

In other words, **Shannon–Hartley theorem** tells the maximum rate of information transmitted over a specified bandwidth channel in the presence of noise.

The Shannon-Hartley theorem is a channel capacity concept which derived a relation between an additive white Gaussian noise (AWGN) channel with a bandwidth of B Hz and signal-to-noise ratio (S/N). According to this theorem, the channel capacity/sec is given by:

$$C = B\log_2\left(1+\frac{S}{N}\right)\text{bits/second} \tag{9.29}$$

where, S/N is the signal to noise ratio, B is bandwidth in Hz and C is measured in bits per second. If S/N is given in dB, conversion is needed for channel capacity calculation.

For example, an S/N ratio of 20 dB resembles a linear power ratio of

$$10\log_{10}\frac{S}{N} = 20 \Rightarrow \frac{S}{N} = 10^{20/10} = 100$$

For error-free communication, bit rate R_b must be less than channel capacity per second ($C = 2BC_s$)

9.16 Shannon Fano Algorithm

Shannon Fano Algorithm is named after its inventor Claude Shannon and Robert Fano. It is an entropy encoding method for lossless data transmission in the communication system.

In this algorithm, a separate code is assigned to each symbol on the basis of their probabilities of occurrence. The code assigned to each individual message stream are of varying length, so it is a variable-length encoding scheme.

The following steps are applied to find the assigned code through the Shannon Fano algorithm:

1. Arrange the set of messages (symbols) in descending order on the basis of their probabilities of occurrence.
2. Divide the list arranged according to step 1 into two groups, with the total probability of both the groups being as close to each other as possible.

3. Allot the value '1' to the lower part and '0' to the upper part.
4. Repeat steps 2 and 3 for each part until all the messages (symbols) are divided into individual subgroups.

The concept of assigning separate codes to each message by Shannon Fano algorithm is understood by an example given below:

SE9.1 A memoryless source emits five messages m_1, m_2, m_3, m_4 and m_5 with probabilities 0.22, 0.28, 0.15, 0.30 and 0.05 respectively. Construct Shannon Fano code.

Sol: Step 1: Arrange the messages according to descending order of probabilities:

Symbol	**Probabilities**
m_4	0.3
m_2	0.28
m_1	0.22
m_3	0.15
m_5	0.05

Step 2: Divide the above list into two groups, with the total probability of both the groups being as close to each other as possible:

S. No.	Groups	Total Probability (P)	Difference b/w probabilities
1	m_4	0.3	0.4
	m_2 m_1 m_3 m_5	0.7	
2	m_4 m_2	0.58	0.16
	m_1 m_3 m_5	0.42	
3	m_4 m_2 m_1	0.8	0.6
	m_3 m_5	0.2	

Groups divided in serial no. 2 are very close to each other as the difference is only 0.16.

Step 3: Assign value 0 to each member of the upper part (group) and 1 to each member of the lower part

Symbol	**Probabilities (Step 1)**	**Step 2**
m_4	0.3	0
m_2	0.28	0
m_1	0.22	1
m_3	0.15	1
m_5	0.05	1

Step 4: Repeat steps 2 and 3 for each part until all the symbols are split into individual subgroups.

Symbol	Probabilities (Step 1)	Step 2	Step 3	Step 4	Final Code
m_4	0.3	0	0		00
m_2	0.28	0	1		01
m_1	0.22	1	0		10
m_3	0.15	1	1	0	110
m_5	0.05	1	1	1	111

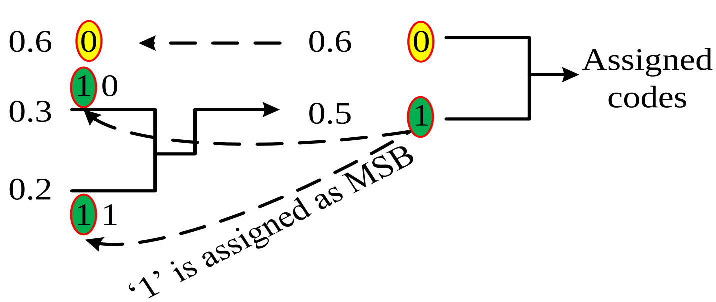
*Assigned value '0' highlighted with yellow is the upper part of the group in the corresponding step while the lower group is highlighted with grey colour with assigned value '1'.

9.17 Huffman Coding (Optimal code)

Huffman coding is a noise-free message (data) transmission (compression) algorithm. In Huffman coding, a variable-length code is assigned to each input message stream. The code length is related to the probabilities of the generated message. A message with a higher probability is encoded with the smallest code, whereas the longer code is assigned to the least probability message.

The following steps are applied to find the assigned code through the Huffman coding algorithm:

1. Create a list of messages (symbols) with decreasing probabilities (frequency counts) order.
2. Combine the probabilities of the two messages having the lowest probabilities and rearrange the order according to step 1.
3. Repeat steps 2 and 1 until the number of messages is reduced to two.
4. Start encoding the message with the last reduction. Assign '0' to upper message and '1' to lower message.
5. Now go back and assign '0' and '1' to the digits which were combined in the previous reduction steps with the value got in the previous step as MSB as given below:



6. Repeat the above steps until the first column is reached.

The concept of assigning separate codes to each message by Huffman coding is understood by an example given below:

SE9.2 A zero memory (memoryless) source emits six messages with probabilities 0.3, 0.25, 0.15, 0.12, 0.1 and 0.08. Find Huffman code. Also, determine its average world length, efficiency and redundancy.

Solution: Step 1: Arrange the messages in descending order according to their probabilities.

Symbol	Probabilities (Step 1)
m_1	0.3
m_2	0.25
m_3	0.15
m_4	0.12
m_5	0.10
m_6	0.08

Step 2: Combine the probabilities of the two messages having the lowest probabilities and rearrange the order according to step 1.

Symbol	Probabilities (Step 1)	S_2 (Step 2)
m_1	0.3	0.3
m_2	0.25	0.25
m_3	0.15	0.18
m_4	0.12	0.15
m_5	0.10	0.12
m_6	0.08	

0.10+0.08= 0.18 > 0.15

Step 3: Repeat steps 2 and 1 until the number of messages is reduced to two.

Symbol S_1	Probabilities (Step 1)	S_2 (Step 2)	S_3	S_4	S_5
m_1	0.3	0.3	0.3	0.43	0.57
m_2	0.25	0.25	0.27	0.3	0.43
m_3	0.15	0.18	0.25	0.27	
m_4	0.12	0.15	0.18		
m_5	0.10	0.12			
m_6	0.08				

Step 4: Start encoding the message with the last reduction. Assign '0' to upper message and '1' to lower message.

Symbols	Probabilities		(Step 1)		(Step 2)		(Step 3)		(Step 4)	
m_1	0.3	**00**	0.3	**00**	0.3	**00**	0.43	**1**	0.57	**0**
m_2	0.25	**10**	0.25	**10**	0.27	**01**	0.3	**00**	0.43	**1**
m_3	0.15	**010**	0.18	**11**	0.25	**10**	0.27	**01**		
m_4	0.12	**011**	0.15	**010**	0.18	**11**				
m_5	0.10	**110**	0.12	**011**						
m_6	0.08	**111**								

Average word length $L = \sum_{i=1}^{N} P_i(n)$

where, N = total no. of messages, n = number of bits in each message, P_i = probability of an i^{th} message. So,

$$L = \sum_{i=1}^{6} P_i(n) = 0.3\times 2 + 0.25\times 2 + 0.15\times 3 + 0.12\times 3 + 0.10\times 3 + 0.08\times 3 = 2.45 \quad \text{binary digits}$$

2 is written as m_1 has code **00** (length is 2)

The entropy $H(m)$ of the source is given by

$$H(m) = -\sum_{i=1}^{6} P_i \log_2 P_i = 2.418\text{bits}$$

Efficiency, $\eta = \frac{H(m)}{L} = \frac{2.418}{2.45} = 0.976$

Redundancy, $\gamma = 1 - \eta = 1 - 0.976 = 0.024$

ADDITIONAL SOLVED EXAMPLES

SE9.3 Twenty four-voice signals are sampled uniformly and then time division multiplexed. The sampling operation uses flat-top samples with 1μs duration. The multiplexing operation includes provision for synchronization by adding an extra pulse of appropriate amplitude and 1μs duration. The highest frequency component of each voice signal is 3.4 kHz.

(i) Assuming a sampling rate of 8 kHz, calculate the spacing between successive pulses of the multiplexed signal.

(ii) Repeat (i) assuming the use of Nyquist rate sampling. **(GATE: 1997)**

Sol: The transmission of 24 multiplexed samples with on synchronization sample is shown in Fig. 9.13.

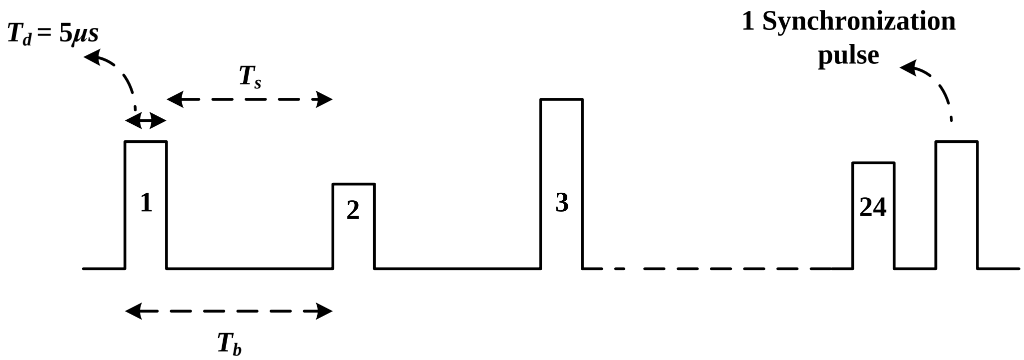


Fig. 9.13 Figure of problem SE9.3

Here, T_b = duration between leading edge of sample,

T_s = spacing between each sample,

T_d = duration of sample = 1μs,

(i) Sampling rate = 8 kHz = 8000 samples/sec

Therefore, the time slot of each sample is

$$\frac{1}{8000} = 125\ \mu s$$

The total number of samples taken in one rotation is

24 (one from each channel) + 1 (synchronization sample) = 25 samples/rotation

Hence, the duration between the leading edge of the sample is

$$T_s = \frac{125}{25} = 5\ \mu s$$

Since the duration of each pulse is 1μs, so, spacing between each sample is

$$T_d = T_s - T_p = 5 - 1 = 4 \mu s$$

(ii) The highest frequency component is 3.4 kHz. Therefore, the Nyquist rate is

$$f_{nyquist} = 2 f_m = 2 \times 3.4 = 6.8\ \text{kHz}$$

Therefore, the time slot of each sample is

$$\frac{1}{6800} = 147\ \mu\text{s}$$

Hence, the duration between the leading edge of the sample is

$$T_s = \frac{147}{25} = 5.88\ \mu\text{s}$$

Since duration of each pulse is 1μs, so, spacing between each sample is

$$T_d = T_s - T_p = 5.88 - 1 = 4.88\ \mu s$$

SE9.4 Two analog message signals $m_1(t)$ and $m_2(t)$ are transmitted over a common communication medium by TDM. If the highest frequency component of the $m_1(t)$ is 5 kHz and that of $m_2(t)$ is 5.5 kHz. What will be the minimum value of the permissible sampling rate?

Sol: the highest frequency component of the multiplexed signals $m_1(t)$ and $m_2(t)$ is 5.5 kHz. Therefore, the minimum value of the permissible sampling rate is

$$(f_s)_{\min} = 2 f_m = 2 \times 5.5 = 11\ \text{kHz}$$

SE9.5 Find the efficiency and redundancy for the probabilities 0.30, 0.25, 0.05, 0.08, 0.20, 0.12 using Shannon-Fano and Huffman coding.

Sol: Let the symbols are m_1, m_2, m_3, m_4, m_5 and m_6. Arrange the symbols with ascending order probabilities.

m_1	0.30
m_2	0.25
m_3	0.20
m_4	0.12
m_5	0.08
m_6	0.05

The entropy $H(m)$ of the source is given by

$$H(m) = -\sum_{i=1}^{6} P_i \log_2 P_i = 2.3601\ \text{bits}$$

Shannon-Fano Coding

Step 1: Make two groups with almost equal probabilities

Group 1: $\left.\begin{array}{l} m_1 = 0.30 \\ m_2 m_3 m_4 m_5 m_6 = 0.25 + 0.20 + 0.12 + 0.08 + 0.05 = 0.70 \end{array}\right\rangle$ difference = 0.40

Group 2: $\left.\begin{aligned} m_1m_2 &= 0.30+0.25=0.55 \\ m_3m_4m_5m_6 &= 0.20+0.12+0.08+0.05=0.45 \end{aligned}\right\rangle$ difference = 0.10

Group 3: $\left.\begin{aligned} m_1m_2m_3 &= 0.30+0.25+0.20=0.75 \\ m_4m_5m_6 &= 0.12+0.08+0.05=0.25 \end{aligned}\right\rangle$ difference = 0.50

Group 2 is most likely equally divided group. Therefore, the messages are arranged into two groups and assign '0' to the upper group and '1' to the lower group as given below

m_1	0.30	0
m_2	0.25	0
m_3	0.20	1
m_4	0.12	1
m_5	0.08	1
m_6	0.05	1

Step 2: Repeat step 1 until all the individual messages are assigned either '0' or '1'. So,

Messages	Probabilities	Step 1	Step 2	Step 3	Step 4	Code word
m_1	0.30	0	0			**00**
m_2	0.25	0	1			**01**
m_3	0.20	1	0			**10**
m_4	0.12	1	1	0		**110**
m_5	0.08	1	1	1	0	**1110**
m_6	0.05	1	1	1	1	**1111**

Average word length $L=\sum_{i=1}^{N} P_i(n)$

So, $L=\sum_{i=1}^{6} P_i(n) = 0.3\times 2+0.25\times 2+0.20\times 2+0.12\times 3+0.08\times 4+0.05\times 4 = 2.38$ binary digits

Efficiency, $\eta = \dfrac{H(m)}{L} = \dfrac{2.3601}{2.38} = 0.9916$

Redundancy, $\gamma = 1-\eta = 1-0.9916 = 0.0084$

Huffman Coding:

Symbol	**Prob./ Code**	**(Step 1)**	**(Step 2)**	**(Step 3)**	**(Step 4)**
***m*₁**	0.3 **00**	0.3 **00**	0.3 **00**	0.45 **1**	0.55 **0**
***m*₂**	0.25 **10**	0.25 **01**	0.25 **01**	0.3 **00**	0.45 **1**
***m*₃**	0.20 **11**	0.20 **11**	0.25 **10**	0.25 **01**	
***m*₄**	0.12 **101**	0.13 **100**	0.20 **11**		
***m*₅**	0.08 **1000**	0.12 **101**			
***m*₆**	0.05 **1001**				

Average word length $L = \sum_{i=1}^{N} P_i(n)$

So, $L = \sum_{i=1}^{6} P_i(n) = 0.3\times 2 + 0.25\times 2 + 0.20\times 2 + 0.12\times 3 + 0.08\times 4 + 0.05\times 4 = 2.38$ binary digits

The entropy $H(m)$ of the source is given by

$$H(m) = -\sum_{i=1}^{6} P_i \log_2 P_i = 2.3601 \text{ bits}$$

Efficiency, $\eta = \frac{H(m)}{L} = \frac{2.3601}{2.38} = 0.9916$

Redundancy, $\gamma = 1-\eta = 1-0.9916 = 0.0084$

SE9.6 Find the Shannon-Fano codes for the symbols with the probabilities 0.22, 0.28, 0.15, 0.30 and 0.05, respectively.

Sol: Arrange the symbols with ascending order probabilities.

Messages	Probabilities	Step 1	Step 2	Step 3
m_1	0.30	0	0	
m_2	0.28	0	1	
m_3	0.22	1	0	
m_4	0.15	1	1	0
m_5	0.05	1	1	1

Messages	Codeword
m_1	00
m_2	01
m_3	10
m_4	110
m_5	111

SE9.7 A source emits one of four messages randomly every 1μsec. The probabilities of these messages are 0.5, 0.3, 0.1 and 0.1. Messages are generated independently. What is the entropy of the source? Obtain a compact binary code and determine the average length of the codeword, the efficiency and the redundancy of the code.

Sol: The entropy $H(m)$ of the source is given by

$$H(m) = -\sum_{i=1}^{4} P_i \log_2 P_i = -\left(0.5\log_2 0.5 + 0.3\log_2 0.3 + 0.1\log_2 0.1 + 0.1\log_2 0.1\right) = 1.6855 \text{ bits}$$

Symbol	Prob./ Code	(Step 1)	(Step 2)
m_1	0.5 **0**	0.5 **0**	0.5 **0**
m_2	0.3 **10**	0.3 **10**	0.5 **1**
m_3	0.1 **110**	0.2 **11**	
m_4	0.1 **111**		

Average word length $L = \sum_{i=1}^{N} P_i(n)$

So, $L=\sum_{i=1}^{4} P_i(n)=0.5\times1+0.3\times2+0.1\times3+0.1\times3=1.7$ binary digits

Efficiency, $\eta=\dfrac{H(m)}{L}=\dfrac{1.6855}{1.7}=0.9915=99.15\%$

Redundancy, $\gamma=1-\eta=1-0.9915=0.0085=0.85\%$

SE9.8 Repeat the above problem for compact ternary code.

Sol: It is known that for *r*-ary code, the reduced code must be r-messages left. It is possible only if the original message has $r+r(k-1)$. If the original message has less than this, then add some dummy message with zero probability to satisfy the above condition. For the 3-ary (r = 3) code, the required number of messages is

$$r+k(r-1)=3+1(3-1)=5 \quad for \quad k=1$$

In the above-mentioned problem, there are four messages. So, one dummy message is added with zero probability. The assigned code to each of the messages would be a combination of 0, 1,….,(r -1) numbers. For 3-ary code, the assigned code for each message would be a combination of 0, 1, 2.

Therefore, the 3-ary code is derived as

Symbol	Prob./ Code		(Step 1)	
m_1	0.5	**0**	0.5	**0**
m_2	0.3	**1**	0.3	**1**
m_3	0.1	**20**	0.2	**2**
m_4	0.1	**21**		
m_5	0.0	**22**		

Average word length $L=0.5\times1+0.3\times1+0.1\times2+0.1\times2=1.2$ bits

The entropy,

$$H(m)=-\sum_{i=1}^{4} P_i\log_3 P_i=-(0.5\log_3 0.5+0.3\log_3 0.3+0.1\log_3 0.1+0.1\log_3 0.1)=1.0634 \text{ bits}$$

Efficiency, $\eta=\dfrac{H(m)}{L}=\dfrac{1.0634}{1.2}=0.8862=88.62\%$

Redundancy, $\gamma=1-0.8862=0.1138=11.38\%$

SE9.9 Determine the Shannon Fano code for the following message with their probabilities given:

Symbol	P_0	P_1	P_2	P_3	P_4	P_5
Probability	0.30	0.10	0.02	0.15	0.40	0.03

Also, determine the entropy, average code length and efficiency.

Sol: Arrange the symbols with ascending order probabilities.

Messages	Probabilities	Step 1	Step 2	Step 3	Step 4	Step 5	Code
P_4	0.40	0					0
P_0	0.30	1	0				10
P_3	0.15	1	1	0			110
P_1	0.10	1	1	1	0		1110
P_5	0.03	1	1	1	1	0	11110
P_2	0.02	1	1	1	1	1	11111

The entropy,

$$H(m) = -\sum_{i=1}^{6} P_i \log_2 P_i = -\begin{pmatrix} 0.4\log_2 0.4 + 0.3\log_2 0.3 + 0.15\log_2 0.15 + 0.1\log_2 0.1 \\ +0.03\log_2 0.03 + 0.02\log_2 0.02 \end{pmatrix} = 2.0572 \text{ bits}$$

Average word length $L = 0.4\times1 + 0.3\times2 + 0.15\times3 + 0.1\times4 + 0.03\times5 + .02\times5 = 2.1$ bits

Efficiency, $\eta = \frac{H(m)}{L} = \frac{2.0572}{2.1} = 0.9796 = 97.96\%$

SE9.10 A discrete memoryless source emits seven messages with probabilities 1/2, 1/4, 1/8, 1/16, 1/32, 1/64 and 1/64, respectively. Find the entropy of the source. Obtain the compact binary code and find the average length of the code. Determine the efficiency and redundancy of the code.

Sol: The compact binary code is Huffman coding. Therefore, the messages are arranged in ascending order of probabilities

Symbol	Prob./ Code	(Step 1)	(Step 2)	(Step 3)	(Step 4)	(Step 5)
m_1	1/2 **0**	1/2 **0**	1/2 **0**	1/2 **0**	1/2 **0**	1/2 **0**
m_2	1/4 **10**	1/4 **10**	1/4 **10**	1/4 **10**	1/4 **10**	1/2 **1**
m_3	1/8 **110**	1/8 **110**	1/8 **110**	1/8 **110**	1/4 **11**	
m_4	1/16 **1110**	1/16 **1110**	1/16 **1110**	1/8 **111**		
m_5	1/32 **11110**	1/32 **11110**	1/16 **1111**			
m_6	1/64 **111110**	1/32 **11111**				
m_7	1/64 **111111**					

Average word length $L = \sum_{i=1}^{N} P_i(n)$

So,

$$L = \sum_{i=1}^{7} P_i(n) = \frac{1}{2}\times1 + \frac{1}{4}\times2 + \frac{1}{8}\times3 + \frac{1}{16}\times4 + \frac{1}{32}\times5 + \frac{1}{64}\times6 + \frac{1}{64}\times6 = \frac{63}{32} = 1.96875 \text{ binary digits}$$

The entropy $H(m)$ of the source is given by

$$H(m) = -\sum_{i=1}^{7} P_i \log_2 P_i = 1.96875 \text{ bits}$$

Efficiency, $\eta = \dfrac{H(m)}{L} = \dfrac{1.96875}{1.96875} = 1 = 100\%$

Redundancy, $\gamma = 1 - \eta = 1 - 1 = 0$

SE9.11 Repeat the above problem for 3-ary code.

Sol: For 3-ary (r = 3) code, the required number of messages is

$$\begin{aligned} &for \quad k=1 \quad r + k(r-1) = 3 + 1(3-1) = 5 \\ &for \quad k=2 \quad r + k(r-1) = 3 + 2(3-1) = 7 \end{aligned}$$

In the above-mentioned problem, there are seven messages. So, no dummy message is required for 3-ary code.

Symbol	Prob./ Code		(Step 1)		(Step 2)	
***m*1**	1/2	**0**	1/2	**0**	1/2	**0**
***m*2**	1/4	**1**	1/4	**1**	1/4	**1**
***m*3**	1/8	**20**	1/8	**20**	1/4	**2**
***m*4**	1/16	**21**	1/16	**21**		
***m*5**	1/32	**220**	1/16	**22**		
***m*6**	1/64	**221**				
***m*7**	1/64	**222**				

Average word length

$$L = \sum_{i=1}^{7} P_i(n) = \frac{1}{2}\times 1 + \frac{1}{4}\times 1 + \frac{1}{8}\times 2 + \frac{1}{16}\times 2 + \frac{1}{32}\times 3 + \frac{1}{64}\times 3 + \frac{1}{64}\times 3 = \frac{42}{32} = 1.3125 \text{ 3-ary digits}$$

The entropy $H(m) = -\sum_{i=1}^{7} P_i \log_3 P_i = 1.242$ 3-ary units

Efficiency, $\eta = \dfrac{H(m)}{L} = \dfrac{1.242}{1.3125} = 0.9463 = 94.63\%$

Redundancy, $\gamma = 1 - \eta = 1 - 0.9463 = 0.0537 = 5.37\%$

SE9.12 What is the Shannon-Hartley theoretical capacity for a signal with a frequency bandwidth of 2 kHz and an SNR = 300?

Sol: The channel capacity is given by

$$C = B * \log_2(1 + \text{SNR})$$

So, $C = 2\times 10^3 \times \log_2(1+300) = 10^3 \times 16.467 = 16467$ bps

SE9.13 Find the channel capacity of the binary symmetric channel (BSC) shown in Fig. 9.14 given below:

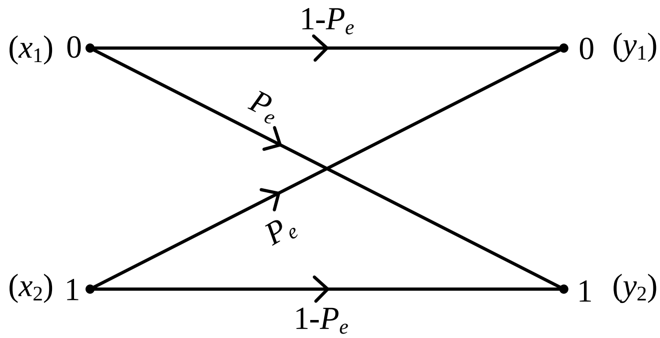


Fig. 9.14 Figure of problem SE9.13.

Sol: The maximum value of mutual information $I(x;y)$ is channel capacity C_s,

$$C_s = \max_{P(x_i)} I(x;y) \qquad \text{bits/symbol}$$

Let the probability of message transmission is

$$P(x_1) = a \qquad \& \qquad P(x_2) = (1-a)$$

The probability of error P_e for $P(y_j \mid x_i)$ is given by

$$P(y_1 \mid x_2) = P(y_2 \mid x_1) = P_e$$
$$P(y_1 \mid x_1) = P(y_2 \mid x_2) = P_e^{'} = (1-P_e)$$

The mutual information $I(x;y)$ is given by

$$I(x;y) = \sum_i \sum_j P(x_i) P(y_j|x_i) \log\left[\frac{P(y_j|x_i)}{\sum_i P(x_i) P(y_j|x_i)}\right]$$

In BSC, the transmitted and received bits are either '1' or '0'. Therefore, $i = j = 2$. So,

$$I(x;y) = \sum_{i=1}^{2}\sum_{j=1}^{2} P(x_i) P(y_j|x_i) \log\left[\frac{P(y_j|x_i)}{\sum_i P(x_i) P(y_j|x_i)}\right]$$

$$= \sum_{i=1}^{2} P(x_i) P(y_1|x_i) \log\left[\frac{P(y_1|x_i)}{\sum_i P(x_i) P(y_1|x_i)}\right] + \sum_{i=1}^{2} P(x_i) P(y_2|x_i) \log\left[\frac{P(y_2|x_i)}{\sum_i P(x_i) P(y_2|x_i)}\right]$$

$$= P(x_1) P(y_1|x_1) \log\left[\frac{P(y_1|x_1)}{P(x_1)P(y_1|x_1) + P(x_2)P(y_1|x_2)}\right] + P(x_2) P(y_1|x_2) \log\left[\frac{P(y_1|x_2)}{P(x_1)P(y_1|x_1) + P(x_2)P(y_1|x_2)}\right]$$
$$+ P(x_1) P(y_2|x_1) \log\left[\frac{P(y_2|x_1)}{P(x_1)P(y_2|x_1) + P(x_2)P(y_2|x_2)}\right] + P(x_2) P(y_2|x_2) \log\left[\frac{P(y_2|x_2)}{P(x_1)P(y_2|x_1) + P(x_2)P(y_2|x_2)}\right]$$

Substitute the values

$$I(x;y) = aP_e^{'} \log\left[\frac{P_e^{'}}{aP_e^{'} + (1-a)P_e}\right] + (1-a)P_e \log\left[\frac{P_e}{aP_e^{'} + (1-a)P_e}\right] + aP_e \log\left[\frac{P_e}{aP_e + (1-a)P_e^{'}}\right]$$
$$+ (1-a)P_e^{'} \log\left[\frac{P_e^{'}}{aP_e + (1-a)P_e^{'}}\right]$$

$$I(x;y) = \left(aP_e' + (1-a)P_e\right)\log\left[\frac{1}{aP_e' + (1-a)P_e}\right] + \left(aP_e'\log P_e' + (1-a)P_e\log P_e\right)$$

$$+\left(aP_e + (1-a)P_e'\right)\log\left[\frac{1}{aP_e + (1-a)P_e'}\right] + \left(aP_e\log P_e + (1-a)P_e'\log P_e'\right)$$

$$= \left(aP_e' + (1-a)P_e\right)\log\left[\frac{1}{aP_e' + (1-a)P_e}\right] + \left(aP_e + (1-a)P_e'\right)\log\left[\frac{1}{aP_e + (1-a)P_e'}\right] + \left(P_e'\log P_e' + P_e\log P_e\right)$$

The above equation is maximum when

$$a = 0.5$$

So, $C_s = \max_{P(x_i)} I(x;y) = 1 - \left[P_e\log\frac{1}{P_e} + (1-P_e)\log\left(\frac{1}{1-P_e}\right)\right]$

SE9.14 Find the expression for the channel capacity of infinite bandwidth.

Sol: The channel capacity is given by

$$C = B\log_2\left(1+\frac{S}{N}\right)$$

It seems that the channel capacity C becomes infinity (∞) as bandwidth goes to infinity (∞). But, this is not true as the noise power is N=NB. So, as B increases, noise power N also increases. Therefore, channel capacity approaches toward a limit. So,

$$\lim_{B\to\infty} C = \lim_{B\to\infty} B\log_2\left(1+\frac{S}{NB}\right)$$

$$= \lim_{B\to\infty}\frac{S}{N}\left[\frac{NB}{S}\log_2\left(1+\frac{S}{NB}\right)\right]$$

Since, $\lim_{x\to\infty} x\log_2\left(1+\frac{1}{x}\right) = \log_2 e = 1.44$

Hence, $\lim_{B\to\infty} C = 1.44\frac{S}{N}$ bits/sec

SE9.15 Consider the binary symmetric channel (BSC) shown in Fig. 9.13 given in problem SE9.13, if the probability of transmission of digit ‘0’ is $P(x_0) = 0.25$ and probability of error $P_e = 0.1$. Calculate all probabilities of $P(x_i, y_j)$ and $P(x_i \mid y_j)$. Also, calculate the value of mutual information $I(x, y)$.

Sol: In BSC, transmitted and received message bits are ‘1’ and ‘0’. Therefore, there are four combinations of $P(x_i, y_j)$ and $P(x_i \mid y_j)$ which are

$$P(x_0, y_0), P(x_0, y_1), P(x_1, y_0) \text{ and } P(x_1, y_1)$$

$$P(x_0 \mid y_0), P(x_0 \mid y_1), P(x_1 \mid y_0) \text{ and } P(x_1 \mid y_1)$$

In the problem, given probabilities are

$$P(x_0) = 0.25\text{ , So, } P(x_1) = 1 - P(x_0) = 1 - 0.25 = 0.75$$

$$P(\mathrm{y}_1 \mid x_0) = P(\mathrm{y}_0 \mid x_1) = P_e = 0.1\text{, So, } P(\mathrm{y}_0 \mid x_0) = P(\mathrm{y}_1 \mid x_1) = 1 - P_e = 1 - 0.1 = 0.9$$

Hence, joint probabilities are given as

$$P(x_0, y_0) = P(x_0)P(y_0 \mid \mathrm{x}_0) = 0.25 \times 0.9 = 0.225$$

$$P(x_0, y_1) = P(x_0)P(y_1 \mid \mathrm{x}_0) = 0.25 \times 0.1 = 0.025$$

$$P(x_1, y_0) = P(x_1)P(y_0 \mid \mathrm{x}_1) = 0.75 \times 0.1 = 0.075$$

$$P(x_1, y_1) = P(x_1)P(y_1 \mid \mathrm{x}_1) = 0.75 \times 0.9 = 0.675$$

The conditional probabilities of x transmitted when y is received are given by

$$P(x_0 \mid y_0) = \frac{P(x_0)P(y_0 \mid \mathrm{x}_0)}{P(y_0)} = \frac{P(x_0)P(y_0 \mid \mathrm{x}_0)}{\sum_i P(x_i)P(y_0 \mid \mathrm{x}_i)} = \frac{P(x_0)P(y_0 \mid \mathrm{x}_0)}{P(x_0)P(y_0 \mid \mathrm{x}_0) + P(x_1)P(y_0 \mid \mathrm{x}_1)}$$

$$= \frac{0.25 \times 0.9}{0.25 \times 0.9 + 0.75 \times 0.1} = \frac{0.225}{0.3} = 0.75$$

$$P(x_0 \mid y_1) = \frac{P(x_0)P(y_1 \mid \mathrm{x}_0)}{P(x_0)P(y_1 \mid \mathrm{x}_0) + P(x_1)P(y_1 \mid \mathrm{x}_1)} = \frac{0.25 \times 0.1}{0.25 \times 0.1 + 0.75 \times 0.9} = \frac{0.025}{0.6975} = 0.0358$$

$$P(x_1 \mid y_0) = \frac{P(x_1)P(y_0 \mid \mathrm{x}_1)}{P(x_0)P(y_0 \mid \mathrm{x}_0) + P(x_1)P(y_0 \mid \mathrm{x}_1)} = \frac{0.75 \times 0.1}{0.25 \times 0.9 + 0.75 \times 0.1} = \frac{0.075}{0.3} = 0.25$$

$$P(x_1 \mid y_1) = \frac{P(x_1)P(y_1 \mid \mathrm{x}_1)}{P(x_0)P(y_1 \mid \mathrm{x}_0) + P(x_1)P(y_1 \mid \mathrm{x}_1)} = \frac{0.75 \times 0.9}{0.25 \times 0.1 + 0.75 \times 0.9} = \frac{0.675}{0.7} = 0.9643$$

Mutual information is given by

$$I(x; y) = \sum_i \sum_j P(x_i, y_j) \log_2 \left[\frac{P(y_j | x_i)}{P(y_j)} \right]$$

$$= P(x_0, \mathrm{y}_0) \log_2 \left[\frac{P(y_0 | x_0)}{P(y_0)} \right] + P(x_0, \mathrm{y}_1) \log_2 \left[\frac{P(y_1 | x_0)}{P(y_1)} \right] + P(x_1, \mathrm{y}_0) \log_2 \left[\frac{P(y_0 | x_1)}{P(y_0)} \right]$$

$$+ P(x_1, \mathrm{y}_1) \log_2 \left[\frac{P(y_1 | x_1)}{P(y_1)} \right]$$

$$= 0.225 \log_2 \frac{0.9}{0.3} + 0.025 \log_2 \frac{0.1}{0.7} + 0.075 \log_2 \frac{0.1}{0.3} + 0.675 \log_2 \frac{0.9}{0.7}$$

$$= 0.3566 - 0.0702 - 0.1189 + 0.2447 = 0.4122 \text{ bits/symbol}$$

PROBLEMS

P9.1 Explain the principle of the time-division multiplexing technique.

P9.2 What is the significance of the sampling theorem in the TDM system?

P9.3 What is multiplexing? Why is it needed? Compare two basic forms of multiplexing.

P9.4 Explain the T1 digital carrier system (OR) PAM/TDM system.

P9.5 Explain the working of the PCM/TDM hierarchy system from T1 to T4.

P9.6 Discuss the concept of synchronization and signaling in T1, TDM, and PCM hierarchy.

P9.7 Write short notes on different types of Synchronization techniques.

P9.8 Define information, entropy and information rate.

P9.9 Write the properties of mutual information and entropy?

P9.10 State and prove the channel capacity theorem/ Shannon-Hartley Law. Calculate the capacity of a Gaussian channel with a bandwidth of 1MHz and S/N ratio of 20dB?

P9.11 Explain the channel capacity of a discrete memoryless channel.

P9.12 Write a short note on Shannon-Fano coding.

P9.13 Write a short note on Huffman coding.

P9.14 Write short notes binary symmetric channel (BSC).

NUMERICAL PROBLEMS

P9.15 Calculate the capacity of a Gaussian channel with a bandwidth of 1MHz and S/N ratio of 20dB?

P9.16 Twelve different message signals, each of bandwidth 10 kHz, are to be multiplexed and transmitted. Determine the minimum bandwidth required for PAM/TDM system.

P9.17 Determine the Huffman code for the following message with their probabilities given:

Symbol	P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Probability	0.1	0.25	0.15	0.05	0.15	0.1	0.05	0.15

P9.18 A memoryless source emits six messages with probabilities 0.3, 0.25, 0.15, 0.12, 0.1 and 0.08. Find the 4-ary Huffman code. Determine its average word length, efficiency, and redundancy.

P9.19 Twenty-four voice signals are sampled uniformly and then time division multiplexed. The highest frequency component for each voice signal is 3.4 kHz.

(i) If the signals are pulse amplitude modulated using Nyquist rate sampling, what is the minimum channel bandwidth?

(ii) If the signals are pulse code modulated with an 8-bit encoder, what is the sampling rate? The bit rate of the system is 1.5 x 10^6 bits/sec.

P9.20 Find the efficiency and redundancy for the probabilities 0.30, 0.25, 0.05, 0.08, 0.20, 0.12 using Shannon-Fano and Huffman coding.

P9.21 A discrete memoryless source emits seven messages with probabilities 1/2, 1/4, 1/8, 1/16, 1/32, 1/64 and 1/64, respectively. Find the entropy of the source. Obtain the compact binary code and find the average length, the efficiency and redundancy of the code.

P9.22 Design a binary Huffman code for a discrete source having seven independent symbols having probabilities 0.25, 0.25, 0.125, 0.125, 0.125, 0.0625 and 0.0625, respectively. Also, calculate the efficiency of this code.

P9.23 Determine $H(\mathrm{x})$, $H(\mathrm{x|y})$, $H(\mathrm{y})$ and $I(x;y)$ for the ternary channel shown in Fig. 9.15.

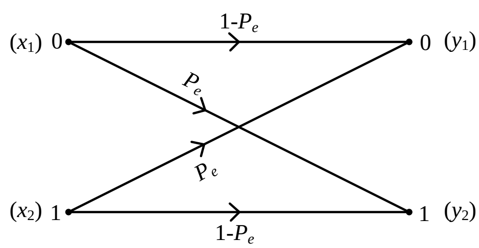


Fig. 9.15 Figure of problem P9.25.

P9.24 A message source generates one of four messages randomly every microsecond. The probabilities of these messages are 0.4, 0.3, 0.2 and 0.1. Each emitted message is independent of the other message in the sequence. What is the source entropy? What is the rate of information generated by these sources (in bits per second)

P9.25 Let *X* and *Y* represent random variables with associated probability distributions $p(x)$ and $p(y)$, respectively. They are not independent. Their conditional probability distributions are $p(x|y)$ and $p(y|x)$, and their joint probability distribution is $p(x,y)$

(i) What is the marginal entropy $H(X)$ of variable *X*, and what is the mutual information of *X* with itself?

(ii) In terms of the probability distributions, what are the conditional entropies $H(X|Y)$ and $H(Y|X)$?

(iii) What is the joint entropy $H(X,Y)$, and what would it be if the random variables *X* and *Y* were independent?

MULTIPLE-CHOICE QUESTIONS

MCQ9.1 …………is analog multiplexing whereas………..digital multiplexing

(a) TDM, FDM
(b) FDM, TDM
(c) Synchronous TDM, Asynchronous TDM
(d) Asynchronous TDM, Synchronous TDM

MCQ9.2 If there are 4 of the ten devices have nothing to send in synchronous TDM, the efficiency is

(a) 40% (c) 100%
(b) 60% (d) 0%

MCQ9.3 The minimum required bandwidth for TDM to transmit N signals each band limited to f Hz is

(a) f (c) f/N
(b) $2f$ (d) Nf

MCQ9.4 A source generates six messages. The entropy of the source will be maximum

(a) Three messages with probabilities 1 and other three messages with 0 probability
(b) One of the probabilities is 0 and the remaining are 1
(c) One of the probabilities is one and the remaining are 0
(d) All probabilities are equal

MCQ9.5 A communication channel with AWGN has a BW of 5 kHz and an SNR of 31. Its channel capacity is

(a) 2.5 kbps (c) 25 kbps
(b) 5 kbps (d) 0.5 kbps

MCQ9.6 A channel has a BW of 4 kHz and SNR of 15. For the same channel capacity, if the SNR is increased to 255, then the new channel bandwidth would be

(a) 2 kHz (c) 8 kHz
(b) 4 kHz (d) 16 kHz

MCQ9.7 The unit of average mutual information is

(a) Bytes (c) Bytes/symbol
(b) Bits (d) Bits/symbol

MCQ9.8 When A and B are statistically independent, then I(A, B)

(a) 0 (c) $\ln(2)$
(b) 1 (d) Cannot be determined

MCQ ANSWERS

MCQ9.1	(b)	MCQ9.5	(c)
MCQ9.2	(b)	MCQ9.6	(a)
MCQ9.3	(d)	MCQ9.7	(b)
MCQ9.4	(d)	MCQ9.8	(a)